

## 2.3 Coordinate geometry of the circle

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 25 marks

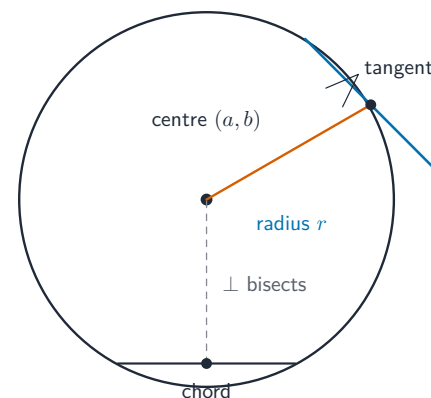
**KEY WORDS** chord • tangent • diameter • midpoint

### Objective

Use the circle equation  $(x - a)^2 + (y - b)^2 = r^2$  in both directions, complete the square on the expanded form, and apply the three examined circle properties. Original teaching questions for specification item P2.3.1; the exam-style part adapts reviewed WMA12 June 2025 demand.

### 1 Key knowledge and worked examples

- Centre  $(a, b)$ , radius  $r$ :  $(x - a)^2 + (y - b)^2 = r^2$ .
- Expanded form  $x^2 + y^2 + fx + gy + c = 0$ : complete both squares – centre  $(-\frac{f}{2}, -\frac{g}{2})$ ,  $r^2 = \frac{f^2 + g^2}{4} - c$ .
- A **diameter** gives the centre as its **midpoint** and  $r^2$  as a quarter of the diameter length squared.
- Properties: the angle in a semicircle is a right angle; the perpendicular from the centre to a **chord** bisects it; a **tangent** is perpendicular to the radius at the contact point.



Tangent  $\perp$  radius at the contact point; the perpendicular from the centre to a chord **bisects** it; the angle in a semicircle is 90.

*Tangent  $\perp$  radius; centre-to-chord perpendicular bisects the chord.*

**Worked example – expanded form.** Find the centre and radius of  $x^2 + y^2 - 6x + 4y - 12 = 0$ .

- Group and complete the square in  $x$  and in  $y$ :

$$(x^2 - 6x) + (y^2 + 4y) = 12 \implies (x - 3)^2 - 9 + (y + 2)^2 - 4 = 12$$

$$(x - 3)^2 + (y + 2)^2 = 25$$

- Centre  $(3, -2)$ , radius 5. Check with the point  $r$  right of the centre,  $(8, -2)$ :  $64 + 4 - 48 - 8 - 12 = 0$  ✓, so it lies on the circle.

### 2 Practice

**2.1** Find the centre and radius of  $x^2 + y^2 + 8x - 2y + 8 = 0$ . [3]

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**2.2** Write the equation of the circle with centre  $(2, -5)$  and radius  $\sqrt{13}$ . [2]

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**2.3** Determine whether  $(5, 2)$  lies on the circle  $(x - 1)^2 + (y - 3)^2 = 17$ . [2]

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**2.4** The points  $(-3, 1)$  and  $(5, 7)$  are the ends of a diameter. Find the circle's equation. [4]

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**2.5** The circle  $x^2 + y^2 = 10$  passes through  $(3, 1)$ . Find the equation of the tangent at that point. [4]

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### 3 Exam-style questions

**3.1** (*Adapted from Edexcel IAL WMA12 June 2025 Q2.*) The line joining  $(-1, -3)$  and  $(7, 1)$  is the diameter of circle  $C$ .

- (a) Find an equation for  $C$ .
- (b) Hence find the exact coordinates of the point on  $C$  that is nearest the  $x$ -axis. [7]

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**3.2** The circle  $(x - 4)^2 + (y - 2)^2 = 25$  meets the line  $x = 0$  at  $A(0, 5)$  and  $B(0, -1)$ . Verify that the perpendicular from the centre to the chord  $AB$  bisects  $AB$ . [3]

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