

1.5 Trigonometric graphs

Name: _____ Class: _____ Date: _____

Total: 21 marks

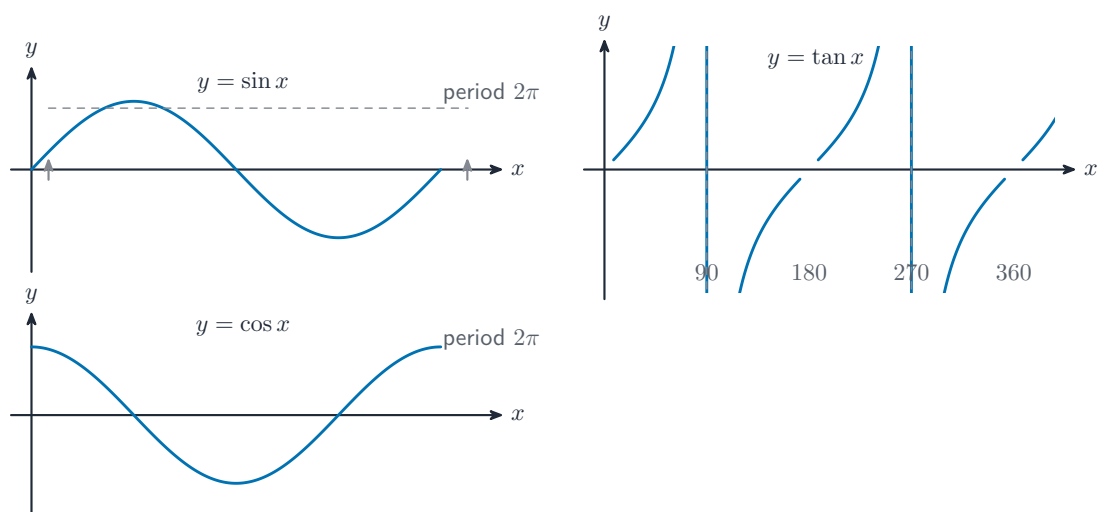
KEY WORDS period • asymptote • periodicity • symmetry

Objective

Use the graphs, symmetries and periodicity of sine, cosine and tangent, solve simple trigonometric equations in a given interval, and link curves like $y = \sin 2x$ or $y = 3 \sin x$ to transformations. Original teaching questions for specification item P1.3.3; the exam-style part adapts reviewed WMA11 June 2025 demand.

1 Key knowledge and worked examples

- $y = \sin x$ and $y = \cos x$: period 360° (2π), values between -1 and 1 .
- $y = \tan x$: period 180° (π), with **asymptotes** at $x = 90^\circ + 180^\circ n$.
- Symmetries: $\cos(-x) = \cos x$ (even); $\sin(180^\circ - x) = \sin x$; $\tan x = \frac{\sin x}{\cos x}$.
- $y = \sin kx$ repeats every $\frac{360^\circ}{k}$; $y = a \sin x$ reaches $\pm a$.



Count periods first, then solutions per period.

Worked example. Solve $\sin x = 0.5$ for $0 \leq x < 360^\circ$.

- The calculator gives the acute angle: $x = 30^\circ$.

- Sine is also positive in the second quadrant, and $\sin(180^\circ - x) = \sin x$:

$$x = 30^\circ \quad \text{or} \quad x = 180^\circ - 30^\circ = 150^\circ$$

- Check on the graph: the line $y = 0.5$ crosses $y = \sin x$ exactly twice in one period ✓.

Worked example – transformed equation. Solve $\sin 2x = 1$ for $0 \leq x \leq 360^\circ$.

- Let $u = 2x$; as x runs from 0 to 360° , u runs from 0° to 720° – two full turns.
- $\sin u = 1$ at $u = 90^\circ + 360^\circ n$: here $u = 90^\circ$ and $u = 450^\circ$.

$$2x = 90^\circ \implies x = 45^\circ \quad 2x = 450^\circ \implies x = 225^\circ$$

- Check: $\sin(2 \times 45^\circ) = \sin 90^\circ = 1$ ✓.

2 Practice

2.1 State the period of $y = \cos 3x$. [1]

2.2 Solve $\cos x = -0.5$ for $0 \leq x \leq 360^\circ$. [3]

2.3 Solve $\tan x = 1$ for $0 \leq x \leq 360^\circ$. [3]

2.4 Describe the transformation mapping $y = \sin x$ onto $y = \sin(x + 30^\circ)$. [2]

2.5 Solve $\sin 2x = 0.5$ for $0 \leq x \leq 360^\circ$. [4]

3 Exam-style questions

3.1 (*Adapted from Edexcel IAL WMA11 June 2025 Q4(c).*) The curve C has equation $y = 3 \sin x$, with x in degrees. Find the number of solutions of $3 \sin x = 1.5$ in the interval $0 < x \leq 1440^\circ$, and list them. [4]

3.2 The graph of $y = k \tan x$ passes through the point $(45^\circ, 5)$.

(a) Find k .

(b) State the equations of the asymptotes of the graph, and the number of asymptotes in the interval $-360^\circ \leq x \leq 360^\circ$. [4]

Solutions

2.1 $360^\circ \div 3 = 120^\circ$.

2.2 $\cos x = -0.5$: calculator gives $x = 120^\circ$; cosine is also negative in the third quadrant: $x = 120^\circ$ and $x = 240^\circ$. Check: $\cos 240^\circ = -0.5$ ✓.

2.3 $\tan 45^\circ = 1$, and tangent's period is 180° : $x = 45^\circ$ and $x = 225^\circ$.

2.4 A translation by $\begin{pmatrix} -30^\circ \\ 0 \end{pmatrix}$ (shifts left by 30°).

2.5 $2x = 30^\circ, 150^\circ, 390^\circ, 510^\circ$ (two turns of the sine curve), so $x = 15^\circ, 75^\circ, 195^\circ, 255^\circ$. Check: $\sin(2 \times 195^\circ) = \sin 390^\circ = \sin 30^\circ = 0.5$ ✓.

3.1 $\sin x = 0.5$, so in each 360° turn $x = 30^\circ$ and $x = 150^\circ$. The interval $0 < x \leq 1440^\circ$ holds four full turns, giving $x = 30^\circ, 150^\circ, 390^\circ, 510^\circ, 750^\circ, 870^\circ, 1110^\circ, 1230^\circ$ – **8 solutions**. Check: $1440/360 = 4$ turns $\times$ 2 solutions per turn = 8 ✓.

3.2 (a) $\tan 45^\circ = 1$, so $k \times 1 = 5$ gives $k = 5$. (b) Asymptotes: $x = 90^\circ + 180^\circ n$ for integer n . In $-360^\circ \leq x \leq 360^\circ$: $x = -270^\circ, -90^\circ, 90^\circ, 270^\circ$ – **4 asymptotes**. Check: each is an odd multiple of 90° , where $\cos x = 0$ and $\tan x$ is undefined ✓.

Local review guidance: correct inverse value or period count; complete solution list in the interval; second-quadrant or period reasoning shown. This is authored classroom guidance.