

1 • Pure Mathematics 1

Pearson Edexcel • International A-Level • Pure Mathematics

Pure Mathematics 1: the toolkit every later unit uses

Pure Mathematics 1 (P1) is the first compulsory unit of both International AS and International A Level Mathematics, and of International A Level Pure Mathematics. This reference covers every P1 topic in the Edexcel IAL specification (Issue 3, April 2019): algebra and functions, coordinate geometry, trigonometry, differentiation and integration.

- The P1 paper (WMA11/01) lasts **1 hour 30 minutes** with **75 marks**, and you must answer all questions. Recent papers have ten questions from 5 to 11 marks each.
- A **calculator** may be used, but not one with symbolic algebra, symbolic differentiation or integration, or retrievable stored formulae.
- The yellow booklet *Mathematical Formulae and Statistical Tables* is provided. The formulae in the “you must know” table below do **not** appear in that booklet, so memorise them.
- Give inexact answers to **3 significant figures** unless told otherwise. When a question says “you must show all stages of your working”, a calculator answer alone scores nothing.

Formulae you must know for P1 (from the specification’s assessment information):

Formula	Use
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	roots of $ax^2 + bx + c = 0$
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	sine rule
$a^2 = b^2 + c^2 - 2bc \cos A$	cosine rule
area = $\frac{1}{2}ab \sin C$	area of a triangle
arc length $s = r\theta$, sector area = $\frac{1}{2}r^2\theta$	θ in radians
$\frac{d}{dx}x^n = nx^{n-1}$	power rule for differentiation
$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$	power rule for integration

Algebra and functions

■ Indices and surds

The **laws of indices**

hold for all rational exponents:

$$a^m \times a^n = a^{m+n} \quad a^m \div a^n = a^{m-n} \quad (a^m)^n = a^{mn}$$

Fractional powers are roots: $a^{m/n} = \sqrt[n]{a^m}$, so $a^{1/2} = \sqrt{a}$ and $a^{1/3} = \sqrt[3]{a}$. A negative power means reciprocal: $a^{-1} = 1/a$. Questions usually ask you to write an answer **in the form** kx^n with k and n fully simplified constants.

Worked example – three index manipulations (adapted from Edexcel IAL WMA11 June 2025 Q1). Given $p = \frac{x^4}{16}$ and $q = \frac{40}{3x}$, write each expression in the form kx^n .

(a) $p^{\frac{1}{2}}$.

- Known: $p = x^4/16$, and a power $\frac{1}{2}$ means square root.

$$p^{\frac{1}{2}} = \sqrt{\frac{x^4}{16}} = \frac{\sqrt{x^4}}{\sqrt{16}} = \frac{x^2}{4}$$

- Check: squaring the answer, $(x^2/4)^2 = x^4/16 = p$.

(b) $(pq)^{-1}$.

- Known: multiply before inverting, using $x^4 \cdot x^{-1} = x^3$.

$$pq = \frac{x^4}{16} \times \frac{40}{3x} = \frac{40x^3}{48} = \frac{5x^3}{6}$$

$$(pq)^{-1} = \left(\frac{5x^3}{6}\right)^{-1} = \frac{6}{5}x^{-3}$$

- Check: $(pq) \times (pq)^{-1} = \frac{5x^3}{6} \times \frac{6}{5x^3} = 1$.

(c) $(pq)^2$.

$$(pq)^2 = \left(\frac{5x^3}{6}\right)^2 = \frac{25x^6}{36}$$

- Check: $5^2 = 25$, $6^2 = 36$, $(x^3)^2 = x^6$, and the constants are already fully simplified.

A **surd** is an exact root such as $\sqrt{2}$. Keep answers exact unless asked for decimals; examiners penalise rounded decimals where a surd is expected. To **rationalise the denominator**, multiply numerator and denominator by a term that removes the root:

Worked example – rationalising. Write $\frac{12}{1 + \sqrt{2}}$ with a rational denominator.

- Known: $(1 + \sqrt{2})(\sqrt{2} - 1) = \sqrt{2} - 1 + 2 - \sqrt{2} = 1$, so multiply by $\frac{\sqrt{2} - 1}{\sqrt{2} - 1}$.

