1.7 Integration

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS integration • constant of integration • power rule

Objective

Integrate x^n and sums, differences and constant multiples ($n \neq -1$), expand brackets and split fractions before integrating, include the constant of integration, and recover $y = f(x)$ from a derivative and one point on the curve. Original teaching questions for specification items P1.5.1 and P1.5.2; the exam-style part adapts reviewed WMA11 June 2025 demand.

1 Key knowledge and worked examples

Integration reverses differentiation:

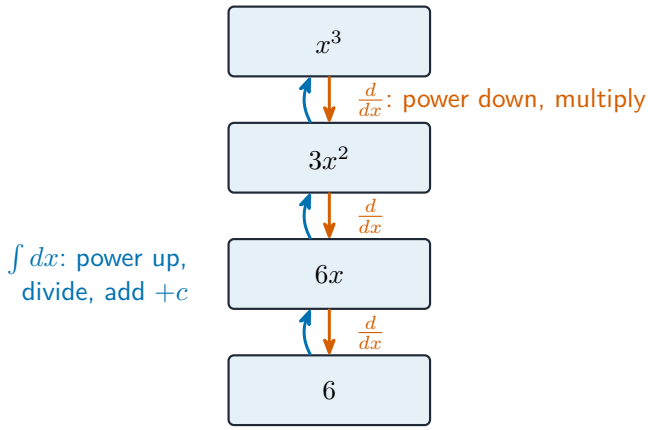
$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \qquad (n \neq -1)$$

Every term needs the **constant of integration** $+c$ – once per answer, after the last term. Raise the power by one, divide by the new power, and always **expand or split first**:

$$\int (x+1)^2 dx = \int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + c$$

$$\int \frac{x^3 + 6x}{\sqrt{x}} \, dx = \int \left(x^{5/2} + 6x^{1/2}\right) dx = \tfrac{2}{7}x^{7/2} + 4x^{3/2} + c$$

Given $\dfrac{dy}{dx}$ **and one point**, find c by substituting the point.



Each integral climbs one rung and picks up an arbitrary constant: $\int 6\,dx = 6x + c$, not just $6x$.

Each integral climbs one rung and picks up an arbitrary constant.

Worked example – recover the curve. A curve passes through $(1, 4)$ and has $\dfrac{dy}{dx} = 3x^2 - 2$. Find its equation.

- Integrate every term:

$$y = \int (3x^2 - 2)\,dx = x^3 - 2x + c$$

- Substitute $(1, 4)$: $1 - 2 + c = 4$, so $c = 5$ and $y = x^3 - 2x + 5$.
- Check by differentiation: $\dfrac{dy}{dx} = 3x^2 - 2$ ✓, and $y(1) = 1 - 2 + 5 = 4$ ✓.

Worked example – split before integrating. Find $\displaystyle\int \frac{x^2 + 2}{\sqrt{x}}\,dx$.

- Divide each term of the numerator by $x^{1/2}$ (subtract the powers):

$$\frac{x^2}{x^{1/2}} = x^{3/2} \qquad \frac{2}{x^{1/2}} = 2x^{-1/2}$$

- Integrate:

$$\int \left(x^{3/2} + 2x^{-1/2}\right) dx = \tfrac{2}{5}x^{5/2} + 2 \times 2x^{1/2} + c = \tfrac{2}{5}x^{5/2} + 4\sqrt{x} + c$$

- Check by differentiating: $\frac{2}{5} \times \frac{5}{2}x^{3/2} = x^{3/2}$ ✓ and $4 \times \frac{1}{2}x^{-1/2} = 2x^{-1/2}$ ✓.

