

Exercise walk-through

2.4

Sequences and series

Pearson Edexcel · International A-Level · Pure Mathematics

Check yourself

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Exhaustion with complete cases; counterexamples; factor/remainder theorems with $(ax - b)$; circles from any form plus the three properties; both series families with proofs and logs; binomial coefficients; log laws with domain checks; identity-reduced trig equations in either unit; stationary points classified and modelled; areas by definite integral and by the trapezium rule with the error direction.

Exercises 2.1·2.3

2.1 For $u_n = 3n - 2$, write down the 10th term. [1]

2.2 For the arithmetic series $7 + 11 + 15 + \dots$, find S_{15} . [3]

2.3 A geometric sequence begins 8, 12, 18, ... Find its 10th term to 3 significant figures. [3]

Exercises 2.1·2.3

2.1 For $u_n = 3n - 2$, write down the 10th term. [1]

2.1 $u_{10} = 3(10) - 2 = 28.$

2.2 For the arithmetic series $7 + 11 + 15 + \dots$, find S_{15} . [3]

2.2 $a = 7, d = 4: S_{15} = \frac{15}{2}\{14 + 14 \times 4\} = 7.5 \times 70 = 525.$

2.3 A geometric sequence begins 8, 12, 18, ... Find its 10th term to 3 significant figures. [3]

2.3 $r = \frac{12}{8} = 1.5: a_{10} = 8(1.5)^9 = 8 \times 38.44 = 308 \text{ (3 s.f.)}.$

Exercises 2.4·2.6

2.4 A geometric series has $S_{\infty} = 60$ and $r = \frac{1}{2}$. Find a . [2]

2.5 The sequence defined by $x_{n+1} = 2x_n - 1$ with $x_1 = 3$: write down the first four terms and classify the sequence. [2]

2.6 The sequence with $x_{n+1} = -x_n$ and $x_1 = 4$: write down the first four terms and state the period. [2]

Exercises 2.4·2.6

2.4 A geometric series has $S_{\infty} = 60$ and $r = \frac{1}{2}$. Find a . [2]

2.4 $a = S_{\infty}(1 - r) = 60 \times \frac{1}{2} = 30.$

2.5 The sequence defined by $x_{n+1} = 2x_n - 1$ with $x_1 = 3$: write down the first four terms and classify the sequence. [2]

2.5 3, 5, 9, 17: increasing (each term exceeds the last).

2.6 The sequence with $x_{n+1} = -x_n$ and $x_1 = 4$: write down the first four terms and state the period. [2]

2.6 4, -4, 4, -4: periodic with period 2.

Exercise 3.1

3.1 (*Adapted from Edexcel IAL WMA12 June 2025 Q7.*) A geometric series begins $18 + 12 + 8 + \dots$

(a) Find the sum to infinity of the series.

(b) Given that the k th term of the series is less than 0.01, use algebra to find the smallest possible value of k . [5]

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(b) Given that the k th term of the series is less than 0.01, use algebra to find the smallest possible value of k . [5]

$$\mathbf{3.1} \text{ (a) } r = \frac{2}{3}, |r| < 1: S_{\infty} = \frac{18}{1 - 2/3} = 54. \text{ (b) } 18 \left(\frac{2}{3}\right)^{k-1} < 0.01 \implies \left(\frac{2}{3}\right)^{k-1} < \frac{1}{1800}.$$

Taking logs and dividing by the negative $\log \frac{2}{3}$ flips the sign:

$$k - 1 > \frac{\log(1/1800)}{\log(2/3)} = 18.48\dots \implies k = 20$$

Check: $18 \left(\frac{2}{3}\right)^{19} = 0.0081 < 0.01 \checkmark$ and $18 \left(\frac{2}{3}\right)^{18} = 0.0122 > 0.01 \checkmark$ (so 19 is excluded).

Exercise 3.2

3.2 (*Adapted from the same question.*) An arithmetic series has first term 500 and common difference -7 . Find the greatest possible value of S_n . [5]

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3.2 Terms stay positive while $500 - 7(n - 1) > 0$, i.e. $n \leq 72$; check the boundary: $u_{72} = 500 - 7(71) = 3 > 0$ and $u_{73} = -4 < 0$, so the maximum partial sum is

$$S_{72} = \frac{72}{2} \{1000 + 71(-7)\} = 36 \times 503 = 18\,108$$

Adding the negative 73rd term would reduce the total ✓.

Local review guidance: correct formula with a and r or d ; exact or correctly rounded value; inequality direction preserved through negative-log division, or the boundary term checked. This is authored classroom guidance.