

2.3 Coordinate geometry of the circle

Name: _____ Class: _____ Date: _____

Total: 25 marks

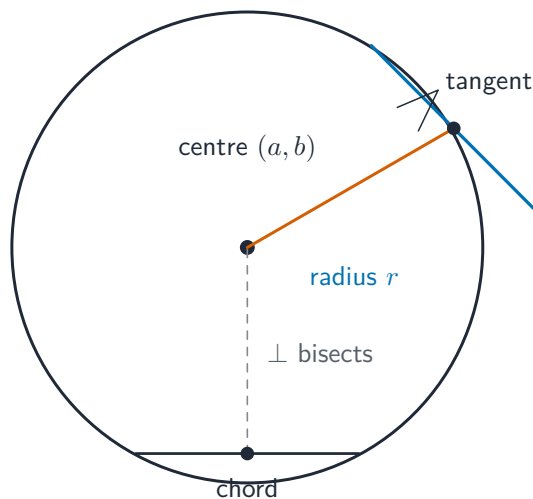
KEY WORDS chord • tangent • diameter • midpoint

Objective

Use the circle equation $(x - a)^2 + (y - b)^2 = r^2$ in both directions, complete the square on the expanded form, and apply the three examined circle properties. Original teaching questions for specification item P2.3.1; the exam-style part adapts reviewed WMA12 June 2025 demand.

1 Key knowledge and worked examples

- Centre (a, b) , radius r : $(x - a)^2 + (y - b)^2 = r^2$.
- Expanded form $x^2 + y^2 + fx + gy + c = 0$: complete both squares – centre $(-\frac{f}{2}, -\frac{g}{2})$, $r^2 = \frac{f^2 + g^2}{4} - c$.
- A **diameter** gives the centre as its **midpoint** and r^2 as a quarter of the diameter length squared.
- Properties: the angle in a semicircle is a right angle; the perpendicular from the centre to a **chord** bisects it; a **tangent** is perpendicular to the radius at the contact point.



Tangent $\perp$ radius at the contact point; the perpendicular from the centre to a chord **bisects** it; the angle in a semicircle is 90.

Tangent $\perp$ radius; centre-to-chord perpendicular bisects the chord.

Worked example – expanded form. Find the centre and radius of $x^2 + y^2 - 6x + 4y - 12 = 0$.

- Group and complete the square in x and in y :

$$(x^2 - 6x) + (y^2 + 4y) = 12 \implies (x - 3)^2 - 9 + (y + 2)^2 - 4 = 12$$

$$(x - 3)^2 + (y + 2)^2 = 25$$

- Centre $(3, -2)$, radius 5. Check with the point r right of the centre, $(8, -2)$: $64 + 4 - 48 - 8 - 12 = 0$ ✓, so it lies on the circle.

2 Practice

2.1 Find the centre and radius of $x^2 + y^2 + 8x - 2y + 8 = 0$.

[3]

2.2 Write the equation of the circle with centre $(2, -5)$ and radius $\sqrt{13}$.

[2]

2.3 Determine whether $(5, 2)$ lies on the circle $(x - 1)^2 + (y - 3)^2 = 17$. [2]

2.4 The points $(-3, 1)$ and $(5, 7)$ are the ends of a diameter. Find the circle's equation. [4]

2.5 The circle $x^2 + y^2 = 10$ passes through $(3, 1)$. Find the equation of the tangent at that point. [4]

3 Exam-style questions

3.1 (*Adapted from Edexcel IAL WMA12 June 2025 Q2.*) The line joining $(-1, -3)$ and $(7, 1)$ is the diameter of circle C .

(a) Find an equation for C .

(b) Hence find the exact coordinates of the point on C that is nearest the x -axis. [7]

3.2 The circle $(x - 4)^2 + (y - 2)^2 = 25$ meets the line $x = 0$ at $A(0, 5)$ and $B(0, -1)$. Verify that the perpendicular from the centre to the chord AB bisects AB . [3]

Solutions

2.1 $(x + 4)^2 + (y - 1)^2 = 9$: centre $(-4, 1)$, radius 3.

2.2 $(x - 2)^2 + (y + 5)^2 = 13$.

2.3 $(5 - 1)^2 + (2 - 3)^2 = 16 + 1 = 17$ – yes, the point lies on the circle.

2.4 Centre: midpoint $= (1, 4)$. Diameter squared: $8^2 + 6^2 = 100$, so $r^2 = 25$.

$$(x - 1)^2 + (y - 4)^2 = 25$$

Check both endpoints: $16 + 9 = 25$ ✓ twice.

2.5 The radius to $(3, 1)$ has gradient $\frac{1}{3}$; the tangent's gradient is -3 :

$$y - 1 = -3(x - 3) \implies y = -3x + 10$$

Check: the tangent passes $(3, 1)$ ✓, and $-3 \times \frac{1}{3} = -1$ (perpendicular) ✓.

3.1 (a) Centre: $(\frac{-1+7}{2}, \frac{-3+1}{2}) = (3, -1)$. Diameter squared: $8^2 + 4^2 = 80$, so $r^2 = 20$:

$$(x - 3)^2 + (y + 1)^2 = 20$$

Check an endpoint: $(-4)^2 + 2^2 = 20$ ✓. (b) Nearest the x -axis means straight below the centre (the centre is 1 unit below the axis, less than $r = \sqrt{20} \approx 4.47$, so the circle crosses the axis; the lowest point is r below the centre):

$$(3, -1 - \sqrt{20}) = (3, -1 - 2\sqrt{5})$$

3.2 The centre is $(4, 2)$. The perpendicular from $(4, 2)$ to the vertical chord $x = 0$ is the horizontal line $y = 2$, meeting AB at $(0, 2)$. The midpoint of $A(0, 5)$ and $B(0, -1)$ is $(0, \frac{5+(-1)}{2}) = (0, 2)$ – the same point, so the perpendicular bisects the chord ✓.

Local review guidance: midpoint/completing-the-square method; exact centre, radius and equation; property applied with the right perpendicular. This is authored classroom guidance.