

## 2.10 The trapezium rule

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

**Total: 22 marks**

**KEY WORDS** trapezium rule • underestimate • overestimate • strip

### Objective

Approximate areas with the trapezium rule from a table of values, understand how strip width and curvature control the error, and scale known integral estimates. Original teaching questions for specification item P2.8.3; the exam-style part adapts reviewed WMA12 June 2025 demand.

### 1 Key knowledge and worked examples

With  $n$  strips of width  $h$  between  $x_0$  and  $x_n$ , and heights  $y_0, y_1, \dots, y_n$ :

$$\int_a^b y \, dx \approx \frac{h}{2} [y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})]$$

- More strips (smaller  $h$ ) improve the estimate.
- A curve bending upwards (**convex**, like  $x^2$ ) leaves the straight strips **above** it: **overestimate**. A curve bending downwards (**concave**, like  $\sqrt{x}$ ) leaves them **below**: **underestimate**.

**Worked example.** Estimate  $\int_0^2 \sqrt{4-x^2} \, dx$  with 4 strips.

- $h = 0.5$ ; heights at  $x = 0, 0.5, 1, 1.5, 2$ : 2, 1.9365, 1.7321, 1.3229, 0.

$$\int_0^2 \sqrt{4-x^2} \, dx \approx \frac{0.5}{2} [2 + 0 + 2(1.9365 + 1.7321 + 1.3229)] = 0.25 \times 11.983 = 3.00$$

- The curve is a quarter circle of radius 2, whose true area is  $\frac{1}{4}\pi(2)^2 = \pi \approx 3.14$ : an **underestimate**, because the quarter circle is concave (bends downwards), so every chord lies below the arc.

## 2 Practice

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**2.1** Estimate  $\int_0^4 x^2 dx$  with 4 strips. [3]

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**2.2** Estimate  $\int_1^3 \frac{1}{x} dx$  with 2 strips, and state whether your answer is an over- or under-estimate. [3]

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**2.3** For a curve that is convex (bends upwards), state whether the trapezium rule over- or underestimates the true area, and why. [2]

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**2.4** Estimate  $\int_0^1 \sqrt{x} dx$  with 4 strips. [3]

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**2.5** State two ways to improve a trapezium-rule estimate. [2]

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### 3 Exam-style questions

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**3.1** (*Adapted from Edexcel IAL WMA12 June 2025 Q5.*) The trapezium rule with five  $y$ -values and strip width  $h = 1.5$  gives the estimate

$$\int_1^5 f(x) dx \approx 12.4$$

Using this result and making your method clear, estimate

(a)  $\int_1^5 \frac{f(x)}{2} dx$

(b)  $\int_1^5 \left( \frac{f(x)}{4} + 3x \right) dx$  [4]

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**3.2** (*Structure adapted from the same paper.*) Use the trapezium rule with 8 strips to estimate  $\int_0^4 \sqrt{x} dx$ , and compare your answer with the exact value  $\frac{16}{3}$ . [5]

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