7.2 The ellipse and the hyperbola

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS ellipse • hyperbola • eccentricity • directrix

The ellipse and the hyperbola

Objective

Use the standard equations and parametric forms of the ellipse and hyperbola, their eccentricity/focus/directrix relations, and tangent conditions (FP3 2.1–2.4).

1 Key knowledge and worked examples

Standard facts. Ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ ($x=a\cos t$, $y=b\sin t$): $b^2=a^2(1-e^2)$, foci $(\pm ae,0)$, directrices $x=\pm\dfrac{a}{e}$. Hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ ($x=a\sec t$, $y=b\tan t$; right branch $x=a\cosh t$, $y=b\sinh t$): $b^2=a^2(e^2-1)$. Tangent condition for $y=mx+c$: $c^2=a^2m^2+b^2$ (ellipse) or $c^2=a^2m^2-b^2$ (hyperbola). Focus property: $PS=e\cdot PN$ at every point.

Worked example – eccentricity, tangent, intercepts (WFM03 June 2025 Q9, condensed). For $\dfrac{x^2}{64}-\dfrac{y^2}{49}=1$: $49=64(e^2-1)$ gives $e=\frac{\sqrt{113}}{8}$, foci $(\pm\sqrt{113},0)$. At $P(8\sec\theta,7\tan\theta)$, $\dfrac{dy}{dx}=\dfrac{7\sec\theta}{8\tan\theta}$ and the tangent is $\frac{x}{8}\sec\theta-\frac{y}{7}\tan\theta=\sec^2\theta-\tan^2\theta=1$, meeting the y-axis at $Q=(0,-7\cot\theta)$.

2 Practice

2.1 For the ellipse $\dfrac{x^2}{25}+\dfrac{y^2}{9}=1$, find the eccentricity, the coordinates of the foci, and the equations of the directrices. [3]

2.2 For the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$, find the eccentricity, the foci and the directrices. [3]

2.3 Find the equations of the tangents to $\dfrac{x^2}{4} + \dfrac{y^2}{3} = 1$ with gradient $\frac{1}{2}$. [4]

2.4 Show that the point $(4\sec\theta, 3\tan\theta)$ lies on $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$ for all values of θ. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WFM03 June 2025 Q9a–c, condensed.)* The hyperbola H has equation $\dfrac{x^2}{64} - \dfrac{y^2}{49} = 1$ and eccentricity e. (a) Show that $e = \dfrac{\sqrt{113}}{8}$. (b) Use calculus to show that the tangent to H at $P(8\sec\theta, 7\tan\theta)$ has equation $\frac{x}{8}\sec\theta - \frac{y}{7}\tan\theta = 1$. (c) Write down the coordinates of the point Q where this tangent meets the y-axis. [7]

3.2 *(Original.)* The hyperbola K has equation $\dfrac{x^2}{49} - \dfrac{y^2}{32} = 1$. (a) Find its eccentricity and the coordinates of its foci. (b) Verify that the point $(7\sec\theta, 4\sqrt{2}\tan\theta)$ lies on K for all θ. [5]