

Exercise walk-through

3.3

Reciprocal trig and harmonic form

Pearson Edexcel · International A-Level · Pure Mathematics

Check yourself

Check yourself

Rational simplification and inverses verified by composing; modulus cases checked; combined transformations; sec, cosec, cot with identities; harmonic form with exact R; compound and double angle proofs; log-graph recovery; growth-decay models with floor; product, quotient, chain and implicit differentiation; recognition integrals; sign-change and iteration methods.

Exercises 2.1·2.3

2.1 Write $\sec \theta$ in terms of $\cos \theta$ and state the range of $\sec \theta$. [2]

2.2 Simplify $\cot \theta \tan \theta$. [1]

2.3 State the restricted domain and range of $\arccos$. [2]

Exercises 2.1·2.3

2.1 Write $\sec \theta$ in terms of $\cos \theta$ and state the range of $\sec \theta$. [2]

2.1 $\sec \theta = \frac{1}{\cos \theta}$; since $|\cos \theta| \leq 1$, $\sec \theta \geq 1$ or $\sec \theta \leq -1$.

2.2 Simplify $\cot \theta \tan \theta$. [1]

2.2 $\cot \theta \tan \theta = 1$ (where both are defined).

2.3 State the restricted domain and range of $\arccos$. [2]

2.3 $\arccos$ maps $[-1, 1]$ onto $[0, \pi]$.

Exercises 2.4·3.1

2.4 Use $\sin 2A \equiv 2 \sin A \cos A$ to write $\sin A \cos A$ as a single term. [2]

2.5 Express $12 \cos \theta + 5 \sin \theta$ in the form $R \cos(\theta - \alpha)$ with exact R and $\alpha = \arctan \frac{5}{12}$. [3]

3.1 (*Adapted from Edexcel IAL WMA13 June 2025 Q2.*) Express $8 \cos x - 15 \sin x$ in the form $R \cos(x + \alpha)$, $R > 0$, $0 < \alpha < \frac{\pi}{2}$, giving R exactly and α to 3 decimal places. Hence write down the maximum value of $8 \cos x - 15 \sin x$ and the smallest positive x at which it occurs. [6]

Exercises 2.4·3.1

2.4 Use $\sin 2A \equiv 2 \sin A \cos A$ to write $\sin A \cos A$ as a single term. [2]

2.4 $\sin A \cos A = \frac{1}{2} \sin 2A.$

2.5 Express $12 \cos \theta + 5 \sin \theta$ in the form $R \cos(\theta - \alpha)$ with exact R and $\alpha = \arctan \frac{5}{12}$. [3]

2.5 $R = \sqrt{144 + 25} = 13$, $\alpha = \arctan \frac{5}{12}$: $12 \cos \theta + 5 \sin \theta = 13 \cos(\theta - \alpha)$. Check: $13 \cos \alpha = 12$ ✓ and $13 \sin \alpha = 5$ ✓ (5-12-13 triangle).

3.1 (*Adapted from Edexcel IAL WMA13 June 2025 Q2.*) Express $8 \cos x - 15 \sin x$ in the form $R \cos(x + \alpha)$, $R > 0$, $0 < \alpha < \frac{\pi}{2}$, giving R exactly and α to 3 decimal places. Hence write down the maximum value of $8 \cos x - 15 \sin x$ and the smallest positive x at which it occurs. [6]

3.1 $R = \sqrt{64 + 225} = 17$; $17 \cos \alpha = 8$, $17 \sin \alpha = 15$ so $\tan \alpha = \frac{15}{8}$, $\alpha = 1.080$ (3 d.p.):

$$8 \cos x - 15 \sin x = 17 \cos(x + 1.080)$$

Maximum = 17 when $x + 1.080 = 0 \pmod{2\pi}$; the smallest positive x is $2\pi - 1.080 = 5.203$ (3 d.p.). Check: $\cos(5.203 + 1.080) = \cos(6.283) = \cos 2\pi = 1$ ✓.

Exercise 3.2

3.2 (*Adapted from Edexcel IAL WMA13 June 2025 Q8a.*) Prove, using the compound angle formulae, that

$$\tan 3x \equiv \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$$

[4]

Exercise 3.2

3.2 (*Adapted from Edexcel IAL WMA13 June 2025 Q8a.*) Prove, using the compound angle formulae, that

$$\tan 3x \equiv \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$$

[4]

3.2 Write $3x = 2x + x$:

$$\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}, \quad \tan 2x = \frac{2t}{1 - t^2} \quad (t = \tan x)$$

Substitute and multiply through by $(1 - t^2)$:

$$\tan 3x = \frac{\frac{2t}{1-t^2} + t}{1 - \frac{2t^2}{1-t^2}} = \frac{2t + t - t^3}{(1 - t^2) - 2t^2} = \frac{3t - t^3}{1 - 3t^2} \quad \blacksquare$$

Local review guidance: correct R and matching of both components; exact or correctly rounded values; maximum read from the amplitude with its location. This is authored classroom guidance.