

Exercise walk-through

2.7

Trigonometric identities and equations

Pearson Edexcel · International A-Level · Pure Mathematics

Check yourself

Check yourself

Exhaustion with complete cases; counterexamples; factor/remainder theorems with $(ax - b)$; circles from any form plus the three properties; both series families with proofs and logs; binomial coefficients; log laws with domain checks; identity-reduced trig equations in either unit; stationary points classified and modelled; areas by definite integral and by the trapezium rule with the error direction.

Exercises 2.1·2.3

2.1 Simplify $\frac{\sin \theta \cos \theta}{\tan \theta}$. [2]

2.2 Solve $\sin \theta = -\frac{1}{2}$ for $0 \leq \theta < 360^\circ$. [3]

2.3 Solve $\tan 2\theta = 1$ for $0 \leq \theta < 180^\circ$. [4]

Exercises 2.1·2.3

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2.1 $\frac{\sin \theta \cos \theta}{\sin \theta / \cos \theta} = \cos^2 \theta$.

2.2 Solve $\sin \theta = -\frac{1}{2}$ for $0 \leq \theta < 360^\circ$. [3]

2.2 Third and fourth quadrants: $\theta = 180^\circ + 30^\circ = 210^\circ$ and $360^\circ - 30^\circ = 330^\circ$.

2.3 Solve $\tan 2\theta = 1$ for $0 \leq \theta < 180^\circ$. [4]

2.3 2θ runs over $[0^\circ, 360^\circ)$: $2\theta = 45^\circ$ or 225° , so $\theta = 22.5^\circ$ or 112.5° .

Exercises 2.4·2.5

2.4 Given $\sin \theta = 0.6$ with θ acute, find $\cos \theta$ and $\tan \theta$. [3]

2.5 Solve $2 \cos^2 \theta + \cos \theta - 1 = 0$ for $0 \leq \theta < 360^\circ$. [4]

Exercises 2.4·2.5

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2.4 $\cos \theta = \sqrt{1 - 0.36} = 0.8$ (acute, so positive); $\tan \theta = 0.6/0.8 = 0.75$.

2.5 Solve $2 \cos^2 \theta + \cos \theta - 1 = 0$ for $0 \leq \theta < 360^\circ$. [4]

2.5 $(2 \cos \theta - 1)(\cos \theta + 1) = 0$: $\cos \theta = \frac{1}{2}$ gives $60^\circ, 300^\circ$; $\cos \theta = -1$ gives 180° . Solutions: $60^\circ, 180^\circ, 300^\circ$.

Exercise 3.1

3.1 (*Adapted from Edexcel IAL WMA12 June 2025 Q8(i).*) Solve, for $0 \leq x < \pi$, the equation $\tan\left(2x - \frac{\pi}{4}\right) = \sqrt{3}$, giving the answers in radians in the form $k\pi$ where k is a rational constant to be found. [4]

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3.1 (*Adapted from Edexcel IAL WMA12 June 2025 Q8(i).*) Solve, for $0 \leq x < \pi$, the equation $\tan\left(2x - \frac{\pi}{4}\right) = \sqrt{3}$, giving the answers in radians in the form $k\pi$ where k is a rational constant to be found. [4]

3.1 $\tan\left(2x - \frac{\pi}{4}\right) = \sqrt{3} = \tan \frac{\pi}{3}$:

$$2x - \frac{\pi}{4} = \frac{\pi}{3} + k\pi \implies 2x = \frac{7\pi}{12} + k\pi \implies x = \frac{7\pi}{24} + \frac{k\pi}{2}$$

In $0 \leq x < \pi$: $x = \frac{7\pi}{24}$ ($k = 0$) and $x = \frac{19\pi}{24}$ ($k = 1$). Check: at $x = \frac{7\pi}{24}$, $2x - \frac{\pi}{4} = \frac{\pi}{3}$ and $\tan \frac{\pi}{3} = \sqrt{3}$ ✓.

Exercise 3.2

3.2 (*Adapted from the same question.*) Solve, for $0 \leq \theta < 360^\circ$, the equation $4 \sin \theta \tan \theta = 3 \cos \theta$, giving your answers to one decimal place. [6]

Exercise 3.2

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3.2 Replace $\tan \theta$ and multiply through by $\cos \theta$:

$$\frac{4 \sin^2 \theta}{\cos \theta} = 3 \cos \theta \implies 4 \sin^2 \theta = 3 \cos^2 \theta \implies 4(1 - \cos^2 \theta) = 3 \cos^2 \theta \implies \cos^2 \theta = \frac{4}{7}$$

$$\cos \theta = \pm \frac{2}{\sqrt{7}} = \pm 0.7559$$

- $\cos \theta = 0.7559$: $\theta = 40.9^\circ$ and $360^\circ - 40.9^\circ = 319.1^\circ$.
- $\cos \theta = -0.7559$: $\theta = 180^\circ - 40.9^\circ = 139.1^\circ$ and $180^\circ + 40.9^\circ = 220.9^\circ$. Solutions: $40.9^\circ, 139.1^\circ, 220.9^\circ, 319.1^\circ$ (1 d.p.). Check 40.9° : $\sin = 0.654$, $\tan = 0.865$: $4(0.654)(0.865) = 2.26$ and $3(0.756) = 2.27$ ✓ (rounding).

Local review guidance: identity applied to reach one function; correct root values; all interval solutions listed with quadrant reasoning. This is authored classroom guidance.