

Exercise walk-through

2.1

Proof: exhaustion and counterexample

Pearson Edexcel · International A-Level · Pure Mathematics

Check yourself

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Exhaustion with complete cases; counterexamples; factor/remainder theorems with $(ax - b)$; circles from any form plus the three properties; both series families with proofs and logs; binomial coefficients; log laws with domain checks; identity-reduced trig equations in either unit; stationary points classified and modelled; areas by definite integral and by the trapezium rule with the error direction.

Exercises 2.1·2.3

2.1 Disprove by counterexample: " $n^2 + 1$ is prime for all positive integers n ." [2]

2.2 Prove that the sum of three consecutive integers is divisible by 3. [3]

2.3 Prove by exhaustion that $n^2 - 1$ is divisible by 3 for every positive integer n that is not a multiple of 3. [4]

Exercises 2.1·2.3

2.1 Disprove by counterexample: " $n^2 + 1$ is prime for all positive integers n ." [2]

2.1 $n = 3$: $9 + 1 = 10 = 2 \times 5$, which is not prime. One counterexample disproves the claim.

2.2 Prove that the sum of three consecutive integers is divisible by 3. [3]

2.2 Let the integers be $(n - 1) + n + (n + 1) = 3n$, which is divisible by 3 for every integer n . ■

2.3 Prove by exhaustion that $n^2 - 1$ is divisible by 3 for every positive integer n that is not a multiple of 3. [4]

2.3 Non-multiples of 3 are $n = 3q + 1$ or $n = 3q + 2$ (complete case list).

- $n = 3q + 1$: $n^2 - 1 = 9q^2 + 6q = 3(3q^2 + 2q)$ ✓
- $n = 3q + 2$: $n^2 - 1 = 9q^2 + 12q + 3 = 3(3q^2 + 4q + 1)$ ✓ Both cases give a multiple of 3. ■

Exercises 2.4·3.1

2.4 Prove that the product of two odd integers is odd. [3]

3.1 (*Adapted from Edexcel IAL WMA12 June 2025 Q10.*) Use algebra to prove by exhaustion that for all positive integers m that are not multiples of 3, the value of $m^2 + 3m + 2$ is always a multiple of 3. [4]

Exercises 2.4·3.1

2.4 Prove that the product of two odd integers is odd. [3]

2.4 Let the integers be $2a+1$ and $2b+1$: their product is $4ab+2a+2b+1 = 2(2ab+a+b)+1$, which is one more than an even number – odd. ■

3.1 (*Adapted from Edexcel IAL WMA12 June 2025 Q10.*) Use algebra to prove by exhaustion that for all positive integers m that are not multiples of 3, the value of $m^2 + 3m + 2$ is always a multiple of 3. [4]

3.1 $m^2 + 3m + 2 = (m + 1)(m + 2)$. Cases: $m = 3q + 1$ or $m = 3q + 2$.

- $m = 3q + 1$: $(3q + 2)(3q + 3) = 3(3q + 2)(q + 1)$ – multiple of 3.
- $m = 3q + 2$: $(3q + 3)(3q + 4) = 3(q + 1)(3q + 4)$ – multiple of 3. Exhausting all non-multiples of 3, the expression is always a multiple of 3. ■ Check: $m = 1$: 6; $m = 2$: 12; $m = 4$: 30; $m = 5$: 42 – all divisible by 3 ✓.

Exercise 3.2

3.2 Prove by exhaustion that no perfect square ends in the digit 8.

[4]

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3.2 Prove by exhaustion that no perfect square ends in the digit 8. [4]

3.2 A square's last digit depends only on its root's last digit. Exhaust the ten endings:

$$0^2 \rightarrow 0, 1^2 \rightarrow 1, 2^2 \rightarrow 4, 3^2 \rightarrow 9, 4^2 \rightarrow 6, 5^2 \rightarrow 5, 6^2 \rightarrow 6, 7^2 \rightarrow 9, 8^2 \rightarrow 4, 9^2 \rightarrow 1$$

The possible endings are 0, 1, 4, 5, 6, 9 – never 8. ■

Local review guidance: complete case structure or a valid counterexample; algebra carried correctly; the conclusion states what was proved, with the case list justified. This is authored classroom guidance.