Exercise walk-through

5.2
Complex roots of cubics and quartics

Pearson Edexcel · International A-Level · Pure Mathematics

Exercises 2.1·2.3

2.1 Given that $2+i$ is a root of $x^2+px+q=0$ with p, q real, find p and q. [2]

2.2 $f(x)=x^3-3x^2+7x-5$ has root $x=1$. Find all roots of $f(x)=0$. [4]

2.3 One root of $x^4-2x^3+6x^2-2x+5=0$ is $1+2i$. Find the other three roots. [5]

Exercises 2.1·2.3

2.1 Given that $2+i$ is a root of $x^2+px+q=0$ with p, q real, find p and q. [2]

2.1 The other root is $2-i$: sum $4=-p$, product $5=q$. So $p=-4$, $q=5$.

2.2 $f(x)=x^3-3x^2+7x-5$ has root $x=1$. Find all roots of $f(x)=0$. [4]

2.2 $f(1)=1-3+7-5=0$ ✓. Divide: $x^3-3x^2+7x-5=(x-1)(x^2-2x+5)$. $x^2-2x+5=0 \Rightarrow x=\dfrac{2\pm\sqrt{4-20}}{2}=1\pm 2i$. Roots: $1,\ 1\pm 2i$. (Product: $1\cdot 5=5$ ✓.)

2.3 One root of $x^4-2x^3+6x^2-2x+5=0$ is $1+2i$. Find the other three roots. [5]

2.3 $1-2i$ is also a root, giving the factor x^2-2x+5. Dividing: $\dfrac{x^4-2x^3+6x^2-2x+5}{x^2-2x+5}=x^2+1$, since $(x^2-2x+5)(x^2+1)=x^4-2x^3+6x^2-2x+5$ ✓. The other roots are $\pm i$.

Exercises 2.4·3.1

2.4 A quartic with real coefficients has roots $1 \pm 3i$ and a repeated real root k, with constant term 50. Find k. [3]

3.1 *(Original.)* $h(z) = 2z^4 + 2z^3 + 7z^2 + 4z + 15$ has a root $\frac{1}{2} + \frac{3}{2}i$. (a) Write down the real quadratic factor coming from this root. (b) Given $h(z) = (2z^2 - 2z + 5)(z^2 + mz + n)$, find m and n. (c) Hence solve $h(z) = 0$ completely. [6]

Exercises 2.4·3.1

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2.4 Product of roots $= (1+3i)(1-3i) \cdot k^2 = 10k^2$. For a monic quartic the product equals the constant term (sign + for degree 4): $10k^2 = 50 \Rightarrow k = \pm\sqrt{5}$.

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3.1 (a) Sum 1, product $\frac{1}{4} + \frac{9}{4} = \frac{5}{2}$: factor $z^2 - z + \frac{5}{2}$, or $2z^2 - 2z + 5$ scaled. (b) Comparing $(2z^2 - 2z + 5)(z^2 + mz + n) = 2z^4 + 2z^3 + 7z^2 + 4z + 15$: constant $5n = 15 \Rightarrow n = 3$; z^3: $2m - 2 = 2 \Rightarrow m = 2$. Check the rest: $(2z^2 - 2z + 5)(z^2 + 2z + 3) = 2z^4 + 2z^3 + 7z^2 + 4z + 15$ ✓. (c) $z^2 + 2z + 3 = 0$ gives $z = -1 \pm i\sqrt{2}$, so the roots are $\frac{1}{2} \pm \frac{3}{2}i$, $-1 \pm i\sqrt{2}$.

Exercise 3.2

3.2 *(Adapted from Edexcel IAL WFM01 June 2025 Q7.)* $f(z) = Pz^4 - 36z^3 + Qz^2 + 192z + 68$ has root $3 + 5i$. Without using a calculator, determine P, Q and the three other roots, giving them in simplest form. [8]

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3.2 *(Adapted from Edexcel IAL WFM01 June 2025 Q7.)* $f(z) = Pz^4 - 36z^3 + Qz^2 + 192z + 68$ has root $3 + 5i$. Without using a calculator, determine P, Q and the three other roots, giving them in simplest form. [8]

3.2 Factor $z^2 - 6z + 34$; comparing coefficients: $c = 2$, $b = 6$, $P = 7$, $Q = 204$; other roots from $7z^2 + 6z + 2 = 0$: $z = \dfrac{-6 \pm \sqrt{-20}}{14} = \dfrac{-3 \pm i\sqrt{5}}{7}$ (and the conjugate $3 - 5i$).

Local review guidance: conjugate stated and the quadratic factor formed from sum and product; coefficient comparison or division carried through exactly; complex roots simplified to $a + bi$ form. This is authored classroom guidance.