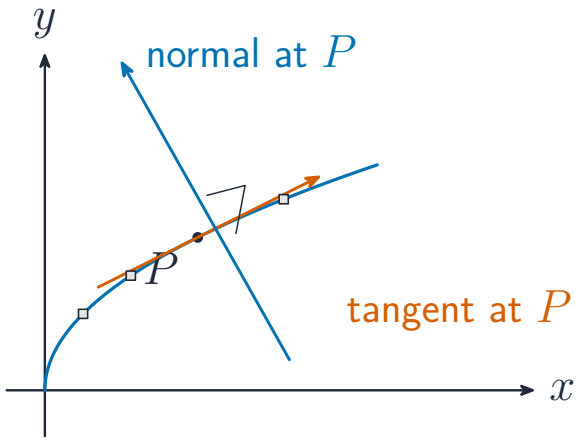
Exercise walk-through

4.2
Parametric equations and differentiation

Pearson Edexcel · International A-Level · Pure Mathematics

Figure: Parametric equations

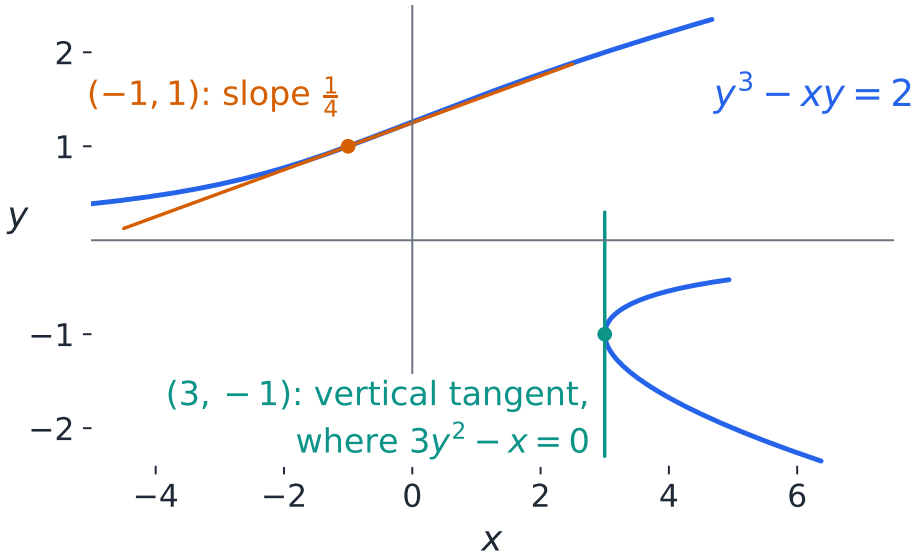


squares mark parameter values

Parametric gradient: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$;
the normal takes its negative reciprocal.

- Tangent: dy/dx = (dy/dt)/(dx/dt).
- Normal: the negative reciprocal gradient.

Figure: Implicit and parametric differentiation



- Where the denominator of $\frac{dy}{dx}$ is zero (and the numerator is not), the tangent is vertical
- Read the axes and units first.

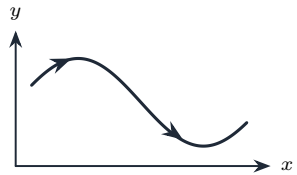
Integration: substitution, parts and partial fractions

- **Substitution** (usually given): rewrite the integral in u, adjust dx via $\frac{du}{dx}$, integrate, substitute back. It is the chain rule reversed.
- **Integration by parts** : $\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$ — the product rule reversed. Know $\int \ln x\,dx = x\ln x - x + c$ and repeated parts for $\int x^2 e^x dx$-type integrals.

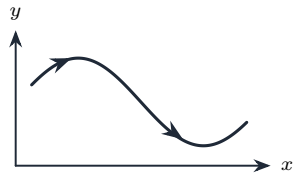
$$\int e^x \cos x\,dx = e^x \sin x - \int e^x \sin x\,dx$$

Exercises 2.1·2.2

2.1 Convert $x = 3t$, $y = t^2$ to a Cartesian equation. [2]

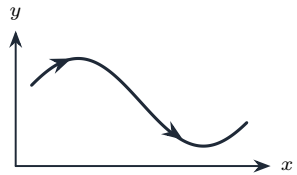


2.2 For $x = t^2$, $y = 2t$ $(t > 0)$, find $\dfrac{dy}{dx}$ at $t = 3$. [3]



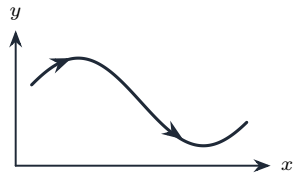
Exercises 2.1·2.2

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2.1 $t = \frac{x}{3} \Rightarrow y = \frac{x^2}{9}$.

