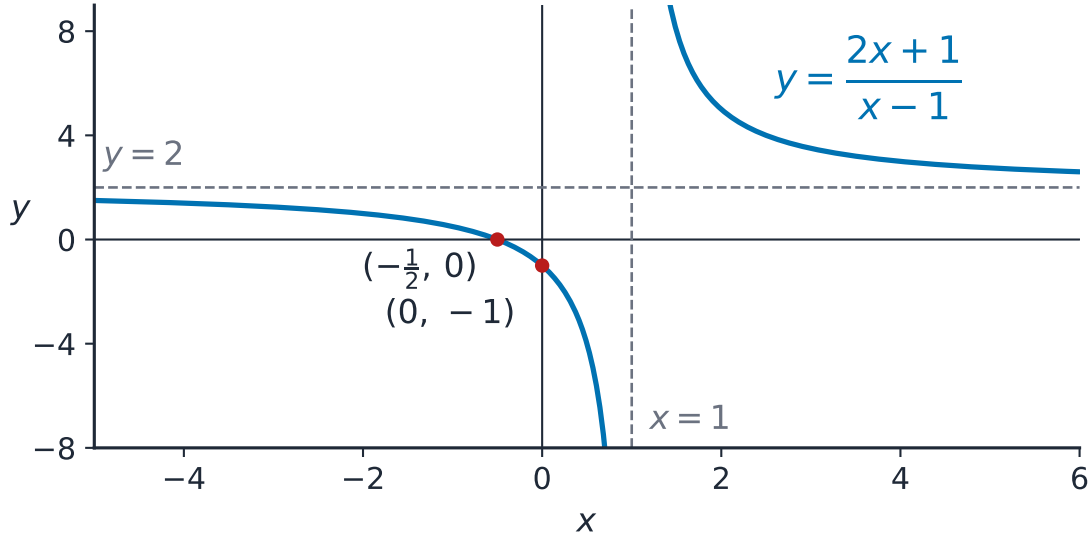
Exercise walk-through

3.6
Integration and numerical methods

Pearson Edexcel · International A-Level · Pure Mathematics

Figure: Algebra: rational expressions and functions



- Asymptotes first, then intercepts, then the two branches
- Read the axes and units first.

Integration: standard forms and recognition

- **Double angle** with $A = 3x$: $\cos 6x \equiv 1 - 2\sin^2 3x$, so $\sin^2 3x = \frac{1}{2}(1 - \cos 6x)$:
- Check by differentiating: $\frac{1}{2} - \frac{6\cos 6x}{12} = \frac{1}{2}(1 - \cos 6x) = \sin^2 3x$ ✓.
- The numerator is the derivative of the denominator ($f' = 6x$, $f = 3x^2 + 4$):
- Check: $\dfrac{d}{dx}\ln(3x^2 + 4) = \dfrac{6x}{3x^2 + 4}$ ✓. The powered form works the same way: with $u = 3x^2 + 4$ and $du = 6x\,dx$,

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c \qquad \int f'(x)\big(f(x)\big)^n\,dx = \frac{\big(f(x)\big)^{n+1}}{n+1} + c \quad (n \neq -1)$$

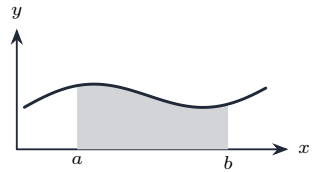
Numerical methods: sign changes and iteration

- **Sign change** : if f is continuous on $[a, b]$ and $f(a)f(b) < 0$, a root lies between a and b. Halve the interval to pin a root to any accuracy.
- **Iteration** : a rearrangement $x_{n+1} = f(x_n)$ from a start value converges to a **fixed point** (or not — always check against a sign-change bound).
- Check the **iteration** converges: $|x_3 - x_2|$ small and the values settle toward 0.43 ✓.
- g is continuous, and the **sign change** between 0.4295 and 0.4305 brackets a root, so the root is 0.430 to 3 d.p. ✓.