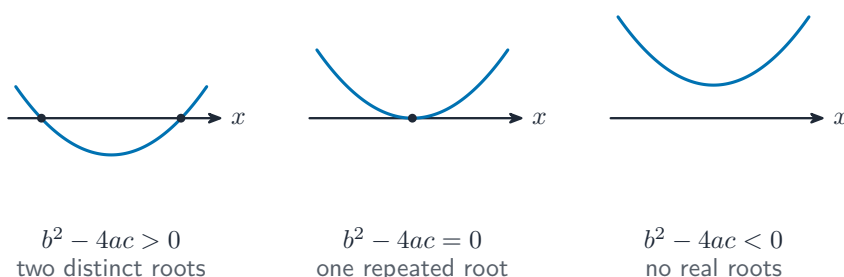
$$\frac{12}{1 + \sqrt{2}} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = \frac{12(\sqrt{2} - 1)}{(1 + \sqrt{2})(\sqrt{2} - 1)} = 12(\sqrt{2} - 1) = 12\sqrt{2} - 12$$

- Check by calculator: $12/(1 + 1.4142\dots) = 4.9706\dots$ and $12\sqrt{2} - 12 = 4.9706\dots$
✓

■ Quadratics: discriminant and completing the square

A **quadratic function** has the form $y = ax^2 + bx + c$; its graph is a parabola opening up ($a > 0$) or down ($a < 0$). The **discriminant** $\Delta = b^2 - 4ac$ counts the roots:

- $\Delta > 0$: two distinct real roots – the curve cuts the x -axis twice;
- $\Delta = 0$: one repeated root – the curve touches the x -axis;
- $\Delta < 0$: no real roots – the curve never meets the x -axis.



The discriminant counts intersections with the x -axis.

Completing the square rewrites $ax^2 + bx + c$ as $a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$, which gives the turning point $\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$ directly. Solve quadratics by factorisation, the formula, completing the square, or a calculator – but "show all stages" questions reject calculator-only working.

Worked example – a hidden quadratic (adapted from Edexcel IAL WMA11 June 2025 Q10). The equation $(k - 1)x^6 + 4x^3 + (k - 4) = 0$, where k is a constant.

(a) For $k = 4.5$, find the exact solutions.

- Known: with $k = 4.5$ the equation is $3.5x^6 + 4x^3 + 0.5 = 0$; multiply by 2 to clear decimals:

$$7x^6 + 8x^3 + 1 = 0$$

- Why substitute: $x^6 = (x^3)^2$, so this is a quadratic in x^3 . Substitute $z = x^3$:

$$7z^2 + 8z + 1 = 0 \implies (7z + 1)(z + 1) = 0 \implies z = -\frac{1}{7} \text{ or } z = -1$$

- Return to x : $x^3 = -\frac{1}{7}$ or $x^3 = -1$, so

$$x = -\sqrt[3]{\frac{1}{7}} = -\frac{\sqrt[3]{7}}{7} \quad \text{or} \quad x = -1$$

- Check: $z = -1$ gives $7 - 8 + 1 = 0$ ✓; $z = -\frac{1}{7}$ gives $7 \cdot \frac{1}{49} - \frac{8}{7} + 1 = \frac{1}{7} - \frac{8}{7} + 1 = 0$ ✓

(b) Find the set of values of k for which the equation has **no real roots**.

- Known: treat the equation as a quadratic in x^3 with $a = k - 1$, $b = 4$, $c = k - 4$. No real roots needs a negative discriminant:

$$b^2 - 4ac = 16 - 4(k - 1)(k - 4) = 16 - 4(k^2 - 5k + 4) = -4k^2 + 20k = 4k(5 - k)$$

$$4k(5 - k) < 0 \implies k < 0 \text{ or } k > 5$$

- Check: at $k = -1$: $4(-1)(6) = -24 < 0$ ✓; at $k = 6$: $4(6)(-1) = -24 < 0$ ✓; at $k = 3$: $4(3)(2) = 24 > 0$ ✓ (two real roots there).

