2.9 Definite integrals and areas

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS definite integral • area under the curve

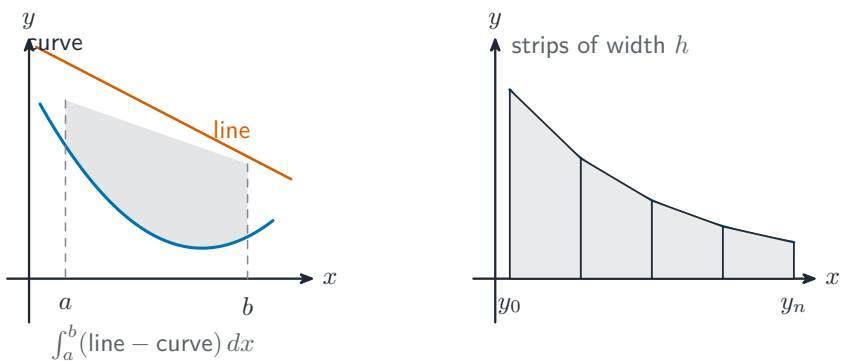
Objective

Evaluate definite integrals with the fundamental theorem, interpret them as areas, and find the area of regions bounded by a curve and a line or by two curves. Original teaching questions for specification items P2.8.1 and P2.8.2; the exam-style part adapts reviewed WMA12 June 2025 demand.

1 Key knowledge and worked examples

$$\int_a^b f(x)\,dx = F(b) - F(a) \quad \text{where } F'(x) = f(x)$$

For $y \geq 0$ this is the **area under the curve** . For a region between a curve and a line (or two curves), sketch first, find the intersections, and integrate **(upper − lower)**:



Integrate (line − curve) when the line is on top.

Worked example – region between curve and line. Find the area enclosed by $y = 4 - x^2$ and $y = x + 2$.

- Intersections: $4 - x^2 = x + 2 \implies x^2 + x - 2 = 0 \implies (x+2)(x-1) = 0$, so $x = -2$ and $x = 1$.

- Which is on top? At $x = 0$: curve gives 4, line gives 2 – the curve is above.

$$\text{Area} = \int_{-2}^{1} \left[(4 - x^2) - (x + 2)\right] dx = \int_{-2}^{1} (-x^2 - x + 2)\, dx = \left[-\frac{x^3}{3} - \frac{x^2}{2} + 2x\right]_{-2}^{1}$$

- At 1: $-\frac{1}{3} - \frac{1}{2} + 2 = \frac{7}{6}$. At -2: $\frac{8}{3} - 2 - 4 = -\frac{10}{3}$.

$$\text{Area} = \frac{7}{6} - \left(-\frac{10}{3}\right) = \frac{7}{6} + \frac{20}{6} = \frac{27}{6} = 4.5$$

- Check: the region is roughly a parabolic cap about 3 wide and up to 2 tall – 4.5 is plausible ✓.

2 Practice

2.1 Evaluate $\displaystyle\int_1^3 2x\, dx$. [2]

2.2 Evaluate $\displaystyle\int_0^4 \sqrt{x}\, dx$. [3]

2.3 Evaluate $\displaystyle\int_0^{\pi/2} \cos x\, dx$. [2]

2.4 Find the area under $y = x^2$ from $x = 0$ to $x = 3$. [2]

2.5 Find the area between $y = \sqrt{x}$, $y = x$ and the lines $x = 0$, $x = 1$. [4]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA12 June 2025 Q3.)* The curve C has equation $y = 2x + \frac{3}{x} - 4$ $(x > 0)$, and the points $P(1, 1)$ and $Q(3, 3)$ lie on the line $y = x$. Find the area of the region bounded by C and the line segment PQ, giving your answer in exact form. [6]

3.2 Find the area of the region bounded by the curve $y = x^2 - 4x + 5$ and the line $y = 5$. [5]