4.1 Proof by contradiction and partial fractions

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

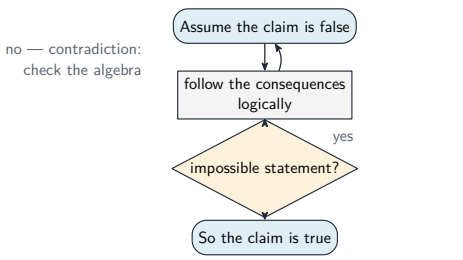
KEY WORDS proof by contradiction • partial fractions • repeated factor • improper fraction

Objective

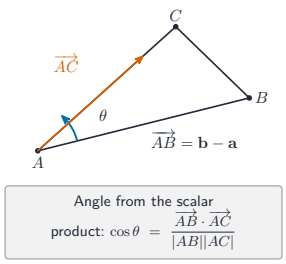
Prove statements by contradiction ($\sqrt{2}$ irrational, infinitely many primes, unfamiliar claims) and decompose rational functions into partial fractions, including repeated linear factors and improper fractions. Original teaching questions for specification items P4.1.1 and P4.2.1; the exam-style part adapts reviewed WMA14 June 2025 demand.

1 Key knowledge and worked examples

Proof by contradiction : assume the negation of the claim, argue logically until an impossibility, conclude the claim. **Partial fractions** split a rational function into sums over its denominator's factors; a repeated factor $(ax+b)^2$ contributes $\dfrac{B}{ax+b}+\dfrac{C}{(ax+b)^2}$; improper fractions divide first.



$\sqrt{2}$ irrational and the infinity of primes both run this loop



Name the assumption and the contradiction explicitly.

Worked example – contradiction. Prove there is no smallest positive rational number.

- Suppose q IS the smallest positive rational. Then $\frac{q}{2}$ is positive, rational, and $\frac{q}{2} < q$ – a smaller one exists, contradicting the assumption. ■

Worked example – repeated factor. Decompose $\dfrac{3x^2+7x+6}{x(x+1)^2}$.

$$\frac{3x^2+7x+6}{x(x+1)^2} = \frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2} \implies 3x^2+7x+6 = A(x+1)^2+Bx(x+1)+Cx$$

- $x=0$: $A=6$. $x=-1$: $2=-C \Rightarrow C=-2$. x^2 coefficients: $3=A+B \Rightarrow B=-3$.

$$\frac{6}{x}-\frac{3}{x+1}-\frac{2}{(x+1)^2}$$

- Check at $x=1$: LHS $=16/4=4$; RHS $=6-\frac{3}{2}-\frac{1}{2}=4$ ✓.

2 Practice

2.1 Prove by contradiction that $\sqrt{3}$ is irrational. [4]

2.2 Decompose $\dfrac{7x+10}{(x+1)(x+3)}$. [3]

2.3 Decompose $\dfrac{5x+7}{(x-1)^2}$. [3]

2.4 Write $\dfrac{x^2+2x+5}{x^2+x}$ in the form $1+\dfrac{A}{x}+\dfrac{B}{x+1}$. [4]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA14 June 2025 Q10.)* Use proof by contradiction to show that $|\sin x+\cos x|<1$ for all angles x with $90^\circ<x<180^\circ$. [4]

3.2 Decompose $\dfrac{2x^2+5x+4}{(x+2)(x+1)^2}$ and verify your decomposition at $x=1$. [5]