1.1 Indices and surds

Name: _____ Class: _____ Date: _____ **Total: 19 marks**

KEY WORDS laws of indices • surd • rationalise the denominator • conjugate

Objective

Use the laws of indices for all rational exponents, write expressions in the form kx^n, and manipulate surds including rationalising a denominator. Original teaching questions for specification items P1.1.1 and P1.1.2; the exam-style part adapts reviewed WMA11 June 2025 demand.

1 Key knowledge and worked examples

The **laws of indices** hold for every rational exponent:

$$a^m \times a^n = a^{m+n} \qquad a^m \div a^n = a^{m-n} \qquad (a^m)^n = a^{mn} \qquad a^{\frac{m}{n}} = \sqrt[n]{a^m} \qquad a^{-1} = \frac{1}{a}$$

Exam questions ask for the form kx^n with k and n fully simplified: one constant times one power of x.

Worked example – combining powers. Write $\dfrac{(16x^{12})^{1/4}}{(27x^{-6})^{1/3}}$ in the form kx^n.

- Apply each outer power to the constant and to x separately:

$$(16x^{12})^{1/4} = 16^{1/4}x^{12/4} = 2x^3 \qquad (27x^{-6})^{1/3} = 27^{1/3}x^{-6/3} = 3x^{-2}$$

- Divide, subtracting the powers:

$$\frac{2x^3}{3x^{-2}} = \frac{2}{3}x^{3-(-2)} = \frac{2}{3}x^5$$

- Check: $2^4 = 16$ ✓, $3^3 = 27$ ✓, and $3 + 2 = 5$ ✓.

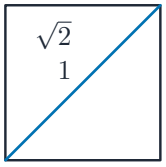
Worked example – rationalising. Write $\dfrac{6}{\sqrt{5}-1}$ in the form $a + b\sqrt{5}$.

- Multiply numerator and denominator by $\sqrt{5}+1$, because $(\sqrt{5}-1)(\sqrt{5}+1) = 5 - 1 = 4$:

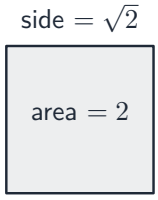
$$\frac{6}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{6(\sqrt{5}+1)}{4} = \frac{3}{2}\sqrt{5} + \frac{3}{2}$$

- Check by calculator: $6/(2.2361-1) = 4.8541$ and $\frac{3}{2}(2.2361) + \frac{3}{2} = 4.8541$ ✓.

A **surd** multiplied by its conjugate kills the roots: $(a+b\sqrt{c})(a-b\sqrt{c}) = a^2 - bc$.



unit square: area 1



square on the diagonal

$\sqrt{2} \times \sqrt{2} = 2$: a surd squared can be a whole number, so $(2\sqrt{2})^2 = 4 \times 2 = 8$ and $(2\sqrt{2})^4 = 8^2 = 64$.

A surd is a real length; squaring it can give a whole number.

2 Practice

2.1 Evaluate $27^{2/3}$. [2]

2.2 Write $(5x^{1/2})(2x^{3/2})$ in the form kx^n. [2]

2.3 Write $(4x^6)^{-1/2}$ in the form kx^n. [2]

2.4 Rationalise the denominator of $\dfrac{10}{\sqrt{6}-1}$, giving your answer in the form $a + b\sqrt{6}$. [3]

2.5 Show that $(2\sqrt{2})^4 = 64$. [2]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA11 June 2025 Q1.)* Given $P = 9x^4$ and $Q = \dfrac{2}{3x}$, express in the form kx^n with fully simplified constants:

(a) $P^{\frac{1}{2}}$

(b) $(PQ)^{-1}$

(c) $(PQ)^2$ [5]

3.2 Show that $(7 + 4\sqrt{3})(2 - \sqrt{3})$ can be written in the form $a + b\sqrt{3}$, where a and b are integers to be found. [3]

Solutions

2.1 $27^{1/3} = 3$, so $27^{2/3} = 3^2 = \mathbf{9}$.

2.2 Multiply constants and add powers: $5 \times 2 = 10$ and $\frac{1}{2} + \frac{3}{2} = 2$, so the answer is $10x^2$.

2.3 Negative power means reciprocal; the outer $-\frac{1}{2}$ applies to both factors: $(4x^6)^{-1/2} = 4^{-1/2}x^{-3} = \frac{1}{2}x^{-3}$.

2.4 $(\sqrt{6}-1)(\sqrt{6}+1) = 6 - 1 = 5$, so $\dfrac{10(\sqrt{6}+1)}{5} = 2\sqrt{6} + 2$.

2.5 $(2\sqrt{2})^2 = 4 \times 2 = 8$, and $8^2 = \mathbf{64}$. Squaring twice is safer than expanding four factors.

3.1 (a) $P^{1/2} = \sqrt{9x^4} = 3x^2$. (b) $PQ = 9x^4 \times \dfrac{2}{3x} = 6x^3$, so $(PQ)^{-1} = \frac{1}{6}x^{-3}$. (c) $(PQ)^2 = 36x^6$. Check (b) against (c): $(PQ)(PQ)^{-1} = 6x^3 \times \frac{1}{6}x^{-3} = 1$ ✓.

3.2 Expand: $7(2) + 7(-\sqrt{3}) + 4\sqrt{3}(2) + 4\sqrt{3}(-\sqrt{3}) = 14 - 7\sqrt{3} + 8\sqrt{3} - 12$. Collect: $14 - 12 = 2$ and $-7\sqrt{3} + 8\sqrt{3} = \sqrt{3}$, so the product is $2 + \sqrt{3}$: $a = 2$, $b = 1$.

Local review guidance: correct index law or conjugate chosen; exact simplified result; the check line. This is authored classroom guidance, not an official scheme.