■ Simultaneous equations and inequalities

Solve **simultaneous equations** analytically by **substitution** : rearrange the linear equation, substitute into the other, solve, then substitute back. A line meeting a curve gives a quadratic whose roots are the intersection x -coordinates – this is the “geometrical interpretation of algebraic solutions” the specification asks for.

Worked example – line meets cubic-squared curve (adapted from Edexcel IAL WMA11 June 2025 Q7). C_1 : $y = 0.2x^2(x - 4)$ and C_2 : $y = 1.2x(8 - x)$ intersect where

$$0.2x^2(x - 4) = 1.2x(8 - x)$$

- Expand both sides: $0.2x^3 - 0.8x^2 = 9.6x - 1.2x^2$.
- Collect terms: $0.2x^3 + 0.4x^2 - 9.6x = 0$, then divide by 0.2: $x^3 + 2x^2 - 48x = 0$.
- Factorise: $x(x^2 + 2x - 48) = x(x + 8)(x - 6) = 0$, so $x = 0, -8, 6$.
- Substitute back into C_2 : $(0, 0)$, $(-8, -153.6)$, $(6, 14.4)$.
- Check with C_1 : $0.2(-8)^2(-12) = 0.2 \cdot 64 \cdot (-12) = -153.6$ ✓ and $0.2(36)(2) = 14.4$ ✓

Inequalities follow the same algebra, but the answer is a range:

- Solve $ax + b > cx + d$ by collecting x (the inequality sign survives adding/subtracting, and survives dividing by a **positive** number; dividing by a negative flips the sign).
- Solve $px^2 + qx + r \leq 0$ by sketching: the solution is where the parabola is on or below the axis, between the roots.
- “Curve below line” inequalities such as $px^2 + qx + r < ax + b$ mean: where is the curve’s height less than the line’s height? Find intersections first.
- When **sketching** the solution region, use a **dotted line** for strict $<$ or $>$ boundaries and a **solid line** for $\leq$ or $\geq$.

Worked example. Solve $x^2 < 3x + 10$.

- Rearrange: $x^2 - 3x - 10 < 0$, and factorise: $(x - 5)(x + 2) < 0$.
- The parabola $y = (x - 5)(x + 2)$ opens upward, so it is below zero **between** its roots:

$$-2 < x < 5$$

- Check: $x = 0$ gives $0 < 10$ ✓; $x = 6$ gives $36 < 28$ × (correctly excluded).

■ Polynomials: expanding and factorising

Expand brackets and collect like terms; factorise by taking out common factors and by grouping. Cubics factorise fully once one root is spotted: $x^3 + 4x^2 + 3x = x(x^2 + 4x + 3) = x(x + 1)(x + 3)$. The notation $f(x)$ names the function, so $f(2)$ means "substitute $x = 2$ ".

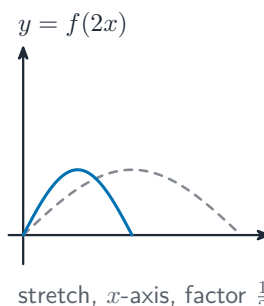
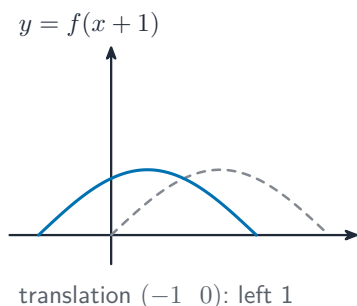
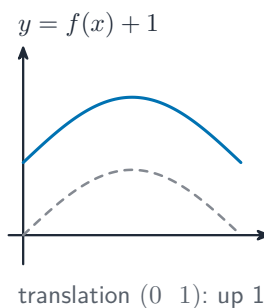
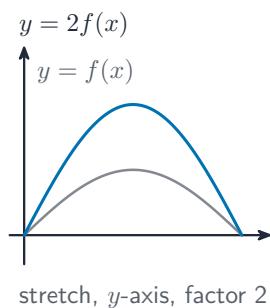
■ Sketching graphs and transformations

Sketch (do not plot) these families: quadratics and simple **cubics** such as $y = x^3 - 4x$; **reciprocal functions** $y = \frac{k}{x}$ and $y = \frac{k}{x^2}$ ($x \neq 0$), which have an **asymptote** – a line the curve approaches but never touches; and the trigonometric graphs below. Mark intercepts and turning points.

