1.3 Straight lines and coordinate geometry

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

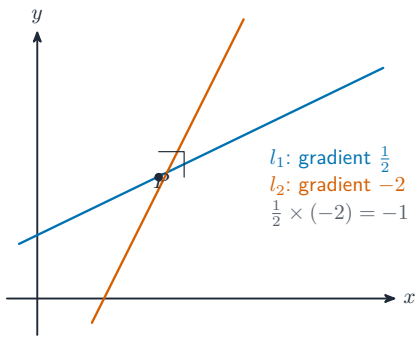
KEY WORDS gradient / • parallel • perpendicular • general form

Objective

Write equations of straight lines in the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$, find lines through two points, and use the parallel and perpendicular gradient conditions. Original teaching questions for specification items P1.2.1 and P1.2.2; the exam-style part adapts reviewed WMA11 June 2025 demand.

1 Key knowledge and worked examples

- Gradient through two points: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.
- Line through (x_1, y_1): $y - y_1 = m(x - x_1)$; the general form $ax + by + c = 0$ hides the gradient in $-a/b$.
- **Parallel** : $m_1 = m_2$. **Perpendicular** : $m_1 m_2 = -1$.
- Distance between points: Pythagoras, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



Perpendicular gradients multiply to −1.

Worked example – the specification's own perpendicular. Find the equation of the line perpendicular to $3x + 4y = 18$ through the point $(2, 3)$.

- Rearrange to gradient form: $4y = -3x + 18$, so $y = -\frac{3}{4}x + \frac{9}{2}$ and $m_1 = -\frac{3}{4}$.
- Perpendicular gradient: $m_2 = -1 \div \left(-\frac{3}{4}\right) = \frac{4}{3}$.

$$y - 3 = \tfrac{4}{3}(x - 2)$$

- Check with the given point: at $x = 2$ the right side is 0, so $y = 3$ ✓.

Worked example – through two points. The line through $(1, 3)$ and $(5, 11)$.

- Gradient: $m = \dfrac{11 - 3}{5 - 1} = \dfrac{8}{4} = 2$.
- Equation: $y - 3 = 2(x - 1) \implies y = 2x + 1$.
- Check with the second point: $2(5) + 1 = 11$ ✓.

2 Practice

2.1 Find the gradient of the line through $(-2, 7)$ and $(4, -5)$. [2]

2.2 Write the equation of the line with gradient $-\frac{1}{2}$ through $(0, 4)$. [2]

2.3 Find the equation of the line parallel to $2x - y = 6$ through $(3, 3)$. [3]

2.4 Find the equation of the line perpendicular to $y = \frac{1}{3}x + 2$ through $(6, 0)$. [3]

2.5 Find the coordinates of the intersection of $y = 2x + 1$ and $y = -x + 7$. [2]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA11 June 2025 Q5.)* The line l_1 has equation $2x + y - 12 = 0$. The line l_2 passes through the origin and is perpendicular to l_1.

(a) Find an equation for l_2.

(b) Find the coordinates of P, the intersection of l_1 and l_2.

(c) Hence find the shortest distance from l_1 to the origin, giving your answer as an exact value. [7]

3.2 Show that the points $A(-1, 4)$, $B(2, 1)$ and $C(4, -1)$ are collinear, and find the coordinates of the point where this line crosses the x-axis. [5]

Solutions

2.1 $m = \dfrac{-5-7}{4-(-2)} = \dfrac{-12}{6} = -2.$

2.2 Through $(0,4)$ (the y-intercept): $y = -\frac{1}{2}x + 4$.

2.3 $2x - y = 6 \implies y = 2x - 6$, so $m = 2$. Parallel keeps $m = 2$: $y - 3 = 2(x-3) \implies y = 2x - 3$.

2.4 $m_1 = \frac{1}{3}$, so $m_2 = -3$: $y - 0 = -3(x-6) \implies y = -3x + 18$.

2.5 $2x + 1 = -x + 7 \implies 3x = 6 \implies x = 2$, and $y = 2(2) + 1 = 5$. The point is $(2,5)$.

3.1 (a) Rearrange l_1: $y = -2x + 12$, so $m_1 = -2$ and $m_2 = \frac{1}{2}$. Through the origin: $y = \frac{1}{2}x$. (b) Substitute: $2x + \frac{1}{2}x - 12 = 0 \implies \frac{5}{2}x = 12 \implies x = \frac{24}{5}$, and $y = \frac{1}{2} \cdot \frac{24}{5} = \frac{12}{5}$. So $P\left(\frac{24}{5}, \frac{12}{5}\right)$. Check in l_1: $2 \cdot \frac{24}{5} + \frac{12}{5} - 12 = \frac{60}{5} - 12 = 0$ ✓. (c) The shortest distance is $|OP|$:

$$|OP| = \sqrt{\left(\tfrac{24}{5}\right)^2 + \left(\tfrac{12}{5}\right)^2} = \frac{\sqrt{576+144}}{5} = \frac{\sqrt{720}}{5} = \frac{12\sqrt{5}}{5}$$

using $\sqrt{720} = \sqrt{144 \times 5} = 12\sqrt{5}$.

3.2 Gradient $AB = \dfrac{1-4}{2-(-1)} = -1$; gradient $BC = \dfrac{-1-1}{4-2} = -1$. The gradients are equal and the segments share B, so A, B, C are collinear. Line through A: $y - 4 = -1(x+1) \implies y = -x + 3$. On the x-axis, $y = 0$: $x = 3$, so the line crosses at $(3,0)$. Check: C fits the line: $-(4) + 3 = -1$ ✓.

Local review guidance: correct gradient method including the sign change for perpendiculars; exact coordinates; collinearity conclusion tied to the shared point. This is authored classroom guidance.