Exercise walk-through

6.1
Inequalities with the modulus sign

Pearson Edexcel · International A-Level · Pure Mathematics

Further Pure Mathematics 2: inequalities, further complex numbers and differential equations

- solve inequalities involving the modulus sign, and sum series by the **method of differences**;
- use **Euler's relation** and **de Moivre's theorem**, and find the nth roots of complex numbers;
- sketch loci in the Argand diagram, and find images under transformations $w = \frac{az+b}{cz+d}$;

Inequalities involving the modulus sign

- split into cases using the definition of the modulus, and keep only the solutions that lie in each case's own range;
- if both sides are moduli (so both are positive or zero), square both sides: $|A| > |B|$ means $A^2 > B^2$.
- Case $x^2 \geq 1$ (that is, $x \leq -1$ or $x \geq 1$): $x^2 - 1 > 2x + 2$, so $x^2 - 2x - 3 > 0$, $(x-3)(x+1) > 0$, giving $x < -1$ or $x > 3$. Both parts lie in this case's range.

Euler's relation and de Moivre's theorem

- **Euler's relation** is
- So a complex number with modulus r and argument θ can be written $z = re^{i\theta}$, and then $z^n = r^n e^{in\theta}$.
- It also gives $\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$ and $\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$.
- **De Moivre's theorem** states that

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Exercises 2.1·2.3

2.1 Solve $|x - 3| < 5$. [2]

2.2 Solve $|2x + 1| \geq 7$. [2]

2.3 Solve $|x - 1| > 2x$. [3]

Exercises 2.1·2.3

2.1 Solve $|x - 3| < 5$. [2]

2.1 $-5 < x - 3 < 5$, so $-2 < x < 8$.

2.2 Solve $|2x + 1| \geq 7$. [2]

2.2 $2x + 1 \geq 7$ or $2x + 1 \leq -7$: $x \geq 3$ or $x \leq -4$.

2.3 Solve $|x - 1| > 2x$. [3]

2.3 Where $x \geq 1$: $x - 1 > 2x$ gives $x < -1$, which contradicts $x \geq 1$ – no solutions. Where $x < 1$: $1 - x > 2x$ gives $x < \frac{1}{3}$. Solution: $x < \frac{1}{3}$.

Exercises 2.4·3.1

2.4 Solve $|x^2 - 4| \leq 3x$. [4]

3.1 *(Original.)* Use algebra to solve $|x + 2| > |2x - 1|$. [4]

Exercises 2.4·3.1

2.4 Solve $|x^2 - 4| \leq 3x$. [4]

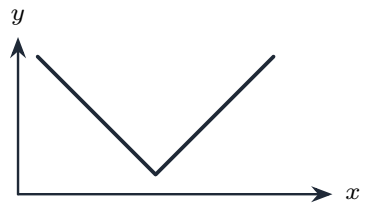
2.4 Where $x^2 \geq 4$ ($x \leq -2$ or $x \geq 2$): $x^2 - 3x - 4 \leq 0$, so $-1 \leq x \leq 4$; keeping the case gives $2 \leq x \leq 4$. Where $x^2 < 4$ ($-2 < x < 2$): $x^2 + 3x - 4 \geq 0$, so $x \leq -4$ or $x \geq 1$; keeping the case gives $1 \leq x < 2$. Union: $1 \leq x \leq 4$. (Check $x = 4$: $|12| = 12 = 3(4)$ ✓; $x = 0.5$: $|-3.75| > 1.5$ ×, outside.)

3.1 *(Original.)* Use algebra to solve $|x + 2| > |2x - 1|$. [4]

3.1 Squaring: $(x + 2)^2 > (2x - 1)^2$, so $x^2 + 4x + 4 > 4x^2 - 4x + 1$, i.e. $3x^2 - 8x - 3 < 0$. Factorising $(3x + 1)(x - 3) < 0$ gives $-\frac{1}{3} < x < 3$.

Exercise 3.2

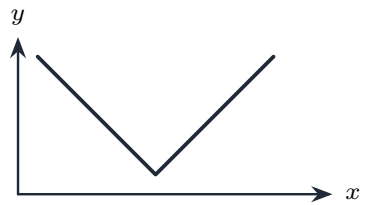
3.2 *(Edexcel IAL WFM02 June 2025 Q8.)* The curve C has equation $y = \dfrac{15x}{|x| + 4}$ and the line l has equation $y = x - 2$. (a) Use algebra to determine the exact values of x for which $x - 2 > \dfrac{15x}{|x| + 4}$. (b) Hence use algebra to determine the exact values of x for which $|x - 2| > \left| \dfrac{15x}{|x| + 4} \right|$. [10]



