

## 1.2.2 Graphs, sketching and transformations

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

**Total: 21 marks**

**KEY WORDS** cubic • reciprocal function • asymptote  
• transformation

### Objective

Sketch cubics, reciprocal curves and their asymptotes, read intersections as solutions, and apply single transformations to graphs. Original teaching questions for specification items P1.1.11 and P1.1.12; the exam-style part adapts reviewed WMA11 June 2025 demand.

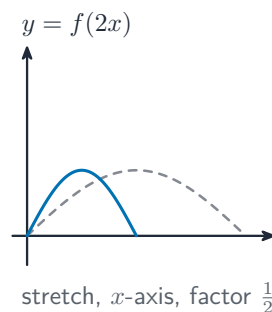
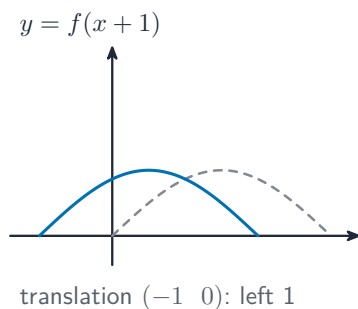
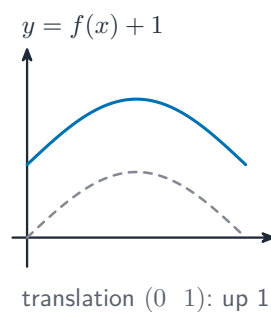
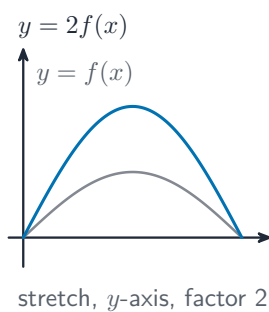
### 1 Key knowledge and worked examples

Sketch these families (sketch means shape + key points, not a plotted table):

- **cubics** such as  $y = (x - 1)(x + 2)(x - 3)$ : mark the three  $x$ -intercepts and the  $y$ -intercept  $(0, (0 - 1)(0 + 2)(0 - 3)) = (0, 6)$ .
- **reciprocal functions**  $y = \frac{k}{x}$  and  $y = \frac{k}{x^2}$  ( $x \neq 0$ ): both branches live either side of an **asymptote** – the  $y$ -axis for these.
- Intersections answer equations:  $y = f(x)$  meets  $y = g(x)$  where  $f(x) = g(x)$ .

One **transformation** at a time on  $y = f(x)$ :

| New equation   | Effect                                                        |
|----------------|---------------------------------------------------------------|
| $y = af(x)$    | stretch parallel to the $y$ -axis, scale factor $a$           |
| $y = f(x) + a$ | translation $\begin{pmatrix} 0 \\ a \end{pmatrix}$            |
| $y = f(x + a)$ | translation $\begin{pmatrix} -a \\ 0 \end{pmatrix}$           |
| $y = f(ax)$    | stretch parallel to the $x$ -axis, scale factor $\frac{1}{a}$ |



*Name direction, type and scale factor (or vector).*

**Worked example.** The graph of  $y = x^2$  is transformed to the graph of  $y = (x + 5)^2$ . Describe the transformation, and state the coordinates of the new turning point.

- Write  $(x + 5)^2 = f(x + 5)$  with  $f(x) = x^2$ : this is a **translation by**  $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$ .
- The turning point  $(0, 0)$  moves to  $(-5, 0)$ .
- Check:  $(-5 + 5)^2 = 0$  ✓ – the new curve touches the  $x$ -axis at  $-5$ .

## 2 Practice

**2.1** Sketch  $y = \frac{1}{x}$ , labelling its asymptotes. [2]

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**2.2** Sketch  $y = x^3 - x$ , marking the intercepts with the axes. [3]

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**2.3** Sketch  $y = \frac{2}{x^2}$  for  $x \neq 0$ , labelling its asymptotes. [2]

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**2.4** Describe the single transformation mapping  $y = \sin x$  onto  $y = 3 \sin x$ . [2]

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**2.5** Describe the single transformation mapping  $y = \sqrt{x}$  onto  $y = \sqrt{x+4}$ , and state where the transformed curve meets the  $x$ -axis. [3]

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### 3 Exam-style questions

**3.1** (*Adapted from Edexcel IAL WMA11 June 2025 Q4.*) The curve  $C$  has equation  $y = 5 \sin x$ , with  $x$  in degrees.

(a) State the period of  $C$ .

(b) State the number of solutions of  $5 \sin x = -5$  in the interval  $0 \leq x \leq 720^\circ$ , justifying your answer. [3]

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**3.2** (*Adapted from Edexcel IAL WMA11 June 2025 Q9.*)

(a) Describe fully the single transformation that maps the graph of  $y = \sqrt{x}$  onto the graph of  $C_1$ :  $y = \sqrt{3x}$ .

(b) The curves  $C_1$ :  $y = \sqrt{3x}$  and  $C_2$ :  $y = 2\sqrt{x}$  intersect at  $P$ . Use algebra to find the exact coordinates of  $P$ . [6]

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## Solutions

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**2.1** Two branches in quadrants 1 and 3, each approaching the  $x$ -axis and  $y$ -axis asymptotes (the axes themselves); the curve passes through  $(1, 1)$  and  $(-1, -1)$ .

**2.2**  $x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1)$ : the curve crosses at  $x = -1, 0, 1$  and passes through the origin with  $y$ -intercept 0.

**2.3** Both branches above the  $x$ -axis (squares are positive), symmetric in the  $y$ -axis, with the  $x$ -axis and  $y$ -axis as asymptotes; through  $(\pm 1, 2)$ .

**2.4** A stretch parallel to the  $y$ -axis with scale factor 3 (amplitude 1 becomes 3).

**2.5**  $y = \sqrt{x + 4} = f(x + 4)$ : a translation by  $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ . The curve meets the  $x$ -axis where  $x + 4 = 0$ , at  $(-4, 0)$ .

**3.1** (a)  $360^\circ$  (the transformation from  $y = \sin x$  is a  $y$ -stretch, which does not change the period). (b)  $5 \sin x = -5 \implies \sin x = -1 \implies x = 270^\circ + 360^\circ n$ . In  $0 \leq x \leq 720^\circ$ :  $x = 270^\circ$  and  $x = 630^\circ$  – **2 solutions**. Check:  $\sin 270^\circ = \sin 630^\circ = -1$  ✓.

**3.2** (a)  $y = \sqrt{3x} = f(3x)$ : a **stretch parallel to the  $x$ -axis with scale factor  $\frac{1}{3}$** . (b) Set the  $y$ -values equal and square both sides (both are non-negative for  $x \geq 0$ ):

$$\sqrt{3x} = 2\sqrt{x} \implies 3x = 4x \implies x = 0$$

The only intersection is  $P = (0, 0)$ : check  $\sqrt{3 \times 0} = 0 = 2\sqrt{0}$  ✓. For  $x > 0$ ,  $\sqrt{3x} = \sqrt{3}\sqrt{x}$  and  $\sqrt{3} < 2$ , so  $C_1$  stays below  $C_2$  – the curves never cross again.

Local review guidance: correct transformation named with direction and factor; exact intersection algebra with both sides' domains respected; justification from the sketch. This is authored classroom guidance.