1.4 Triangle trigonometry and radians

Name: _____ Class: _____ Date: _____ **Total: 20 marks**

KEY WORDS sine rule • cosine rule • radian • sector • ambiguous case

Objective

Use the sine rule (including the ambiguous case), the cosine rule and the area formula $\frac{1}{2}ab\sin C$, and work in radians with $s = r\theta$ and sector area $\frac{1}{2}r^2\theta$. Original teaching questions for specification items P1.3.1 and P1.3.2; the exam-style part adapts reviewed WMA11 June 2025 demand.

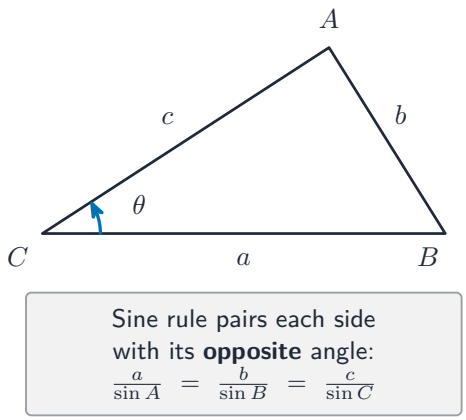
1 Key knowledge and worked examples

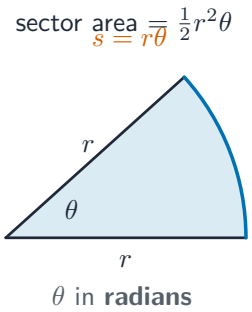
In triangle ABC with sides a, b, c opposite angles A, B, C:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc\cos A \qquad \text{area} = \tfrac{1}{2}ab\sin C$$

Quote the formula first: the method mark follows the quoted formula. The sine rule has an **ambiguous case**: $\sin\theta = \sin(180° - \theta)$, so both the acute and the obtuse angle can fit the same sine value – check the diagram.

Radians: $180° = \pi$ rad. With θ in radians, **arc length** $s = r\theta$ and **sector area** $= \frac{1}{2}r^2\theta$.





Each side pairs with its opposite angle.

Worked example – cosine rule. In triangle ABC, $b = 5$, $c = 7$ and $A = 60°$. Find a.

- Known: two sides and the included angle, so the cosine rule finds the third side:

$$a^2 = b^2 + c^2 - 2bc\cos A = 25 + 49 - 2(5)(7)\cos 60° = 74 - 70 \times \tfrac{1}{2} = 74 - 35 = 39$$

$$a = \sqrt{39} \approx 6.24$$

- Check: the answer sits between $|7 - 5| = 2$ and $7 + 5 = 12$ ✓.

Worked example – ambiguous sine rule. In triangle ABC, $A = 35°$, $a = 6$ and $b = 8$. Find the two possible values of B.

$$\frac{\sin B}{8} = \frac{\sin 35°}{6} \implies \sin B = \frac{8\sin 35°}{6} = 0.7648$$

$$B = 49.9° \quad \text{or} \quad B = 180° - 49.9° = 130.1°$$

- Both are possible because $130.1° + 35° = 165.1° < 180°$ leaves room for angle C.
- Check: with $B = 130.1°$, $C = 14.9°$, and $\sin B = \sin 49.9°$ ✓ – one sine, two angles.

2 Practice

2.1 Convert 135° to radians, giving an exact answer in terms of π. [2]

2.2 A circle has radius 10 cm. Find the arc length for a sector angle of 0.5 radians. [2]

2.3 Find the area of a sector with radius 4 cm and angle $\frac{\pi}{6}$ radians, giving an exact answer. [2]

2.4 Two sides of a triangle are 5 cm and 8 cm with included angle 32°. Find the area. [2]

2.5 In triangle ABC, $A = 80°$, $a = 12$ and $C = 50°$. Find c. [3]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA11 June 2025 Q6.)* A sector of a circle has radius 9 m and angle 1.2 radians.

(a) Find the area of the sector.

(b) Find the perimeter of the sector. [4]

3.2 A triangular garden plot has sides of 11 m and 15 m with included angle 0.9 radians.

(a) Find the area of the plot.

(b) Find the length of the third side, giving your answer to 3 significant figures. [5]

Solutions

2.1 $135^\circ = \dfrac{135}{180}\pi = \dfrac{3\pi}{4}$ radians.

2.2 $s = r\theta = 10 \times 0.5 = 5$ cm.

2.3 Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 16 \times \frac{\pi}{6} = \dfrac{4\pi}{3}$ cm^2.

2.4 Area $= \frac{1}{2}(5)(8)\sin 32^\circ = 20\sin 32^\circ = 10.6$ cm^2 (3 s.f.).

2.5 Sine rule: $\dfrac{c}{\sin 50^\circ} = \dfrac{12}{\sin 80^\circ}$, so $c = \dfrac{12\sin 50^\circ}{\sin 80^\circ} = \dfrac{12 \times 0.7660}{0.9848} = 9.34$ (3 s.f.).

3.1 (a) Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 81 \times 1.2 = 48.6$ m^2. (b) Arc $= r\theta = 9 \times 1.2 = 10.8$ m, so perimeter $= 10.8 + 9 + 9 = 28.8$ m. Check: the two straight edges are radii ✓.

3.2 (a) Area $= \frac{1}{2}(11)(15)\sin 0.9 = 82.5 \times 0.7833 = 64.6$ m^2 (3 s.f.). (b) Cosine rule (angle 0.9 rad is included between the given sides):

$$x^2 = 11^2 + 15^2 - 2(11)(15)\cos 0.9 = 121 + 225 - 330(0.6216) = 346 - 205.1 = 140.9$$

$$x = \sqrt{140.9} = 11.9 \text{ m (3 s.f.)}$$

Check: $\cos 0.9 \approx \cos 51.6^\circ > 0$ (acute), so the $-2bc\cos A$ term subtracts ✓; the side sits between 4 and 26 ✓.

Local review guidance: correct formula quoted and substituted; accuracy with units and 3 s.f.; ambiguous case or unit handling stated. This is authored classroom guidance.