Exercise walk-through

2.2
Factor and remainder theorems

Pearson Edexcel · International A-Level · Pure Mathematics

Check yourself

Check yourself

Exhaustion with complete cases; counterexamples; factor/remainder theorems with $(ax - b)$; circles from any form plus the three properties; both series families with proofs and logs; binomial coefficients; log laws with domain checks; identity-reduced trig equations in either unit; stationary points classified and modelled; areas by definite integral and by the trapezium rule with the error direction.

Exercises 2.1·2.3

2.1 Show that $(x+1)$ is a factor of $x^3 - 7x - 6$. [2]

2.2 Hence factorise $x^3 - 7x - 6$ completely. [3]

2.3 Find the remainder when $x^3 + x^2 - 2$ is divided by $(x-1)$, and say what your answer implies. [2]

Exercises 2.1·2.3

2.1 Show that $(x+1)$ is a factor of $x^3 - 7x - 6$. [2]

2.1 $f(-1) = -1 + 7 - 6 = 0$, so $(x+1)$ is a factor by the factor theorem.

2.2 Hence factorise $x^3 - 7x - 6$ completely. [3]

2.2 Divide: $x^3 - 7x - 6 = (x+1)(x^2 - x - 6) = (x+1)(x-3)(x+2)$. Check: $f(0) = -6$ and $(1)(-3)(2) = -6$ ✓.

2.3 Find the remainder when $x^3 + x^2 - 2$ is divided by $(x-1)$, and say what your answer implies. [2]

2.3 $f(1) = 1 + 1 - 2 = 0$: the remainder is 0, so $(x-1)$ is itself a factor.

Exercises 2.4·3.1

2.4 Find the remainder when $3x^3 - 5x^2 + 7$ is divided by $(2x - 1)$. [3]

2.5 $f(x) = x^3 + ax + b$ has $(x - 2)$ as a factor and leaves remainder 15 when divided by $(x - 3)$. Find a and b. [5]

3.1 *(Adapted from Edexcel IAL WMA12 June 2025 Q4.)* $f(x) = 2x^3 + px^2 + 11x + q$, where $(x - 1)$ is a factor of $f(x)$ and $f(x)$ leaves remainder -36 when divided by $(x + 2)$. Find p and q, and verify both conditions. [5]

Exercises 2.4·3.1

2.4 Find the remainder when $3x^3 - 5x^2 + 7$ is divided by $(2x - 1)$. [3]

2.4 $f\left(\frac{1}{2}\right) = 3\left(\frac{1}{8}\right) - 5\left(\frac{1}{4}\right) + 7 = \frac{3}{8} - \frac{5}{4} + 7 = \frac{49}{8}$.

2.5 $f(x) = x^3 + ax + b$ has $(x - 2)$ as a factor and leaves remainder 15 when divided by $(x - 3)$. Find a and b. [5]

2.5 $f(2) = 8 + 2a + b = 0$ and $f(3) = 27 + 3a + b = 15$. Subtracting: $19 + a = 15$, so $a = -4$; then $8 - 8 + b = 0$ gives $b = 0$. $f(x) = x^3 - 4x$. Check: $f(2) = 0$ ✓ and $f(3) = 27 - 12 = 15$ ✓.

3.1 *(Adapted from Edexcel IAL WMA12 June 2025 Q4.)* $f(x) = 2x^3 + px^2 + 11x + q$, where $(x - 1)$ is a factor of $f(x)$ and $f(x)$ leaves remainder -36 when divided by $(x + 2)$. Find p and q, and verify both conditions. [5]

3.1 $f(1) = 2 + p + 11 + q = 0 \implies p + q = -13$. $f(-2) = -16 + 4p - 22 + q = -36 \implies 4p + q = 2$. Subtracting: $3p = 15$, so $p = 5$ and $q = -18$. Verify: $f(1) = 2 + 5 + 11 - 18 = 0$ ✓; $f(-2) = -16 + 20 - 22 - 18 = -36$ ✓.

Exercise 3.2

3.2 Using the factor theorem, fully factorise $x^3 + 4x^2 + x - 6$. [4]

Exercise 3.2

3.2 Using the factor theorem, fully factorise $x^3 + 4x^2 + x - 6$. [4]

3.2 Test $f(-2) = -8 + 16 - 2 - 6 = 0$: $(x + 2)$ is a factor. Divide: $x^3 + 4x^2 + x - 6 = (x + 2)(x^2 + 2x - 3) = (x + 2)(x + 3)(x - 1)$. Check: $f(0) = -6$ and $(2)(3)(-1) = -6$ ✓.

Local review guidance: correct theorem applied at the right root; exact quotient and remainder; verification by rebuilding or by f evaluated. This is authored classroom guidance.