Intersections answer equations: the curves $y = f(x)$ and $y = g(x)$ meet where $f(x) = g(x)$.

One **transformation** at a time, applied to $y = f(x)$:

New equation	What it does to the graph
$y = af(x)$	stretch parallel to the y -axis, scale factor a
$y = f(x) + a$	translation by $\begin{pmatrix} 0 \\ a \end{pmatrix}$ (moves up by a)
$y = f(x + a)$	translation by $\begin{pmatrix} -a \\ 0 \end{pmatrix}$ (moves left by a)
$y = f(ax)$	stretch parallel to the x -axis, scale factor $\frac{1}{a}$



Name the direction, the type and the scale factor (or vector).

Worked example – describe two transformations (adapted from Edexcel IAL WMA11 June 2025 Q9).

- $y = \sqrt{2x}$ from $y = \sqrt{x}$: write $\sqrt{2x} = f(2x)$, so this is a **stretch parallel to the x-axis with scale factor $\frac{1}{2}$** .
- $y = 12 - \sqrt{x}$ from $y = -\sqrt{x}$: write $12 - \sqrt{x} = -\sqrt{x} + 12$, a **translation by $\begin{pmatrix} 0 \\ 12 \end{pmatrix}$** (up by 12).
- Check with a point: $y = \sqrt{x}$ passes (4, 2); after the first transformation the curve passes (2, 2) (halved x), and $y = \sqrt{2 \times 2} = 2$ ✓. After the second, $y = -\sqrt{4} + 12 = 10$, so (4, 10) lies on it ✓.

Coordinate geometry in the (x, y) plane

A line through (x_1, y_1) with gradient m has equation $y - y_1 = m(x - x_1)$; the form $ax + by + c = 0$ hides the gradient in $-\frac{a}{b}$. Two points give the gradient: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

- **Parallel** lines share their gradient: $m_1 = m_2$.
- **Perpendicular** lines have gradients multiplying to -1 : $m_1 m_2 = -1$.

Worked example – perpendicular through the origin (adapted from Edexcel IAL WMA11 June 2025 Q5). The line l_1 has equation $x - 2y + 25 = 0$; l_2 passes through the origin and is perpendicular to l_1 .

(a) Equation of l_2 .

- Known: rearrange l_1 : $2y = x + 25$, so $y = \frac{1}{2}x + \frac{25}{2}$ and $m_1 = \frac{1}{2}$.
- Why: perpendicular gradients multiply to -1 , so $m_2 = -1 \div \frac{1}{2} = -2$.

$$y - 0 = -2(x - 0) \implies y = -2x$$

(b) l_1 and l_2 intersect at P .

- Substitute $y = -2x$ into $x - 2y + 25 = 0$:

$$x - 2(-2x) + 25 = 0 \implies 5x + 25 = 0 \implies x = -5, \quad y = -2(-5) = 10$$

- $P = (-5, 10)$. Check in l_1 : $-5 - 20 + 25 = 0 \checkmark$

(c) Shortest distance from l_1 to the origin, as a simplified surd.

- The shortest distance is $|OP|$ along the perpendicular:

$$|OP| = \sqrt{(-5 - 0)^2 + (10 - 0)^2} = \sqrt{25 + 100} = \sqrt{125} = 5\sqrt{5}$$

- Check: $\sqrt{125} = \sqrt{25 \times 5} = 5\sqrt{5} \approx 11.18 \checkmark$

Trigonometry

■ The sine rule, cosine rule and area of a triangle

In triangle ABC with sides a, b, c opposite angles A, B, C :

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cos A \quad \text{area} = \frac{1}{2}ab \sin C$$

Use the sine rule for pairs (side, opposite angle); the cosine rule when no pair is opposite. The sine rule has an **ambiguous case** : $\sin \theta = \sin(180^\circ - \theta)$, so an angle and its supplement can both fit – check which the diagram allows.

