

Exercise walk-through

2.10

The trapezium rule

Pearson Edexcel · International A-Level · Pure Mathematics

Check yourself

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Exhaustion with complete cases; counterexamples; factor/remainder theorems with $(ax - b)$; circles from any form plus the three properties; both series families with proofs and logs; binomial coefficients; log laws with domain checks; identity-reduced trig equations in either unit; stationary points classified and modelled; areas by definite integral and by the trapezium rule with the error direction.

Exercises 2.1·2.3

2.1 Estimate $\int_0^4 x^2 dx$ with 4 strips. [3]

2.2 Estimate $\int_1^3 \frac{1}{x} dx$ with 2 strips, and state whether your answer is an over- or under-estimate. [3]

2.3 For a curve that is convex (bends upwards), state whether the trapezium rule over- or underestimates the true area, and why. [2]

Exercises 2.1·2.3

2.1 Estimate $\int_0^4 x^2 dx$ with 4 strips. [3]

2.1 $h = 1$; heights 0, 1, 4, 9, 16: $\frac{1}{2}[0 + 16 + 2(1 + 4 + 9)] = \frac{1}{2}(44) = 22$. (True value $64/3 \approx 21.3$: an overestimate, as x^2 is convex and the chords sit above.)

2.2 Estimate $\int_1^3 \frac{1}{x} dx$ with 2 strips, and state whether your answer is an over- or under-estimate. [3]

2.2 $h = 1$; heights at $x = 1, 2, 3$: 1, 0.5, 0.3333: $\frac{1}{2}[1 + 0.3333 + 2(0.5)] = \frac{4}{3} = 1.333$. $y = 1/x$ is convex (bends upwards), so the chords lie above the curve: an **overestimate** (true value $\ln 3 = 1.099$).

2.3 For a curve that is convex (bends upwards), state whether the trapezium rule over- or underestimates the true area, and why. [2]

2.3 Overestimate: a convex curve bends upwards, so every chord (strip top) sits above the curve and the strips count extra area between chord and curve.

Exercises 2.4·2.5

2.4 Estimate $\int_0^1 \sqrt{x} \, dx$ with 4 strips.

[3]

2.5 State two ways to improve a trapezium-rule estimate.

[2]

Exercises 2.4·2.5

2.4 Estimate $\int_0^1 \sqrt{x} \, dx$ with 4 strips. [3]

2.4 $h = 0.25$; heights 0, 0.5, 0.7071, 0.8660, 1: $0.125[0 + 1 + 2(2.0731)] = 0.125 \times 5.146 = 0.643$.

2.5 State two ways to improve a trapezium-rule estimate. [2]

2.5 Use more strips (smaller h); the rule is also exact when the graph is straight, so flatter regions estimate better – in general, more strips and checking the curvature direction.

Exercise 3.1

3.1 (*Adapted from Edexcel IAL WMA12 June 2025 Q5.*) The trapezium rule with five y -values and strip width $h = 1.5$ gives the estimate

$$\int_1^5 f(x) dx \approx 12.4$$

Using this result and making your method clear, estimate

(a) $\int_1^5 \frac{f(x)}{2} dx$

(b) $\int_1^5 \left(\frac{f(x)}{4} + 3x \right) dx$

[4]

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Full solution 3.1

3.1 (a) Constants factor out of integrals:

$$\int_1^5 \frac{f(x)}{2} dx = \frac{1}{2} \times 12.4 = 6.2$$

(b) Split the integral and scale each part:

$$\int_1^5 \left(\frac{f(x)}{4} + 3x \right) dx = \frac{1}{4}(12.4) + 3 \left[\frac{x^2}{2} \right]_1^5 = 3.1 + 3(12 - 0.5) = 3.1 + 34.5 = 37.6$$

Check: $\frac{5^2 - 1^2}{2} = 12 \checkmark$.

Exercise 3.2

3.2 (*Structure adapted from the same paper.*) Use the trapezium rule with 8 strips to estimate $\int_0^4 \sqrt{x} \, dx$, and compare your answer with the exact value $\frac{16}{3}$. [5]

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3.2 $h = 0.5$; heights at $x = 0, 0.5, \dots, 4$: 0, 0.7071, 1, 1.2247, 1.4142, 1.5811, 1.7321, 1.8708, 2

$$\frac{0.5}{2} [0 + 2 + 2(0.7071 + 1 + 1.2247 + 1.4142 + 1.5811 + 1.7321 + 1.8708)] = 0.25 [2 + 19.060] = 5.265$$

So the estimate is 5.27 (3 s.f.), below the exact $\frac{16}{3} = 5.333$: an underestimate, because $\sqrt{x}$ is concave (it rises steeply then flattens), so each chord lies below the curve and the strips undercount.

Local review guidance: correct trapezium bracket structure with $h/2$; arithmetic to the requested accuracy; error direction tied to the curvature. This is authored classroom guidance.