2.6 Exponentials and logarithms

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS logarithm • laws of logarithms • domain • asymptote

Objective

Use the graph of $y = a^x$, apply the laws of logarithms to combine and split log expressions, solve equations of the form $a^x = b$, and check every candidate against the domain. Original teaching questions for specification items P2.5.1 to P2.5.3; the exam-style part adapts reviewed WMA12 June 2025 demand.

1 Key knowledge and worked examples

$y = a^x$ $(a > 0,\ a \neq 1)$ passes through $(0, 1)$; for $a > 1$ it increases without bound, with the x-axis as an **asymptote**. The **logarithm** inverts it: $\log_a x = y \iff a^y = x$. The three laws:

$$\log_a x + \log_a y \equiv \log_a(xy) \qquad \log_a x - \log_a y \equiv \log_a \frac{x}{y} \qquad k \log_a x \equiv \log_a(x^k)$$

Also know $\log_a a = 1$ and $\log_a \frac{1}{x} = -\log_a x$. To solve $a^x = b$, take logs: $x = \log_a b = \dfrac{\log b}{\log a}$ (change of base). **Logs need positive arguments** – always test each solution in the original equation.

Worked example. Solve $\log_3 x + \log_3(x - 2) = 1$.

- Combine the left side into one log:

$$\log_3\{x(x-2)\} = 1 = \log_3 3 \implies x^2 - 2x = 3 \implies x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0 \implies x = 3 \text{ or } x = -1$$

- Domain check: $\log_3(x - 2)$ needs $x > 2$, so $x = -1$ is rejected. $x = 3$.
- Check: $\log_3 3 + \log_3 1 = 1 + 0 = 1$ ✓.

2 Practice

2.1 Solve $5^x = 125$. [2]

2.2 Solve $4^{x-1} = 8$ by writing both sides as powers of 2. [3]

2.3 Write $2\log_{10} 3 + \log_{10} 8 - \log_{10} 6$ as a single logarithm. [3]

2.4 Solve $\log_2(x + 6) - \log_2 x = 1$. [3]

2.5 Describe the key features of the graph of $y = 2^x$. [2]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA12 June 2025 Q6(i).)* Solve $2\log_2(x-4) = \log_2(x+8)$. [4]

3.2 *(Adapted from the same question.)* The curves $y = 3^{2x}$ and $y = 4 \times 3^x$ meet at the point P. Find the exact coordinates of P, giving your answer in the form $(\log_3 a,\ b)$ where a and b are integers. [5]

Solutions

2.1 $5^x = 5^3$, so $x = 3$.

2.2 $4^{x-1} = 2^{2x-2}$ and $8 = 2^3$: $2x - 2 = 3$, so $x = \frac{5}{2}$.

2.3 $\log_{10}\left(\dfrac{3^2 \times 8}{6}\right) = \log_{10} 12$.

2.4 $\log_2 \dfrac{x+6}{x} = 1 \implies \dfrac{x+6}{x} = 2 \implies x + 6 = 2x \implies x = 6$. Domain: both $x + 6$ and x positive ✓. Check: $\log_2 12 - \log_2 6 = \log_2 2 = 1$ ✓.

2.5 Passes through $(0, 1)$; increases for all x; the x-axis ($y = 0$) is an asymptote as $x \to -\infty$.

3.1 $2\log_2(x-4) = \log_2(x-4)^2 = \log_2(x+8)$, so

$$(x-4)^2 = x + 8 \implies x^2 - 8x + 16 = x + 8 \implies x^2 - 9x + 8 = 0$$

$$(x-1)(x-8) = 0 \implies x = 1 \text{ or } x = 8$$

Domain: $x - 4 > 0$ needs $x > 4$, so $x = 1$ is rejected and $x = 8$. Check: $2\log_2 4 = 4$ and $\log_2 16 = 4$ ✓.

3.2 Set the expressions equal and substitute $u = 3^x$:

$$3^{2x} = 4 \times 3^x \implies u^2 = 4u \implies u = 4 \ (u \neq 0) \implies 3^x = 4 \implies x = \log_3 4$$

$$y = 3^{2x} = (3^x)^2 = 4^2 = 16$$

$P = (\log_3 4,\ 16)$: $a = 4$, $b = 16$. Check on the other curve: $4 \times 3^{\log_3 4} = 4 \times 4 = 16$ ✓.

Local review guidance: correct log law applied; quadratic solved exactly; domain rejection stated and the surviving root checked. This is authored classroom guidance.