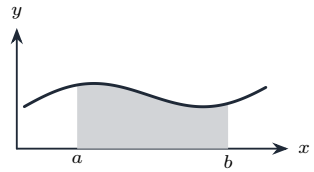
$$x_2 = \tfrac{1}{3}\arctan\left(\tfrac{3}{0.8}\right) = \tfrac{1}{3}\arctan(3.75) = 0.4367 \qquad x_4 = 0.4307 \text{ (4 d.p.)}$$

Exercises 2.1·2.2

2.1 Find $\int e^{5x}\,dx$. [2]

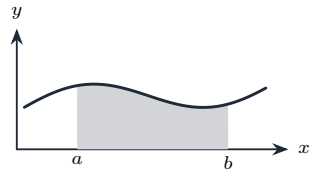


2.2 Find $\int \frac{3}{x}\,dx$ $(x > 0)$. [2]



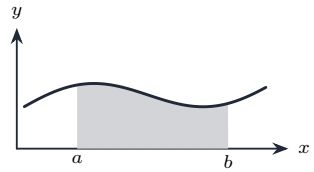
Exercises 2.1·2.2

2.1 Find $\int e^{5x}\,dx$. [2]



2.1 $\frac{1}{5}e^{5x} + c$.

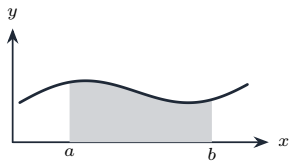
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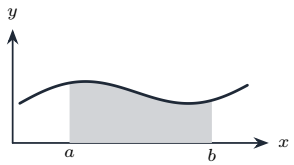
2.2 $3\ln x + c$.

Exercises 2.3·2.4

2.3 Find $\int \cos 4x \, dx$. [2]

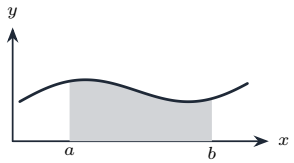


2.4 Find $\int \dfrac{2x+2}{x^2+2x+5} \, dx$. [3]



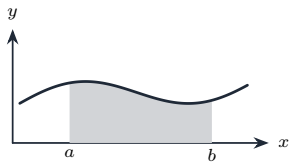
Exercises 2.3·2.4

2.3 Find $\int \cos 4x \, dx$. [2]



2.3 $\frac{1}{4}\sin 4x + c$.

2.4 Find $\displaystyle\int \frac{2x+2}{x^2+2x+5}\, dx$. [3]

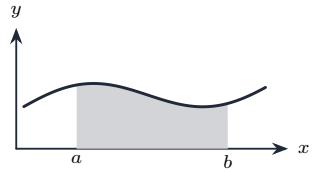


2.4 Numerator $= (x^2+2x+5)'$: $\ln(x^2+2x+5) + c$ (the quadratic is always positive, so no modulus needed).

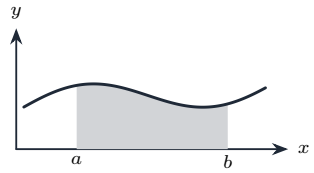
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Exercises 2.5·2.6

2.5 Find $\int 3x^2(x^3+1)^4\,dx$. [3]

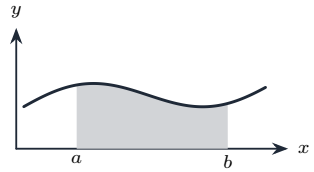


2.6 Show that $x^3 - 3x + 1 = 0$ has a root between 0 and 1. [2]



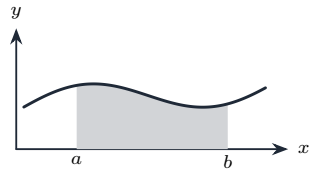
Exercises 2.5·2.6

2.5 Find $\int 3x^2(x^3+1)^4\,dx$. [3]



2.5 With $u = x^3+1$, $du = 3x^2dx$: $\frac{1}{5}(x^3+1)^5 + c$.

2.6 Show that $x^3 - 3x + 1 = 0$ has a root between 0 and 1. [2]



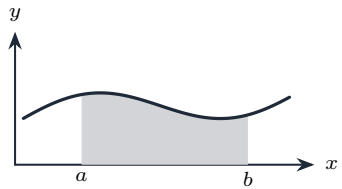
2.6 $f(x) = x^3 - 3x + 1$: $f(0) = 1 > 0$, $f(1) = -1 < 0$; f is continuous (polynomial), and the sign change over $[0, 1]$ guarantees a root in $(0, 1)$.

Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA13 June 2025 Q5 and Q7.)*

(a) Find $\int \cos^2 2x \, dx$.

(b) Find $\int \dfrac{x}{x^2 + 4} \, dx$. [5]

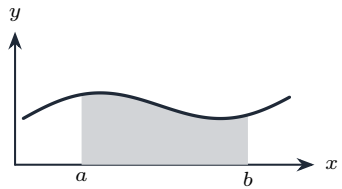


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(a) Find $\displaystyle\int \cos^2 2x\,dx$.

(b) Find $\displaystyle\int \frac{x}{x^2+4}\,dx$. [5]



3.1 (a) $\cos^2 2x = \frac{1}{2}(1+\cos 4x)$:

$$\int \cos^2 2x\,dx = \frac{x}{2} + \frac{\sin 4x}{8} + c$$

Check: $\frac{1}{2} + \frac{4}{8}\cos 4x = \frac{1}{2}(1+\cos 4x) = \cos^2 2x$ ✓. (b) Numerator $x = \frac{1}{2}(x^2+4)'$: $\frac{1}{2}\ln(x^2+4) + c$.

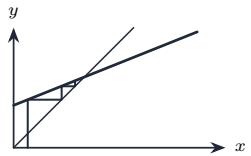
Exercise 3.2

3.2 *(Adapted from Edexcel IAL WMA13 June 2025 Q7.)* The equation $x = \mathrm{e}^{-x}$ has exactly one positive root.

(a) Show the root lies between 0.5 and 0.6.

(b) Use the iteration $x_{n+1} = \mathrm{e}^{-x_n}$ from $x_1 = 0.5$ to find x_3 to 3 decimal places.

(c) Verify the root is 0.567 to 3 decimal places. [7]



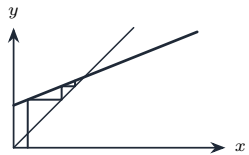
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(c) Verify the root is 0.567 to 3 decimal places. [7]



3.2 (a) $g(x) = x - \mathrm{e}^{-x}$: $g(0.5) = 0.5 - 0.6065 = -0.1065 < 0$ and $g(0.6) = 0.6 - 0.5488 = 0.0512 > 0$; g is continuous, so a root lies in $(0.5, 0.6)$. (b) $x_2 = \mathrm{e}^{-0.5} = 0.6065$; $x_3 = \mathrm{e}^{-0.6065} = 0.545$ (3 d.p.). (c) $g(0.5665) = 0.5665 - \mathrm{e}^{-0.5665} = 0.5665 - 0.5675 = -0.0010 < 0$ and $g(0.5675) = 0.5675 - 0.5669 = +0.0006 > 0$: the sign change between 0.5665 and 0.5675 fixes the root as 0.567 to 3 d.p. ✓ (True root: the omega constant $0.5671\ldots$ ✓.)

Local review guidance: correct standard form or recognition of f' over f; exact integrals with $+c$; sign-change statement naming continuity, and the iteration carried with enough steps. This is authored classroom guidance.