2 Practice

2.1 Find $\int 8x^3\,dx$. [2]

2.2 Find $\int x^{1/2}\,dx$. [2]

2.3 Find $\int (3x - 2x^2)\,dx$. [2]

2.4 Find $\int (2x+3)(x-1)\,dx$. [3]

2.5 A curve has $f'(x) = 4x - 1$ and passes through $(1, 3)$. Find $f(x)$. [3]

2.6 Find $\int \frac{x^3 + 6x}{\sqrt{x}}\,dx$. [4]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA11 June 2025 Q3.)* Find $\displaystyle\int \frac{(x+1)^2}{2\sqrt{x}}\,dx$, giving each coefficient exactly. [5]

3.2 *(Adapted from Edexcel IAL WMA11 June 2025 Q8(c).)* A curve passes through $(4, 20)$ and has $\dfrac{dy}{dx} = 6\sqrt{x} - 2x$. Find the equation of the curve. [5]

Solutions

2.1 $\displaystyle\int 8x^3\,dx = \frac{8x^4}{4} + c = 2x^4 + c.$

2.2 $\displaystyle\int x^{1/2}\,dx = \tfrac{2}{3}x^{3/2} + c.$

2.3 $\tfrac{3}{2}x^2 - \tfrac{2}{3}x^3 + c.$

2.4 Expand first: $(2x+3)(x-1) = 2x^2 + x - 3$, so $\displaystyle\int = \tfrac{2}{3}x^3 + \tfrac{1}{2}x^2 - 3x + c.$

2.5 $f(x) = \displaystyle\int (4x-1)\,dx = 2x^2 - x + c$. Substituting $(1,3)$: $2 - 1 + c = 3 \implies c = 2$, so $f(x) = 2x^2 - x + 2$.

2.6 Split: $\dfrac{x^3}{x^{1/2}} = x^{5/2}$ and $\dfrac{6x}{x^{1/2}} = 6x^{1/2}$, so

$$\int \left(x^{5/2} + 6x^{1/2}\right) dx = \tfrac{2}{7}x^{7/2} + 6 \times \tfrac{2}{3}x^{3/2} + c = \tfrac{2}{7}x^{7/2} + 4x^{3/2} + c$$

3.1 Expand the numerator: $(x+1)^2 = x^2 + 2x + 1$; divide each term by $2x^{1/2}$:

$$\frac{x^2}{2x^{1/2}} = \tfrac{1}{2}x^{3/2} \qquad \frac{2x}{2x^{1/2}} = x^{1/2} \qquad \frac{1}{2x^{1/2}} = \tfrac{1}{2}x^{-1/2}$$

Integrate term by term:

$$\int \left(\tfrac{1}{2}x^{3/2} + x^{1/2} + \tfrac{1}{2}x^{-1/2}\right) dx = \tfrac{1}{2} \times \tfrac{2}{5}x^{5/2} + \tfrac{2}{3}x^{3/2} + \tfrac{1}{2} \times 2x^{1/2} + c$$

$$= \frac{1}{5}x^{5/2} + \frac{2}{3}x^{3/2} + \sqrt{x} + c$$

Check by differentiating: $\tfrac{1}{5} \times \tfrac{5}{2} = \tfrac{1}{2}$ ✓, $\tfrac{2}{3} \times \tfrac{3}{2} = 1$ ✓, $\tfrac{1}{2}$ ✓ – the integrand returns exactly.

3.2 Integrate with $x^{1/2} = \sqrt{x}$:

$$y = \int \left(6x^{1/2} - 2x\right) dx = 6 \times \tfrac{2}{3}x^{3/2} - x^2 + c = 4x^{3/2} - x^2 + c$$

Substitute $(4, 20)$: $4(4)^{3/2} - 16 + c = 4 \times 8 - 16 + c = 16 + c = 20$, so $c = 4$:

$$y = 4x^{3/2} - x^2 + 4$$

Check: $y(4) = 32 - 16 + 4 = 20$ ✓ and $\dfrac{dy}{dx} = 6x^{1/2} - 2x$ ✓.

Local review guidance: power raised correctly after expanding or splitting; exact coefficients; $+c$ present and evaluated from the point. This is authored classroom guidance.