1.5 Trigonometric graphs

Name: _____ Class: _____ Date: _____ **Total: 21 marks**

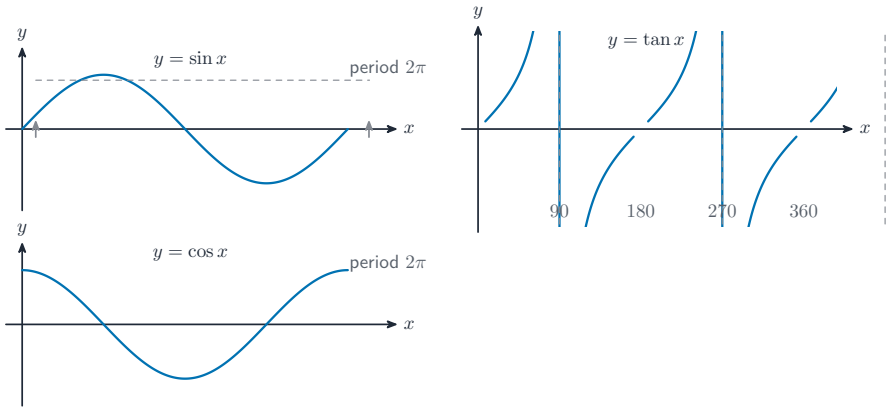
KEY WORDS period • asymptote • periodicity • symmetry

Objective

Use the graphs, symmetries and periodicity of sine, cosine and tangent, solve simple trigonometric equations in a given interval, and link curves like $y = \sin 2x$ or $y = 3\sin x$ to transformations. Original teaching questions for specification item P1.3.3; the exam-style part adapts reviewed WMA11 June 2025 demand.

1 Key knowledge and worked examples

- $y = \sin x$ and $y = \cos x$: period $360°$ (2π), values between -1 and 1.
- $y = \tan x$: period $180°$ (π), with **asymptotes** at $x = 90° + 180°n$.
- Symmetries: $\cos(-x) = \cos x$ (even); $\sin(180° - x) = \sin x$; $\tan x = \dfrac{\sin x}{\cos x}$.
- $y = \sin kx$ repeats every $\dfrac{360°}{k}$; $y = a\sin x$ reaches $\pm a$.



Count periods first, then solutions per period.

Worked example. Solve $\sin x = 0.5$ for $0 \le x < 360°$.

- The calculator gives the acute angle: $x = 30°$.
- Sine is also positive in the second quadrant, and $\sin(180° - x) = \sin x$:

$$x = 30° \quad \text{or} \quad x = 180° - 30° = 150°$$

- Check on the graph: the line $y = 0.5$ crosses $y = \sin x$ exactly twice in one period ✓.

Worked example – transformed equation. Solve $\sin 2x = 1$ for $0 \le x \le 360°$.

- Let $u = 2x$; as x runs from 0 to $360°$, u runs from $0°$ to $720°$ – two full turns.
- $\sin u = 1$ at $u = 90° + 360°n$: here $u = 90°$ and $u = 450°$.

$$2x = 90° \implies x = 45° \qquad 2x = 450° \implies x = 225°$$

- Check: $\sin(2 \times 45°) = \sin 90° = 1$ ✓.

2 Practice

2.1 State the period of $y = \cos 3x$. [1]

2.2 Solve $\cos x = -0.5$ for $0 \le x \le 360°$. [3]

2.3 Solve $\tan x = 1$ for $0 \le x \le 360°$. [3]

2.4 Describe the transformation mapping $y = \sin x$ onto $y = \sin(x + 30°)$. [2]

2.5 Solve $\sin 2x = 0.5$ for $0 \leq x \leq 360°$. [4]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA11 June 2025 Q4(c).)* The curve C has equation $y = 3\sin x$, with x in degrees. Find the number of solutions of $3\sin x = 1.5$ in the interval $0 < x \leq 1440°$, and list them. [4]

3.2 The graph of $y = k\tan x$ passes through the point $(45°, 5)$.

(a) Find k.

(b) State the equations of the asymptotes of the graph, and the number of asymptotes in the interval $-360° \leq x \leq 360°$. [4]

Solutions

2.1 $360° \div 3 = 120°$.

2.2 $\cos x = -0.5$: calculator gives $x = 120°$; cosine is also negative in the third quadrant: $x = 120°$ and $x = 240°$. Check: $\cos 240° = -0.5$ ✓.

2.3 $\tan 45° = 1$, and tangent's period is $180°$: $x = 45°$ and $x = 225°$.

2.4 A translation by $\begin{pmatrix} -30° \\ 0 \end{pmatrix}$ (shifts left by $30°$).

2.5 $2x = 30°, 150°, 390°, 510°$ (two turns of the sine curve), so $x = 15°, 75°, 195°, 255°$. Check: $\sin(2 \times 195°) = \sin 390° = \sin 30° = 0.5$ ✓.

3.1 $\sin x = 0.5$, so in each $360°$ turn $x = 30°$ and $x = 150°$. The interval $0 < x \leq 1440°$ holds four full turns, giving $x = 30°, 150°, 390°, 510°, 750°, 870°, 1110°, 1230°$ – **8 solutions**. Check: $1440/360 = 4$ turns $\times$ 2 solutions per turn $= 8$ ✓.

3.2 (a) $\tan 45° = 1$, so $k \times 1 = 5$ gives $k = 5$. (b) Asymptotes: $x = 90° + 180°n$ for integer n. In $-360° \leq x \leq 360°$: $x = -270°, -90°, 90°, 270°$ – **4 asymptotes**. Check: each is an odd multiple of $90°$, where $\cos x = 0$ and $\tan x$ is undefined ✓.

Local review guidance: correct inverse value or period count; complete solution list in the interval; second-quadrant or period reasoning shown. This is authored classroom guidance.