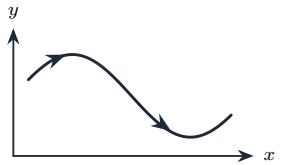
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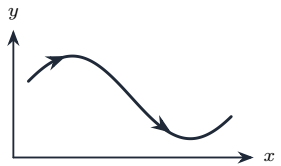
2.2 $\dfrac{dx}{dt} = 2t = 6$, $\dfrac{dy}{dt} = 2$: $\dfrac{dy}{dx} = \frac{2}{6} = \frac{1}{3}$.

Exercises 2.3·2.4

2.3 For $x = \cos t$, $y = \sin t$, find $\dfrac{dy}{dx}$ and state the curve. [3]

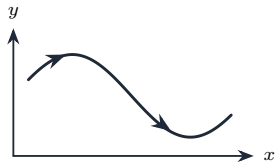


2.4 A curve has $x = t + 1$, $y = t^2 - 3t$. Find the equation of the tangent at $t = 2$. [4]



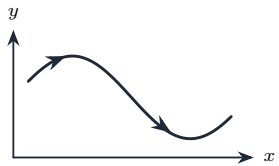
Exercises 2.3·2.4

2.3 For $x = \cos t$, $y = \sin t$, find $\dfrac{dy}{dx}$ and state the curve. [3]



2.3 $\dfrac{dy}{dx} = \dfrac{\cos t}{-\sin t} = -\cot t$; eliminating t gives $x^2 + y^2 = 1$ (the unit circle).

2.4 A curve has $x = t + 1$, $y = t^2 - 3t$. Find the equation of the tangent at $t = 2$. [4]



2.4 $\dfrac{dy}{dx} = \dfrac{2t-3}{1}$; at $t = 2$: gradient 1, point $(3, -2)$. Tangent: $y + 2 = x - 3 \implies y = x - 5$.

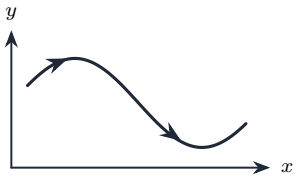
exercises.lol exercises.com.cn

Exercise 3.1

3.1 *(Demand adapted from Edexcel IAL WMA14 June 2025.)* A curve is given by $x = 4t$, $y = \dfrac{4}{t}$ $(t > 0)$.

(a) Find $\dfrac{dy}{dx}$ in terms of t.

(b) Find the equation of the normal at $t = 2$, in the form $ax + by + c = 0$ with integer coefficients. [6]

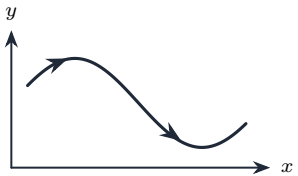


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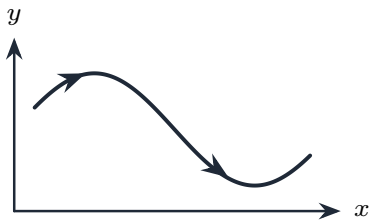
3.1 (a) $\dfrac{dx}{dt} = 4$, $\dfrac{dy}{dt} = -\dfrac{4}{t^2}$:

$$\frac{dy}{dx} = -\frac{1}{t^2}$$

(b) At $t = 2$: gradient $-\frac{1}{4}$, point $(8, 2)$. Normal gradient 4: $y - 2 = 4(x - 8) \implies 4x - y - 30 = 0$. Check: $4(8) - 2 - 30 = 0$ ✓; the gradients multiply to $-\frac{1}{4} \times 4 = -1$ ✓.

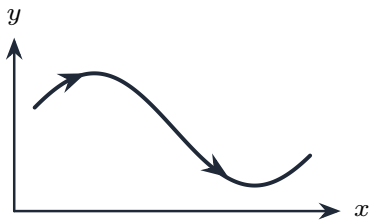
Exercise 3.2

3.2 A curve has $x = t^2$, $y = t^3$. Find the Cartesian equation and the tangent at $t = -1$. [6]



Exercise 3.2

3.2 A curve has $x = t^2$, $y = t^3$. Find the Cartesian equation and the tangent at $t = -1$. [6]



3.2 Cartesian: $t^2 = x$, $t^3 = y \Rightarrow y^2 = t^6 = (t^2)^3 = x^3$, so $y^2 = x^3$. At $t = -1$: point $(1, -1)$, $\dfrac{dy}{dx} = \dfrac{3t^2}{2t} = \frac{3t}{2} = -\frac{3}{2}$. Tangent: $y + 1 = -\frac{3}{2}(x - 1) \implies 3x + 2y - 1 = 0$. Check: $3(1) + 2(-1) - 1 = 0$ ✓; and $(1, -1)$ satisfies $y^2 = x^3$ ✓.

Local review guidance: both derivatives with respect to t found; gradient value at the parameter; line through the correct point in the requested form. This is authored classroom guidance.