7.3 Differentiating hyperbolic and inverse functions

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS inverse function • chain rule

Differentiating hyperbolic and inverse functions

Objective

Differentiate hyperbolic expressions by the chain, product and quotient rules, and inverse trigonometric/hyperbolic functions by implicit inversion, respecting domains (FP3 3.1–3.2).

1 Key knowledge and worked examples

Derivatives. $\frac{d}{dx}\sinh x = \cosh x$, $\frac{d}{dx}\cosh x = \sinh x$, $\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$; $\frac{d}{dx}\operatorname{arsinh} x = \frac{1}{\sqrt{1+x^2}}$, $\frac{d}{dx}\operatorname{arcosh} x = \frac{1}{\sqrt{x^2-1}}$, $\frac{d}{dx}\operatorname{artanh} x = \frac{1}{1-x^2}$, $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$. Where a domain like $x > 0$ is given, use it to resolve $\sqrt{x^2} = x$.

Worked example – inverse derivatives on a restricted domain (WFM03 June 2025 Q4). For $y = \operatorname{arsinh} x + \operatorname{arsinh}\frac{1}{x}$, $x > 0$: $\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}} + \frac{-1/x^2}{\sqrt{1+1/x^2}} = \frac{1}{\sqrt{1+x^2}} - \frac{1}{x\sqrt{1+x^2}} = \frac{x-1}{x\sqrt{1+x^2}}$. The stationary point is $x = 1$, where $y = 2\operatorname{arsinh} 1 = 2\ln(1+\sqrt{2})$.

2 Practice

2.1 Differentiate $x\sinh 2x$. [4]

2.2 Differentiate $\cosh(3x^2 + 1)$. [3]

2.3 Differentiate $x^2 \operatorname{artanh} x$. [3]

2.4 Show that $\dfrac{d}{dx}(e^x \cosh x) = e^{2x}$. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WFM03 June 2025 Q4.)* $y = \operatorname{arsinh} x + \operatorname{arsinh} \dfrac{1}{x}$, $x > 0$. (a) Show that $\dfrac{dy}{dx} = \dfrac{x-1}{x\sqrt{1+x^2}}$. (b) Hence determine the exact value of y for which $\dfrac{dy}{dx} = 0$, giving your answer as a simplified natural logarithm. [6]

3.2 *(Original.)* (a) Differentiate $\operatorname{arcosh} 2x$. (b) Find the equation of the tangent to $y = \operatorname{arcosh} 2x$ at the point where $x = 1$, giving exact values. [6]