Exercise 3.3 (a)

3.3 The figure shows part of the curve with equation $y = \mathrm{e}^{\frac{1}{2}x} - x$. The region R is bounded by the curve, the coordinate axes and the line $x = 2$.

x	**0**	**0.5**	**1**	**1.5**	**2**
y	1		0.6487		0.7183

(a) Complete the table, giving the values of y to 4 decimal places. [2]

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y	1		0.6487		0.7183

(a) Complete the table, giving the values of y to 4 decimal places. [2]

(a)

- $x = 0.5$: $\mathrm{e}^{0.25} - 0.5 = \mathbf{0.7840}$; $x = 1.5$: $\mathrm{e}^{0.75} - 1.5 = \mathbf{0.6170}$

Exercise 3.3 (b)

3.3 The figure shows part of the curve with equation $y = \mathrm{e}^{\frac{1}{2}x} - x$. The region R is bounded by the curve, the coordinate axes and the line $x = 2$.

x	**0**	**0.5**	**1**	**1.5**	**2**
y	1		0.6487		0.7183

(b) Use the trapezium rule with all the values of y in the table to find an estimate for the area of R, to 4 decimal places. [3]

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(b)

- trapezium rule with $h = 0.5$, $A \approx \frac{h}{2}\left[y_0 + y_4 + 2(y_1 + y_2 + y_3)\right]$:

$$A \approx \frac{0.5}{2}\left[1 + 0.7183 + 2(0.7840 + 0.6487 + 0.6170)\right] = 0.25(5.8177) = \mathbf{1.4544}$$

Exercise 3.3 (c)

3.3 The figure shows part of the curve with equation $y = \mathrm{e}^{\frac{1}{2}x} - x$. The region R is bounded by the curve, the coordinate axes and the line $x = 2$.

x	**0**	**0.5**	**1**	**1.5**	**2**
y	1		0.6487		0.7183

(c) Use integration to find the exact area of R, in the form $a\mathrm{e} + b$. [3]

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(c)

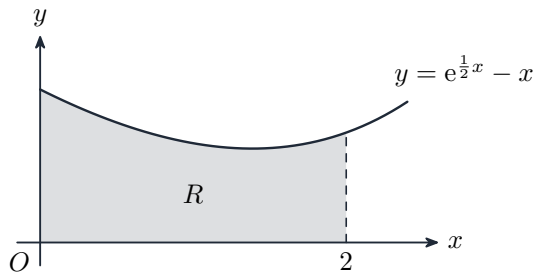
- using $\int \mathrm{e}^{kx}\,dx = \frac{1}{k}\mathrm{e}^{kx}$: $\int_0^2 (\mathrm{e}^{x/2} - x)\,dx = [2\mathrm{e}^{x/2} - \frac{1}{2}x^2]_0^2 = (2\mathrm{e} - 2) - 2 = \mathbf{2}\mathrm{e} - \mathbf{4}$

Exercise 3.3 (d)

3.3 The figure shows part of the curve with equation $y = e^{\frac{1}{2}x} - x$. The region R is bounded by the curve, the coordinate axes and the line $x = 2$.

x	**0**	**0.5**	**1**	**1.5**	**2**
y	1		0.6487		0.7183

(d) Find the percentage error in the estimate in (b), and explain, with reference to the shape of the curve, why the trapezium rule gives an overestimate. [3]

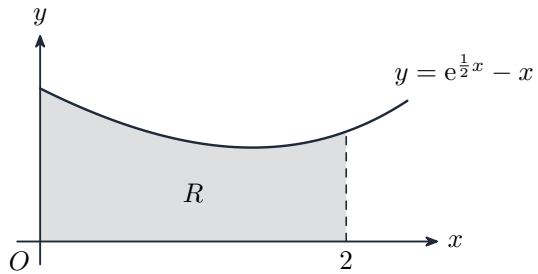


Exercise 3.3 (d)

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y	1		0.6487		0.7183

(d) Find the percentage error in the estimate in (b), and explain, with reference to the shape of the curve, why the trapezium rule gives an overestimate. [3]



(d)

- percentage error $= \frac{\text{estimate}-\text{exact}}{\text{exact}} \times 100 = \frac{1.4544-1.4366}{1.4366} \times 100 = \mathbf{1.24}\%$
- the curve is convex (bends upwards), so each trapezium's top edge lies above it