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Full solution on next slide

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Full solution 3.2

3.2 (a) The denominator $|x| + 4 > 0$ always. For $x \geq 0$: $(x-2)(x+4) > 15x \Rightarrow x^2 - 13x - 8 > 0$, roots $\frac{13 \pm \sqrt{201}}{2}$, so $x > \frac{13+\sqrt{201}}{2}$ (the other root is negative). For $x < 0$: $(x-2)(4-x) > 15x \Rightarrow -x^2 - 9x - 8 > 0 \Rightarrow (x+1)(x+8) < 0$, so $-8 < x < -1$. Solution: $-8 < x < -1$ or $x > \frac{13+\sqrt{201}}{2}$. (b) Both sides non-negative, so squaring preserves the inequality: $(x-2)^2 (|x|+4)^2 > 225x^2$ factorises as $\left[(x-2)(|x|+4) - 15x\right]\left[(x-2)(|x|+4) + 15x\right] > 0$, so the two brackets P and Q must have the same sign (not "$P > 0$ or $Q < 0$"). For $x \geq 0$: $P = x^2 - 13x - 8$ and $Q = x^2 + 17x - 8$; both positive for $x > \frac{13+\sqrt{201}}{2}$, both negative for $0 \leq x < \frac{\sqrt{321}-17}{2}$. For $x < 0$: $Q = -x^2 + 21x - 8 < 0$ for every $x < 0$ (the roots $\frac{21 \pm \sqrt{409}}{2}$ are both positive), so we need $P < 0$, i.e. $(x+1)(x+8) > 0$: $x < -8$ or $-1 < x < 0$. Solution: $x < -8$ or $-1 < x < \frac{\sqrt{321}-17}{2}$ or $x > \frac{13+\sqrt{201}}{2}$. (Check $x = -5$: $7 < \frac{75}{9}$, so it is correctly excluded.)

Local review guidance: cases named with their defining condition; valid multiplication/squaring step with sign justified; exact surd roots carried through; "hence" reuses part (a) rather than restarting. This is authored classroom guidance.

Exercise 3.3 (a)

3.3 The figure shows the graphs of $y = |x^2 - 2x - 3|$ and $y = x + 1$.

(a) Find the x-coordinates of all the points where the two graphs meet. [4]

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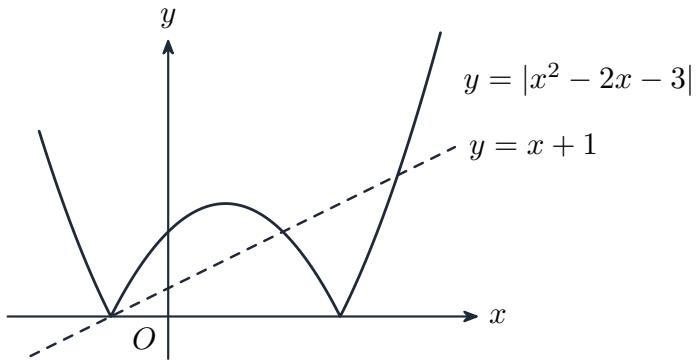
(a)

- $x^2 - 2x - 3 = (x - 3)(x + 1)$, which is negative only for $-1 < x < 3$
- where $x^2 - 2x - 3 \geq 0$: $x^2 - 2x - 3 = x + 1 \Rightarrow x^2 - 3x - 4 = 0 \Rightarrow (x - 4)(x + 1) = 0$, so $x = 4$ or $x = -1$
- where $x^2 - 2x - 3 < 0$: $-(x^2 - 2x - 3) = x + 1 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x - 2)(x + 1) = 0$, so $x = 2$
- the graphs meet at $x = \mathbf{-1}$, $x = \mathbf{2}$ and $x = \mathbf{4}$ (at $x = -1$ both are zero)

Exercise 3.3 (b)

3.3 The figure shows the graphs of $y = |x^2 - 2x - 3|$ and $y = x + 1$.

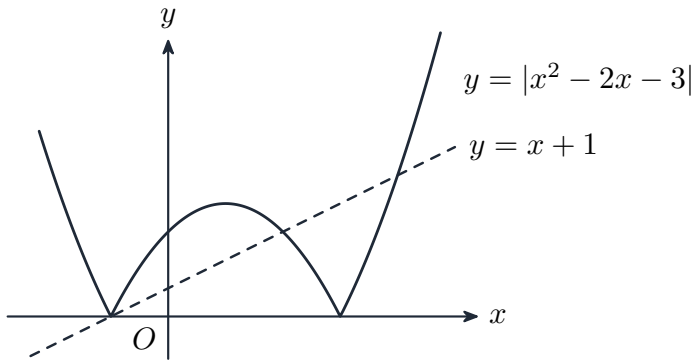
(b) Hence solve the inequality $|x^2 - 2x - 3| > x + 1$. [2]



Exercise 3.3 (b)

3.3 The figure shows the graphs of $y = |x^2 - 2x - 3|$ and $y = x + 1$.

(b) Hence solve the inequality $|x^2 - 2x - 3| > x + 1$. [2]



(b)

- the curve is above the line except between the meeting points 2 and 4, and at $x = -1$, where they are equal
- $\boldsymbol{x < -1}$, $\boldsymbol{-1 < x < 2}$ or $\boldsymbol{x > 4}$

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