Exercise walk-through

3.5 Differentiation rules and implicit work

Pearson Edexcel · International A-Level · Pure Mathematics

Check yourself

Check yourself

Rational simplification and inverses verified by composing; modulus cases checked; combined transformations; sec, cosec, cot with identities; harmonic form with exact R; compound and double angle proofs; log-graph recovery; growth-decay models with floor; product, quotient, chain and implicit differentiation; recognition integrals; sign-change and iteration methods.

Exercises 2.1·2.3

2.1 Differentiate $y = \sin 5x$. [1]

2.2 Differentiate $y = \mathrm{e}^{-3x}$. [1]

2.3 Differentiate $y = x^2 \cos x$ using the product rule. [3]

Exercises 2.1·2.3

2.1 Differentiate $y = \sin 5x$. [1]

2.1 $\dfrac{dy}{dx} = 5\cos 5x$.

2.2 Differentiate $y = \mathrm{e}^{-3x}$. [1]

2.2 $\dfrac{dy}{dx} = -3\mathrm{e}^{-3x}$.

2.3 Differentiate $y = x^2 \cos x$ using the product rule. [3]

2.3 $\dfrac{dy}{dx} = 2x\cos x - x^2 \sin x$.

Exercises 2.4·2.6

2.4 Differentiate $y = \dfrac{e^x}{x}$ using the quotient rule. [3]

2.5 Differentiate $y = \ln(4x^3 + 1)$ using the chain rule. [2]

2.6 Find $\dfrac{dy}{dx}$ for $x = e^{2y}$. [3]

Exercises 2.4·2.6

2.4 Differentiate $y = \dfrac{e^x}{x}$ using the quotient rule. [3]

2.4 $\dfrac{dy}{dx} = \dfrac{e^x \cdot x - e^x \cdot 1}{x^2} = \dfrac{e^x(x-1)}{x^2}$.

2.5 Differentiate $y = \ln(4x^3 + 1)$ using the chain rule. [2]

2.5 $\dfrac{dy}{dx} = \dfrac{12x^2}{4x^3+1}$ (chain: outer ln, inner $4x^3 + 1$).

2.6 Find $\dfrac{dy}{dx}$ for $x = e^{2y}$. [3]

2.6 $\dfrac{dx}{dy} = 2e^{2y} = 2x$, so $\dfrac{dy}{dx} = \dfrac{1}{2x}$.

Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA13 June 2025 Q7.)* The curve C has equation $y = \mathrm{e}^{-x^2} \sin 3x$ for $0 \leq x \leq 3$. Show, using calculus, that the x-coordinates of the stationary points of C satisfy

$$x = \tfrac{1}{3} \arctan \frac{3}{2x}$$

[4]

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[4]

3.1 From the worked derivative: stationary points have

$$\mathrm{e}^{-x^2}\big(3\cos 3x - 2x\sin 3x\big) = 0$$

$\mathrm{e}^{-x^2} > 0$ always, so $3\cos 3x = 2x\sin 3x$. Dividing by $\cos 3x$ (at a stationary point $\cos 3x \neq 0$, since that would force $\sin 3x = \pm 1 \neq 0$ and then $0 = \pm 2x$ fails for $x > 0$):

$$3 = 2x\tan 3x \implies \tan 3x = \frac{3}{2x} \implies 3x = \arctan\frac{3}{2x} \implies x = \tfrac{1}{3}\arctan\frac{3}{2x} \qquad \blacksquare$$

Exercise 3.2

3.2 A cylindrical tank drains so that the volume of water is $V = 500\mathrm{e}^{-0.1t}$ litres after t minutes. Find the rate at which water leaves at $t = 10$, and the time at which the rate is 5 litres per minute. [5]

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3.2 $\dfrac{dV}{dt} = -50\mathrm{e}^{-0.1t}$; at $t = 10$: $-50\mathrm{e}^{-1} = -18.4$ litres per minute (leaving at 18.4 L/min). Rate = 5: $50\mathrm{e}^{-0.1t} = 5 \Rightarrow \mathrm{e}^{-0.1t} = 0.1 \Rightarrow t = 10\ln 10 = 23.0$ minutes (3 s.f.). Check: $V(23.0) = 500\mathrm{e}^{-2.303} = 50.0$ litres, one tenth of the initial – matching the rate being one tenth of the initial 50 L/min ✓.

Local review guidance: correct rule with both parts differentiated; simplified derivative; exponential-rate equation solved with logs. This is authored classroom guidance.