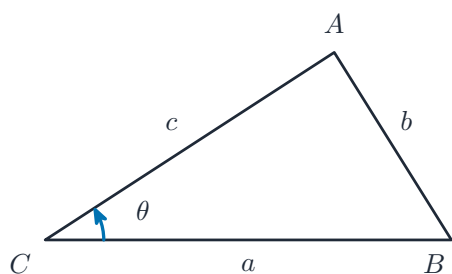
■ Radian measure, arc length and sector area

One **radian** is the angle subtended by an arc equal in length to the radius; $180^\circ = \pi$ radians. With θ in radians:

$$\text{arc length } s = r\theta \quad \text{sector area} = \frac{1}{2}r^2\theta$$

The **arc length** is $s = r\theta$ and the **sector area** is $\frac{1}{2}r^2\theta$.

A **major sector** is the larger share of the circle: angle $2\pi - \theta$ when the minor angle is θ .

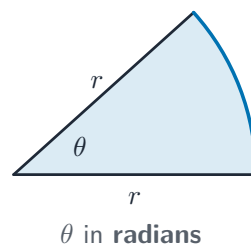


Sine rule pairs each side with its **opposite** angle:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{sector area} = \frac{1}{2}r^2\theta$$

$$s = r\theta$$



Quote the formula first – the method mark follows the formula.

Worked example – a helipad from a major sector and a triangle (adapted from Edexcel IAL WMA11 June 2025 Q6). A landing pad is the major sector AOB of a circle centre O joined to triangle AOC : $AO = OB = 15$ m, $BC = 2$ m, CBO is a straight line, and angle $ACO = 0.6$ radians.

(a) Show angle $COA = 1.847$ radians to 3 decimal places.

- Known: $OC = OB + BC = 15 + 2 = 17$ m; in triangle AOC , side $AO = 15$ is opposite the given angle 0.6 , and side $OC = 17$ is opposite angle CAO .
- Sine rule (quote it first):

$$\frac{\sin CAO}{17} = \frac{\sin 0.6}{15} \implies \sin CAO = \frac{17 \sin 0.6}{15} = 0.6399 \dots \implies CAO = 0.6944 \dots$$

$$COA = \pi - 0.6 - 0.6944 \dots = 1.8471 \dots = 1.847 \text{ (3 d.p.)}$$

- Check: angles sum to π : $0.6 + 0.6944 + 1.8471 \approx 3.1416 \checkmark$

(b) Total area of the pad.

- Major sector angle: $2\pi - 1.847 = 4.436$ radians.

$$\text{sector} = \frac{1}{2}r^2\theta = \frac{1}{2} \times 15^2 \times 4.436 = 499.1 \text{ m}^2 \text{ (awrt 499)}$$

$$\text{triangle} = \frac{1}{2}ab \sin C = \frac{1}{2} \times 15 \times 17 \times \sin 1.847 = 122.7 \text{ m}^2$$

$$\text{total} = 499.1 + 122.7 = 622 \text{ m}^2 \text{ (3 s.f.)}$$

- Check: $\sin 1.847 = \sin(\pi - 1.847) = \sin 1.2946 = 0.962 \checkmark$

(c) Perimeter of the pad.

- The boundary is the major arc AB plus sides AC and BC .

$$\text{arc} = r\theta = 15 \times 4.436 = 66.5 \text{ m}$$

- Cosine rule for AC (quote it first):

