

1.2.1 Simultaneous equations and inequalities

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS simultaneous equations

• substitution

• inequality

• dotted boundary

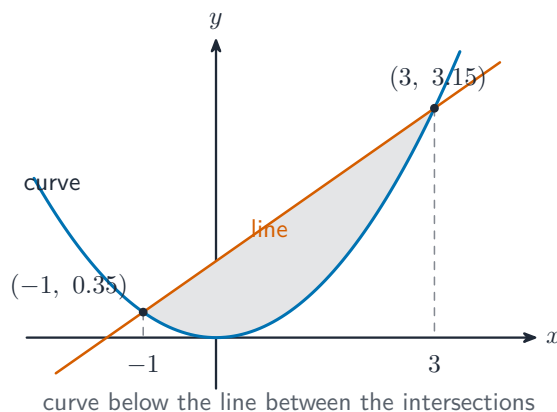
Objective

Solve simultaneous equations by substitution (including a line meeting a curve) and solve linear and quadratic inequalities, interpreting them graphically. Original teaching questions for specification items P1.1.6 to P1.1.9; the exam-style part adapts reviewed WMA11 June 2025 demand.

1 Key knowledge and worked examples

Simultaneous equations by **substitution** : rearrange the linear equation for one variable, substitute into the other equation, solve, then substitute back. A line meeting a curve gives a quadratic in one unknown; its roots are the intersection x -coordinates – the geometrical meaning of the algebra.

Inequalities keep their sign under add/subtract and under division by a positive number; dividing by a negative number reverses the sign. For a quadratic inequality, sketch the parabola and read off where it sits above or below the axis. When **sketching** a region, use a **dotted boundary** for strict inequalities and a **solid boundary** for $\leq$ or $\geq$.



Between the intersections the curve sits below the line.

Worked example – line meets curve. Find the intersections of $y = x + 1$ and $y = x^2 - 1$.

- Substitute the line's y into the curve:

$$x + 1 = x^2 - 1 \implies x^2 - x - 2 = 0 \implies (x - 2)(x + 1) = 0 \implies x = 2 \text{ or } x = -1$$

- Substitute back into $y = x + 1$: the points are $(2, 3)$ and $(-1, 0)$.
- Check in the curve: $2^2 - 1 = 3$ ✓ and $(-1)^2 - 1 = 0$ ✓.

Worked example – quadratic inequality. Solve $x^2 + 4x - 5 \geq 0$.

- Factorise: $(x + 5)(x - 1) \geq 0$. The parabola opens upward with roots -5 and 1 .
- Upward parabolas sit on or above the axis **outside** their roots:

$$x \leq -5 \quad \text{or} \quad x \geq 1$$

- Check $x = 0$ (excluded): $-5 \geq 0$ false ✓; $x = 2$ (included): $4 + 8 - 5 = 7 \geq 0$ ✓.

2 Practice

2.1 Show that the line $y = 2x - 3$ does not meet the curve $y = x^2$. [3]

2.2 Solve $3x - 2 > x + 8$. [2]

2.3 Solve $x^2 \leq 9$. [2]

2.4 Solve $x^2 > 4x + 5$. [3]

2.5 Find the range of x for which the curve $y = x^2$ is below the line $y = 2x + 3$. [3]

3 Exam-style questions

3.1 (*Adapted from Edexcel IAL WMA11 June 2025 Q7(c).*) The curves C_1 : $y = x^2 - 4x + 5$ and C_2 : $y = 2x - 1$ intersect at two points. Use algebra to find the exact coordinates of both. [6]

3.2 Solve the inequality $\frac{2}{x} < x$, where $x \neq 0$. [5]

Solutions

2.1 Substitute: $2x - 3 = x^2 \implies x^2 - 2x + 3 = 0$. The discriminant is $4 - 12 = -8 < 0$, so there are no real solutions: the line never meets the curve.

2.2 $2x > 10$, so $x > 5$.

2.3 $|x| \leq 3$: the answer is $-3 \leq x \leq 3$ (a parabola below the axis between its roots).

2.4 Rearrange: $x^2 - 4x - 5 > 0 \implies (x - 5)(x + 1) > 0$. Outside the roots: $x < -1$ or $x > 5$.

2.5 The curve is below the line where $x^2 < 2x + 3$: $x^2 - 2x - 3 < 0 \implies (x - 3)(x + 1) < 0$, so $-1 < x < 3$.

3.1 Set the expressions equal: $x^2 - 4x + 5 = 2x - 1 \implies x^2 - 6x + 6 = 0$.

- Quadratic formula with $a = 1$, $b = -6$, $c = 6$:

$$x = \frac{6 \pm \sqrt{36 - 24}}{2} = \frac{6 \pm \sqrt{12}}{2} = 3 \pm \sqrt{3}$$

- Substitute into $y = 2x - 1$: $y = 2(3 \pm \sqrt{3}) - 1 = 5 \pm 2\sqrt{3}$.
- Intersections: $(3 + \sqrt{3}, 5 + 2\sqrt{3})$ and $(3 - \sqrt{3}, 5 - 2\sqrt{3})$.
- Check in C_1 for $x = 3 + \sqrt{3}$: $x^2 - 4x + 5 = (12 + 6\sqrt{3}) - (12 + 4\sqrt{3}) + 5 = 5 + 2\sqrt{3}$ ✓ (using $(3 + \sqrt{3})^2 = 9 + 6\sqrt{3} + 3 = 12 + 6\sqrt{3}$).

3.2 Move everything to one side: $\frac{2}{x} - x < 0 \implies \frac{2 - x^2}{x} < 0$. The fraction changes sign at $x = -\sqrt{2}$, $x = 0$ and $x = \sqrt{2}$. Test each interval:

Interval	Sign of $2 - x^2$	Sign of x	Fraction
$x < -\sqrt{2}$	—	—	+
$-\sqrt{2} < x < 0$	+	—	—
$0 < x < \sqrt{2}$	+	+	+
$x > \sqrt{2}$	—	+	—

Solution: $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$. Check $x = -1$: $2/(-1) = -2 < -1$ ✓. Check $x = 2$: $1 < 2$ ✓. Check $x = -2$: $-1 < -2$ false ✓ (correctly excluded). Multiplying both sides by x without cases reverses the inequality for $x < 0$ – that is the trap.

Local review guidance: valid substitution or rearrangement; exact coordinates/ranges; interval tested or region justified. This is authored classroom guidance.