Exercise walk-through

6.1 Inequalities with the modulus sign

Pearson Edexcel · International A-Level · Pure Mathematics

Exercises 2.1·2.3

2.1 Solve $|x-3|<5$. [2]

2.2 Solve $|2x+1|\geq 7$. [2]

2.3 Solve $|x-1|>2x$. [3]

Exercises 2.1·2.3

2.1 Solve $|x-3|<5$. [2]

2.1 $-5<x-3<5$, so $-2<x<8$.

2.2 Solve $|2x+1|\geq 7$. [2]

2.2 $2x+1\geq 7$ or $2x+1\leq -7$: $x\geq 3$ or $x\leq -4$.

2.3 Solve $|x-1|>2x$. [3]

2.3 Where $x\geq 1$: $x-1>2x$ gives $x<-1$, which contradicts $x\geq 1$ – no solutions. Where $x<1$: $1-x>2x$ gives $x<\frac{1}{3}$. Solution: $x<\frac{1}{3}$.

Exercises 2.4·3.1

2.4 Solve $|x^2 - 4| \leq 3x$. [4]

3.1 *(Original.)* Use algebra to solve $|x + 2| > |2x - 1|$. [4]

Exercises 2.4·3.1

2.4 Solve $|x^2 - 4| \leq 3x$. [4]

2.4 Where $x^2 \geq 4$ ($x \leq -2$ or $x \geq 2$): $x^2 - 3x - 4 \leq 0$, so $-1 \leq x \leq 4$; keeping the case gives $2 \leq x \leq 4$. Where $x^2 < 4$ ($-2 < x < 2$): $x^2 + 3x - 4 \geq 0$, so $x \leq -4$ or $x \geq 1$; keeping the case gives $1 \leq x < 2$. Union: $1 \leq x \leq 4$. (Check $x = 4$: $|12| = 12 = 3(4)$ ✓; $x = 0.5$: $|-3.75| > 1.5$ ×, outside.)

3.1 *(Original.)* Use algebra to solve $|x + 2| > |2x - 1|$. [4]

3.1 Squaring: $(x + 2)^2 > (2x - 1)^2$, so $x^2 + 4x + 4 > 4x^2 - 4x + 1$, i.e. $3x^2 - 8x - 3 < 0$. Factorising $(3x + 1)(x - 3) < 0$ gives $-\frac{1}{3} < x < 3$.

Exercise 3.2

3.2 *(Edexcel IAL WFM02 June 2025 Q8.)* The curve C has equation $y = \dfrac{15x}{|x|+4}$ and the line l has equation $y = x - 2$. (a) Use algebra to determine the exact values of x for which $x - 2 > \dfrac{15x}{|x|+4}$. (b) Hence use algebra to determine the exact values of x for which $|x-2| > \left|\dfrac{15x}{|x|+4}\right|$. [10]

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3.2 *(Edexcel IAL WFM02 June 2025 Q8.)* The curve C has equation $y = \dfrac{15x}{|x|+4}$ and the line l has equation $y = x - 2$. (a) Use algebra to determine the exact values of x for which $x - 2 > \dfrac{15x}{|x|+4}$. (b) Hence use algebra to determine the exact values of x for which $|x-2| > \left|\dfrac{15x}{|x|+4}\right|$. [10]

3.2 (a) The denominator $|x| + 4 > 0$ always. For $x \geq 0$: $(x-2)(x+4) > 15x \Rightarrow x^2 - 13x - 8 > 0$, roots $\frac{13 \pm \sqrt{201}}{2}$, so $x > \frac{13+\sqrt{201}}{2}$ (the other root is negative). For $x < 0$: $(x-2)(4-x) > 15x \Rightarrow -x^2 - 9x - 8 > 0 \Rightarrow (x+1)(x+8) < 0$, so $-8 < x < -1$. Solution: $-8 < x < -1$ or $x > \frac{13+\sqrt{201}}{2}$. (b) Both sides non-negative, so squaring preserves the inequality: $(x-2)^2(|x|+4)^2 > 225x^2$ factorises as $\big[(x-2)(|x|+4) - 15x\big]\big[(x-2)(|x|+4) + 15x\big] > 0$, i.e. the union of part (a) with $(x-2)(|x|+4) < -15x$. New branch, $x \geq 0$: $x^2 + 17x - 8 < 0$ gives $0 \leq x < \frac{\sqrt{321}-17}{2}$. New branch, $x < 0$: $-x^2 + 21x - 8 < 0 \Rightarrow x^2 - 21x + 8 > 0$, whose roots $\frac{21 \pm \sqrt{409}}{2}$ are both positive, so every $x < 0$ qualifies. Solution: $x < \frac{\sqrt{321}-17}{2}$ or $x > \frac{13+\sqrt{201}}{2}$.

Local review guidance: cases named with their defining condition; valid multiplication/squaring step with sign justified; exact surd roots carried through; “hence” reuses part (a) rather than restarting. This is authored classroom guidance.