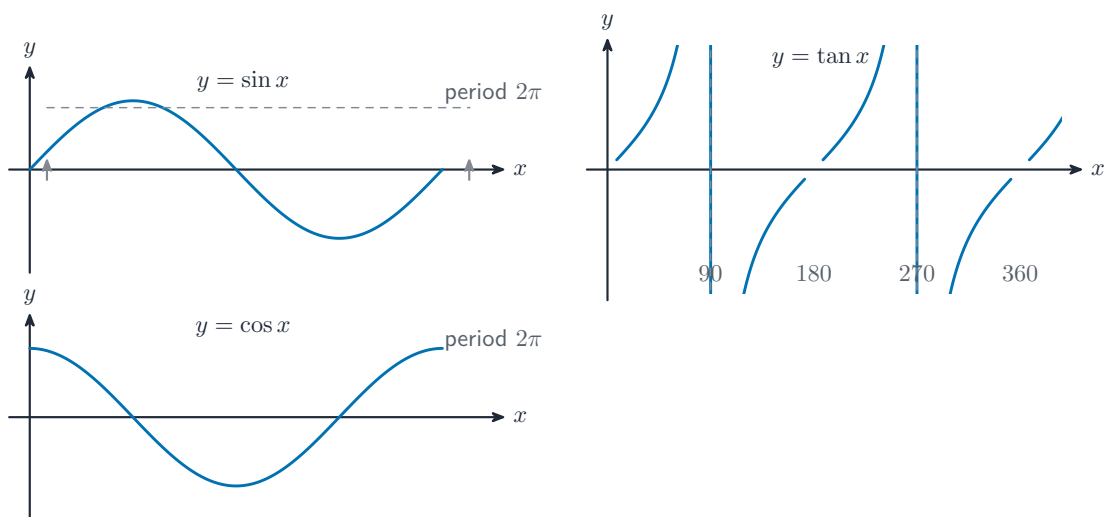
$$AC^2 = 15^2 + 17^2 - 2(15)(17) \cos 1.847 = 225 + 289 + 139.9 = 653.9 \implies AC = 25.6 \text{ m}$$

$$\text{perimeter} = 66.5 + 25.6 + 2 = 94.1 \text{ m (3 s.f.)}$$

- Check: $\cos 1.847 < 0$ (the angle is obtuse), so the $-2bc \cos A$ term adds $\checkmark$

■ Sine, cosine and tangent graphs

$y = \sin x$ and $y = \cos x$ repeat every 2π (360°); $y = \tan x$ repeats every π (180°) with asymptotes at $x = 90^\circ + 180^\circ n$. Know the symmetries: $\cos$ is even, $\sin(180^\circ - x) = \sin x$, and $\tan x = \sin x / \cos x$. Curves such as $y = 3 \sin x$, $y = \sin 2x$ and $y = \sin(x + \frac{\pi}{6})$ follow from the transformation rules above. Solve equations **graphically** by counting intersections in a given interval.



Count periods first, then solutions per period.

Worked example – counting solutions (adapted from Edexcel IAL WMA11 June 2025 Q4). For $y = 4 \cos x$ with x in degrees: how many solutions does $5 + 4 \cos x = 1$ have for $-720^\circ < x \leq 720^\circ$?

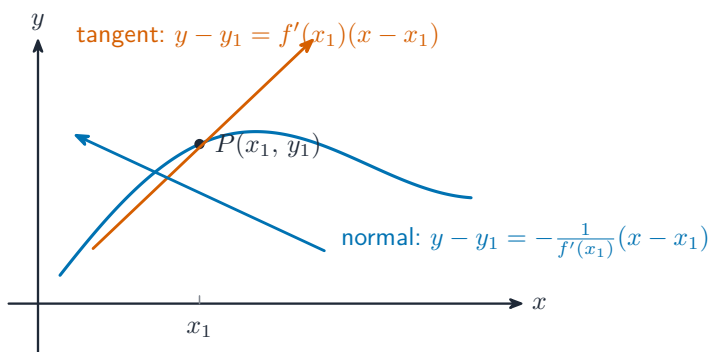
- Rearrange: $4 \cos x = -4$, so $\cos x = -1$, giving $x = 180^\circ + 360^\circ n$.
- In $-720^\circ < x \leq 720^\circ$: $x = -540^\circ, -180^\circ, 180^\circ, 540^\circ$ – **4 solutions**.
- Check: $\cos(\pm 180^\circ) = -1 \checkmark$, and $5 + 4(-1) = 1 \checkmark$. The interval is half-open, so $x = -720^\circ$ (where $\cos = 1$) is excluded correctly.

