Exercise walk-through

4.6
Vectors

Pearson Edexcel · International A-Level · Pure Mathematics

Check yourself

Check yourself

Contradiction proofs with the assumption named; partial fractions with repeated factors and improper cases; parametric conversions and gradients; the general binomial series with validity; implicit tangents; separable equations with conditions; parts, substitution and partial-fraction integrals; exact volumes of revolution; vector angles from the scalar product.

Exercises 2.1·2.3

2.1 Find $|3\mathbf{i} + 4\mathbf{j}|$ and the unit vector in that direction. [2]

2.2 For $P(1, 2, 3)$ and $Q(4, 6, 3)$, find $\overrightarrow{PQ}$ and its magnitude. [3]

2.3 Determine whether $\mathbf{a} = 2\mathbf{i} + (-1)\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = (-4)\mathbf{i} + 2\mathbf{j} + (-6)\mathbf{k}$ are parallel. [2]

Exercises 2.1·2.3

2.1 Find $|3\mathbf{i} + 4\mathbf{j}|$ and the unit vector in that direction. [2]

2.1 Magnitude 5; unit vector $\frac{1}{5}(3\mathbf{i} + 4\mathbf{j})$.

2.2 For $P(1, 2, 3)$ and $Q(4, 6, 3)$, find $\overrightarrow{PQ}$ and its magnitude. [3]

2.2 $\overrightarrow{PQ} = 3\mathbf{i} + 4\mathbf{j}$, magnitude 5.

2.3 Determine whether $\mathbf{a} = 2\mathbf{i} + (-1)\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = (-4)\mathbf{i} + 2\mathbf{j} + (-6)\mathbf{k}$ are parallel. [2]

2.3 $\mathbf{b} = -2\mathbf{a}$: yes, parallel (one is a scalar multiple of the other).

Exercise 2.4

2.4 Find the angle between $1\mathbf{i} + 1\mathbf{k}$ and $1\mathbf{j} + 1\mathbf{k}$, giving an exact value. [4]

Exercise 2.4

2.4 Find the angle between $1\mathbf{i} + 1\mathbf{k}$ and $1\mathbf{j} + 1\mathbf{k}$, giving an exact value. [4]

2.4 Dot product $= 0 + 0 + 1 = 1$; magnitudes $\sqrt{2}$ each:

$$\cos\theta = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2} \implies \theta = 60^\circ$$

Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA14 June 2025 Q6.)* Relative to O: A has position vector $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and B has position vector $3\mathbf{i} + \mathbf{j} + 6\mathbf{k}$. The point C has position vector $2\mathbf{i} + \beta\mathbf{j} + 4\mathbf{k}$, and angle BAC is $60°$. Find the exact possible values of β. [6]

Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA14 June 2025 Q6.)* Relative to O: A has position vector $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and B has position vector $3\mathbf{i} + \mathbf{j} + 6\mathbf{k}$. The point C has position vector $2\mathbf{i} + \beta\mathbf{j} + 4\mathbf{k}$, and angle BAC is $60°$. Find the exact possible values of β. [6]

3.1 $\overrightarrow{AB} = 2\mathbf{i}+2\mathbf{j}+4\mathbf{k}$, $|AB| = \sqrt{24}$; $\overrightarrow{AC} = 1\mathbf{i}+(\beta+1)\mathbf{j}+2\mathbf{k}$; dot product $= 2+2(\beta+1)+8 = 2\beta + 12$.

$$\cos 60° = \frac{2\beta + 12}{\sqrt{24}\sqrt{1 + (\beta+1)^2 + 4}} = \frac{1}{2}$$

$$2(2\beta + 12) = \sqrt{24}\sqrt{(\beta+1)^2 + 5} \implies 16(\beta+6)^2 = 24((\beta+1)^2 + 5)$$

$$2(\beta+6)^2 = 3((\beta+1)^2 + 5) \implies 2\beta^2 + 24\beta + 72 = 3\beta^2 + 6\beta + 18$$

$$\beta^2 - 18\beta - 54 = 0 \implies \beta = \frac{18 \pm \sqrt{540}}{2} = 9 \pm 3\sqrt{15}$$

- Check $\beta = 9 + 3\sqrt{15} \approx 20.62$: dot $= 53.2$; $|AC| = \sqrt{1 + 467.4 + 4} = 21.7$; $\cos\theta = 53.2/(4.90 \times 21.7) = 0.500 = \frac{1}{2}$ ✓ exactly.

Exercise 3.2

3.2 The line segment RS has midpoint $M(1, 1, 0)$; R has position vector $3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$. Find $\overrightarrow{RS}$ and the distance RS. [4]

Exercise 3.2

3.2 The line segment RS has midpoint $M(1,1,0)$; R has position vector $3\mathbf{i}-\mathbf{j}+4\mathbf{k}$. Find $\overrightarrow{RS}$ and the distance RS. [4]

3.2 $\overrightarrow{OR} = 3\mathbf{i}+(-1)\mathbf{j}+4\mathbf{k}$; $\overrightarrow{OM} = \frac{1}{2}(\overrightarrow{OR}+\overrightarrow{OS}) \Rightarrow \overrightarrow{OS} = 2\overrightarrow{OM}-\overrightarrow{OR} = (-1)\mathbf{i}+3\mathbf{j}+(-4)\mathbf{k}$.

$$\overrightarrow{RS} = \overrightarrow{OS} - \overrightarrow{OR} = (-4)\mathbf{i} + 4\mathbf{j} + (-8)\mathbf{k}, \qquad RS = \sqrt{16+16+64} = \sqrt{96} = 4\sqrt{6}$$

- Check: M is the midpoint of R and S: $\frac{1}{2}(3+(-1)) = 1$, $\frac{1}{2}(-1+3) = 1$, $\frac{1}{2}(4+(-4)) = 0$ ✓.

Local review guidance: correct $\overrightarrow{AB}$, $\overrightarrow{AC}$ and scalar product; exact β or distance; angle equation set up with magnitudes. This is authored classroom guidance.