

Exercise walk-through

5.6

Matrices, transformations, series and induction

Pearson Edexcel · International A-Level · Pure Mathematics

Exercises 2.1·2.3

2.1 For $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, find $\mathbf{AB}$, $\det \mathbf{A}$ and $\mathbf{A}^{-1}$. [4]

2.2 Describe fully the single geometrical transformation represented by (a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; (b) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$; (c) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$; (d) $\begin{pmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{pmatrix}$. [4]

2.3 A triangle of area 2 is transformed by $\mathbf{M}$ with $\det \mathbf{M} = -7$. Find the area of the image and state what the sign tells you. [2]

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2.1 $\mathbf{AB} = \begin{pmatrix} 1 & -3 \\ 4 & -2 \end{pmatrix}$; $\det \mathbf{A} = 10$; $\mathbf{A}^{-1} = \frac{1}{10} \begin{pmatrix} 4 & -1 \\ -2 & 3 \end{pmatrix}$.

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2.2 (a) Reflection in $y = x$; (b) enlargement, scale factor 3, centre O ; (c) stretch, factor 2, parallel to the x -axis; (d) rotation 30° anticlockwise about O .

2.3 A triangle of area 2 is transformed by $\mathbf{M}$ with $\det \mathbf{M} = -7$. Find the area of the image and state what the sign tells you. [2]

2.3 Image area $= 2 \times 7 = 14$; the negative determinant means the transformation reverses orientation (includes a reflection).

Exercises 2.4·2.5

2.4 Use standard formulae to show that $\sum_{r=1}^n (r+1)^2 = \frac{n}{6}(2n^2 + 9n + 13)$. [4]

2.5 Prove by induction that $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$ for all positive integers n . [3]

Exercises 2.4·2.5

2.4 Use standard formulae to show that $\sum_{r=1}^n (r+1)^2 = \frac{n}{6}(2n^2 + 9n + 13)$. [4]

$$\begin{aligned} \text{2.4 } \sum (r^2 + 2r + 1) &= \frac{n(n+1)(2n+1)}{6} + n(n+1) + n = \frac{n(n+1)(2n+1) + 6n(n+1) + 6n}{6} = \frac{n(2n^2 + 3n + 1 + 6n + 6 + 6)}{6} \\ &= \frac{n(2n^2 + 9n + 13)}{6} \quad \checkmark \quad (\text{check } n = 1: 4 \text{ and } \frac{24}{6} = 4). \end{aligned}$$

2.5 Prove by induction that $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$ for all positive integers n . [3]

2.5 $n = 1$: both sides $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \checkmark$. If true for k : $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & k+1 \\ 0 & 1 \end{pmatrix} \checkmark$. True for $n = 1$ and $n = k \Rightarrow n = k + 1$; by induction, true for all positive integers.

Exercise 3.1

3.1 (*Adapted from Edexcel IAL WFM01 June 2025 Q1.*) $\mathbf{M} = \begin{pmatrix} 1 & a \\ 2 & -3 \end{pmatrix}$, $a \neq -\frac{3}{2}$. (a)

Determine $\mathbf{M}^{-1}$ in terms of a . (b) Given that $\mathbf{M}^{-1} = 2\mathbf{M} + 4\mathbf{I}$, determine the value of a . [6]

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3.1 (a) $\det \mathbf{M} = -3 - 2a$, so $\mathbf{M}^{-1} = \frac{1}{-(3+2a)} \begin{pmatrix} -3 & -a \\ -2 & 1 \end{pmatrix}$. (b) $2\mathbf{M} + 4\mathbf{I} = \begin{pmatrix} 6 & 2a \\ 4 & -2 \end{pmatrix}$.

Entry (2, 2): $\frac{1}{-(3+2a)} = -2 \Rightarrow 3 + 2a = \frac{1}{2} \Rightarrow a = -\frac{5}{4}$. Every other entry confirms it:

(2, 1): $\frac{-2}{-1/2} = 4 \checkmark$; (1, 1): $\frac{-3}{-1/2} = 6 \checkmark$; (1, 2): $\frac{-a}{-1/2} = 2a \checkmark$ (identically true).

Exercise 3.2

3.2 (*Adapted from Edexcel IAL WFM01 June 2025 Q8.*) (a) Using standard summation formulae, show that $\sum_{r=1}^{2n} (2r^2 - 1) = \frac{4}{3}n(n+1)(4n-1)$. (b) Prove by induction that $\sum_{r=1}^n r(3r-2)^2 = \frac{1}{4}n^2(n+1)(9n-7)$. (c) Hence solve the equation $8 \sum_{r=1}^n r(3r-2)^2 = 15 \sum_{r=1}^{2n} (2r^2 - 1)$. [13]

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3.2 (a) $2 \sum_{r=1}^{2n} r^2 - 2n = \frac{2(2n)(2n+1)(4n+1)}{6} - 2n = \frac{2n(2n+1)(4n+1) - 6n}{3} = \frac{2n(8n^2 + 6n - 2)}{3} = \frac{4n(4n^2 + 3n - 1)}{3} = \frac{4}{3}n(n+1)(4n-1)$ (since $4n^2 + 3n - 1 = (n+1)(4n-1)$). (b) Base $n = 1$: $1 = \frac{1}{4} \cdot 1 \cdot 2 \cdot 2$ ✓. Step: adding $(k+1)(3k+1)^2$ to $\frac{1}{4}k^2(k+1)(9k-7)$ gives $\frac{1}{4}(k+1)[9k^3 + 29k^2 + 24k + 4] = \frac{1}{4}(k+1)(k+1)(k+2)(9k+2)$, the target for $n = k+1$; close with the induction statement. (c) $8 \cdot \frac{1}{4}n^2(n+1)(9n-7) = 15 \cdot \frac{4}{3}n(n+1)(4n-1)$ – cancel $n(n+1)$ (positive): $2n(9n-7) = 20(4n-1) \Rightarrow 9n^2 - 47n + 10 = 0 \Rightarrow (9n-2)(n-5) = 0$, so $n = 5$ (n a positive integer).

Local review guidance: inverse formula and entry-by-entry comparison; exact values; induction with base case, inductive step and closing sentence. This is authored classroom guidance.