Differentiation

The **derivative** $f'(x) = \frac{dy}{dx}$ is the **gradient of the tangent** to $y = f(x)$ at a point – the limit of the chord gradients as the two points merge. It is the **rate of change** of y with respect to x . The **second derivative** $f''(x)$ differentiates again. (The chain rule is **not** required in P1.)

Differentiate term by term with the power rule, expanding products and splitting fractions first:

$$y = (2x + 5)(x - 1) = 2x^2 + 3x - 5 \implies \frac{dy}{dx} = 4x + 3 \quad y = \frac{3x^2 + 5}{x^2} = 3 + 5x^{-2} \implies \frac{dy}{dx} = -10x^{-3}$$



$$\text{Tangent gradient} = f'(x_1); \text{normal gradient} = -1/f'(x_1).$$

Worked example – gradient condition on a curve (adapted from Edexcel IAL WMA11 June 2025 Q2). $C: y = 2x^{5/2} - 4x + 3$; the point P on C has x -coordinate 2^k and the gradient at P is 16. Find k .

- Differentiate (powers drop by one): $\frac{dy}{dx} = 2 \cdot \frac{5}{2}x^{3/2} - 4 = 5x^{3/2} - 4$.
- Set the gradient to 16 at $x = 2^k$:

$$5(2^k)^{3/2} - 4 = 16 \implies (2^k)^{3/2} = 4 = 2^2 \implies 2^{3k/2} = 2^2 \implies \frac{3k}{2} = 2 \implies k = \frac{4}{3}$$

- Check: $2^{4/3} = 2.5198\dots$ and $5(2.5198)^{3/2} - 4 = 5(4.0) - 4 = 16 \checkmark$

Worked example – tangent, normal work and second derivative (adapted from Edexcel IAL WMA11 June 2025 Q8). A curve has $f'(x) = 3\sqrt{x} + kx^2$ and passes through $P(4, 12)$; the tangent at P is $y = 10x + c$.

(a) Show $k = \frac{1}{4}$ and find c .

- Known: the tangent's gradient equals $f'(4)$:

$$f'(4) = 3\sqrt{4} + 16k = 6 + 16k = 10 \implies k = \frac{1}{4}$$

- The tangent passes through $(4, 12)$: $12 = 10(4) + c \implies c = -28$.
- Check: $f'(4) = 6 + 16 \cdot \frac{1}{4} = 10 \checkmark$ and $10(4) - 28 = 12 \checkmark$

(b) $f''(x)$ at P .

$$f''(x) = \frac{3}{2}x^{-1/2} + 2kx = \frac{3}{2\sqrt{x}} + \frac{x}{2} \implies f''(4) = \frac{3}{4} + 2 = \frac{11}{4}$$

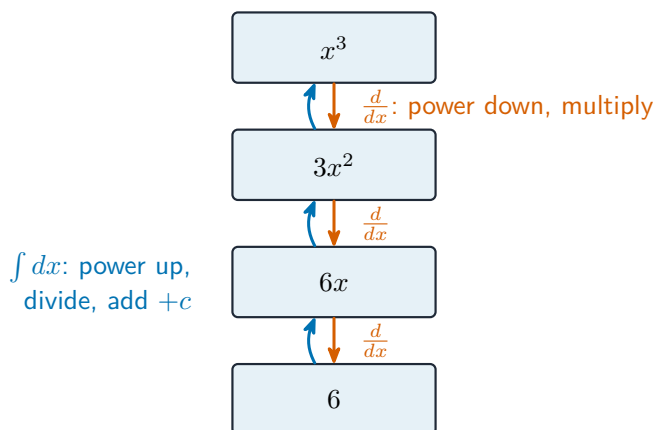
- Check: $\frac{3}{2\sqrt{4}} = \frac{3}{4} \checkmark$

Integration

Integration reverses differentiation: if $\frac{dy}{dx} = x^n$ ($n \neq -1$), then

$$y = \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

The **constant of integration** c is required – every vertical shift shares the same derivative. Raise each power by one, divide by the new power, and integrate term by term after expanding. Given $f'(x)$ **and one point** on the curve, find c and recover $y = f(x)$ exactly.



