

Exercise walk-through

3.6

Integration and numerical methods

Pearson Edexcel · International A-Level · Pure Mathematics

Check yourself

Check yourself

Rational simplification and inverses verified by composing; modulus cases checked; combined transformations; sec, cosec, cot with identities; harmonic form with exact R; compound and double angle proofs; log-graph recovery; growth-decay models with floor; product, quotient, chain and implicit differentiation; recognition integrals; sign-change and iteration methods.

Exercises 2.1·2.3

2.1 Find $\int e^{5x} dx$. [2]

2.2 Find $\int \frac{3}{x} dx$ ($x > 0$). [2]

2.3 Find $\int \cos 4x dx$. [2]

Exercises 2.1·2.3

2.1 Find $\int e^{5x} dx$. [2]

2.1 $\frac{1}{5}e^{5x} + c$.

2.2 Find $\int \frac{3}{x} dx$ ($x > 0$). [2]

2.2 $3 \ln x + c$.

2.3 Find $\int \cos 4x dx$. [2]

2.3 $\frac{1}{4} \sin 4x + c$.

Exercises 2.4·2.6

2.4 Find $\int \frac{2x+2}{x^2+2x+5} dx$. [3]

2.5 Find $\int 3x^2(x^3+1)^4 dx$. [3]

2.6 Show that $x^3 - 3x + 1 = 0$ has a root between 0 and 1. [2]

Exercises 2.4·2.6

2.4 Find $\int \frac{2x+2}{x^2+2x+5} dx$. [3]

2.4 Numerator = $(x^2 + 2x + 5)'$: $\ln(x^2 + 2x + 5) + c$ (the quadratic is always positive, so no modulus needed).

2.5 Find $\int 3x^2(x^3+1)^4 dx$. [3]

2.5 With $u = x^3 + 1$, $du = 3x^2 dx$: $\frac{1}{5}(x^3 + 1)^5 + c$.

2.6 Show that $x^3 - 3x + 1 = 0$ has a root between 0 and 1. [2]

2.6 $f(x) = x^3 - 3x + 1$: $f(0) = 1 > 0$, $f(1) = -1 < 0$; f is continuous (polynomial), and the sign change over $[0, 1]$ guarantees a root in $(0, 1)$.

Exercise 3.1

3.1 (*Adapted from Edexcel IAL WMA13 June 2025 Q5 and Q7.*)

(a) Find $\int \cos^2 2x \, dx$.

(b) Find $\int \frac{x}{x^2 + 4} \, dx$.

Exercise 3.1

3.1 (*Adapted from Edexcel IAL WMA13 June 2025 Q5 and Q7.*)

(a) Find $\int \cos^2 2x \, dx$.

(b) Find $\int \frac{x}{x^2 + 4} \, dx$.

[5]

3.1 (a) $\cos^2 2x = \frac{1}{2}(1 + \cos 4x)$:

$$\int \cos^2 2x \, dx = \frac{x}{2} + \frac{\sin 4x}{8} + c$$

Check: $\frac{1}{2} + \frac{4}{8} \cos 4x = \frac{1}{2}(1 + \cos 4x) = \cos^2 2x \checkmark$. (b) Numerator $x = \frac{1}{2}(x^2 + 4)'$:
 $\frac{1}{2} \ln(x^2 + 4) + c$.

Exercise 3.2

3.2 (*Adapted from Edexcel IAL WMA13 June 2025 Q7.*) The equation $x = e^{-x}$ has exactly one positive root.

- (a) Show the root lies between 0.5 and 0.6.
- (b) Use the iteration $x_{n+1} = e^{-x_n}$ from $x_1 = 0.5$ to find x_3 to 3 decimal places.
- (c) Verify the root is 0.567 to 3 decimal places.

[7]

Exercise 3.2

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- (a) Show the root lies between 0.5 and 0.6.
- (b) Use the iteration $x_{n+1} = e^{-x_n}$ from $x_1 = 0.5$ to find x_3 to 3 decimal places.
- (c) Verify the root is 0.567 to 3 decimal places. [7]

3.2 (a) $g(x) = x - e^{-x}$: $g(0.5) = 0.5 - 0.6065 = -0.1065 < 0$ and $g(0.6) = 0.6 - 0.5488 = 0.0512 > 0$; g is continuous, so a root lies in $(0.5, 0.6)$. (b) $x_2 = e^{-0.5} = 0.6065$; $x_3 = e^{-0.6065} = 0.545$ (3 d.p.). (c) $g(0.5665) = 0.5665 - e^{-0.5665} = 0.5665 - 0.5675 = -0.0010 < 0$ and $g(0.5675) = 0.5675 - e^{-0.5675} = 0.5675 - 0.5669 = +0.0006 > 0$: the sign change between 0.5665 and 0.5675 fixes the root as 0.567 to 3 d.p. ✓ (True root: the omega constant 0.5671... ✓.)

Local review guidance: correct standard form or recognition of f' over f ; exact integrals with $+c$; sign-change statement naming continuity, and the iteration carried with enough steps. This is authored classroom guidance.