Exercise walk-through

4.4 Implicit differentiation and differential equations

Pearson Edexcel · International A-Level · Pure Mathematics

Check yourself

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Contradiction proofs with the assumption named; partial fractions with repeated factors and improper cases; parametric conversions and gradients; the general binomial series with validity; implicit tangents; separable equations with conditions; parts, substitution and partial-fraction integrals; exact volumes of revolution; vector angles from the scalar product.

Exercises 2.1·2.3

2.1 Find $\dfrac{dy}{dx}$ for $x^2 + y^2 = 25$. [2]

2.2 Find $\dfrac{dy}{dx}$ for $x^2 + 2xy = 3$. [3]

2.3 A cube's edge grows at 0.1 cm/s. Find $\dfrac{dV}{dt}$ when the edge is 5 cm. [3]

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2.2 Find $\dfrac{dy}{dx}$ for $x^2 + 2xy = 3$. [3]

2.2 $2x + 2y + 2x\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{x+y}{x}$.

2.3 A cube's edge grows at 0.1 cm/s. Find $\dfrac{dV}{dt}$ when the edge is 5 cm. [3]

2.3 $V = x^3$: $\dfrac{dV}{dt} = 3x^2\dfrac{dx}{dt} = 3(25)(0.1) = 7.5\ \text{cm}^3/\text{s}$.

Exercises 2.4·3.1

2.4 Solve $\dfrac{dy}{dx} = \dfrac{2x}{y}$ with $y = 3$ at $x = 0$. [4]

3.1 *(Adapted from Edexcel IAL WMA14 June 2025 Q2.)* A sealed spherical balloon loses air at a constant 12 cm^3 per second. Show that the rate of decrease of its radius is inversely proportional to the square of the radius, and find that rate when $r = 3$ cm. [5]

Exercises 2.4·3.1

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2.4 $\int y\,dy = \int 2x\,dx \Rightarrow \frac{y^2}{2} = x^2 + c$; $y(0) = 3$ gives $c = \frac{9}{2}$, so $y^2 = 2x^2 + 9$.

3.1 *(Adapted from Edexcel IAL WMA14 June 2025 Q2.)* A sealed spherical balloon loses air at a constant 12 cm^3 per second. Show that the rate of decrease of its radius is inversely proportional to the square of the radius, and find that rate when $r = 3$ cm. [5]

3.1 $V = \frac{4}{3}\pi r^3$, $\dfrac{dV}{dr} = 4\pi r^2$; chain rule with $\dfrac{dV}{dt} = -12$:

$$-12 = 4\pi r^2 \frac{dr}{dt} \implies \frac{dr}{dt} = -\frac{3}{\pi r^2}$$

The rate of decrease $\dfrac{3}{\pi r^2}$ is a constant multiple of $\dfrac{1}{r^2}$ – inversely proportional to r^2. ■

At $r = 3$:

$$\frac{3}{9\pi} = \frac{1}{3\pi} \approx 0.106 \text{ cm/s (3 s.f.)}$$

Check: units $\text{cm}^3/\text{s} \div \text{cm}^2 = \text{cm/s}$ ✓.

Exercise 3.2

3.2 *(Adapted from Edexcel IAL WMA14 June 2025 Q3.)* Solve the differential equation

$$y\frac{dy}{dx} = 6\sec 3x \tan 3x, \qquad y = 2 \text{ at } x = 0$$

giving your answer in the form $y^2 = g(x)$. [6]

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giving your answer in the form $y^2 = g(x)$. [6]

3.2 Separate: $\displaystyle\int y\,dy = \int 6\sec 3x \tan 3x\,dx$ with $\int \sec 3x \tan 3x\,dx = \frac{1}{3}\sec 3x$:

$$\frac{y^2}{2} = 2\sec 3x + c \implies y^2 = 4\sec 3x + C$$

Condition $y = 2$ at $x = 0$: $4 = 4(1) + C \Rightarrow C = 0$:

$$y^2 = 4\sec 3x$$

Check: $\sqrt{4} = 2$ at $x = 0$ ✓; differentiating back: $2y\dfrac{dy}{dx} = 12\sec 3x \tan 3x \Rightarrow y\dfrac{dy}{dx} = 6\sec 3x \tan 3x$ ✓.

Local review guidance: correct implicit/rate structure; exact derivative or particular solution; condition applied and the solution checked. This is authored classroom guidance.