Each integral climbs one rung and picks up an arbitrary constant: $\int 6 dx = 6x + c$, not just $6x$.

Differentiation slides down; integration climbs back up, plus c .

Worked example – expand before integrating (adapted from Edexcel IAL WMA11 June 2025 Q3). Find $\int \frac{(x+3)^2}{3\sqrt{x}} dx$.

- Expand the numerator: $(x+3)^2 = x^2 + 6x + 9$; the denominator is $3x^{1/2}$.
- Divide term by term (powers of x subtract):

$$\frac{x^2}{3x^{1/2}} = \frac{1}{3}x^{3/2} \quad \frac{6x}{3x^{1/2}} = 2x^{1/2} \quad \frac{9}{3x^{1/2}} = 3x^{-1/2}$$

- Integrate each term (power up one, divide by the new power):

$$\begin{aligned}\int \left(\frac{1}{3}x^{3/2} + 2x^{1/2} + 3x^{-1/2}\right) dx &= \frac{1}{3} \cdot \frac{2}{5}x^{5/2} + 2 \cdot \frac{2}{3}x^{3/2} + 3 \cdot 2x^{1/2} + c \\ &= \frac{2}{15}x^{5/2} + \frac{4}{3}x^{3/2} + 6\sqrt{x} + c\end{aligned}$$

- Check by differentiating: $\frac{2}{15} \cdot \frac{5}{2}x^{3/2} = \frac{1}{3}x^{3/2} \checkmark$, $\frac{4}{3} \cdot \frac{3}{2} = 2 \checkmark$, $6 \cdot \frac{1}{2} = 3 \checkmark$ – the integrand returns exactly.

Worked example – recover $f(x)$ from $f'(x)$ and a point (adapted from Edexcel IAL WMA11 June 2025 Q8, continued). With $k = \frac{1}{4}$, so $f'(x) = 3x^{1/2} + \frac{1}{4}x^2$, and $P(4, 12)$ on the curve:

$$f(x) = \int \left(3x^{1/2} + \frac{1}{4}x^2\right) dx = 3 \cdot \frac{2}{3}x^{3/2} + \frac{1}{4} \cdot \frac{1}{3}x^3 + d = 2x^{3/2} + \frac{x^3}{12} + d$$

- Substitute P : $2(4)^{3/2} + \frac{64}{12} + d = 12$, and $4^{3/2} = 8$:

$$16 + \frac{16}{3} + d = 12 \implies d = -\frac{28}{3} \implies f(x) = 2x^{3/2} + \frac{x^3}{12} - \frac{28}{3}$$

- Check: $f(4) = 16 + \frac{16}{3} - \frac{28}{3} = 16 - 4 = 12 \checkmark$, and differentiating the answer gives $f'(x) = 3x^{1/2} + \frac{x^2}{4}$, so $f'(4) = 6 + 4 = 10 \checkmark$ (the tangent gradient from before).

Checklist before you call this unit done

- Manipulate $a^{m/n}$, negative powers and surds exactly, and rationalise denominators.
- Read $b^2 - 4ac$ three ways, complete the square, and solve hidden quadratics by substitution.
- Solve linear–quadratic simultaneous equations by substitution and interpret them as intersections.
- Solve and sketch linear and quadratic inequalities with correct dotted/solid boundaries.
- Sketch cubics, reciprocals (with asymptotes) and transformed curves, naming direction, type and scale factor of each single transformation.
- Write line equations in both forms; use $m_1m_2 = -1$ for perpendiculars.
- Quote sine rule, cosine rule and $\frac{1}{2}ab \sin C$; handle the ambiguous case and major sectors in radians.
- Count trigonometric-graph solutions over an interval before listing them.
- Differentiate and integrate x^n term by term, after expanding; always add $+c$.
- Turn $f'(x)$ plus one point into $f(x)$, and check by substituting the point back.