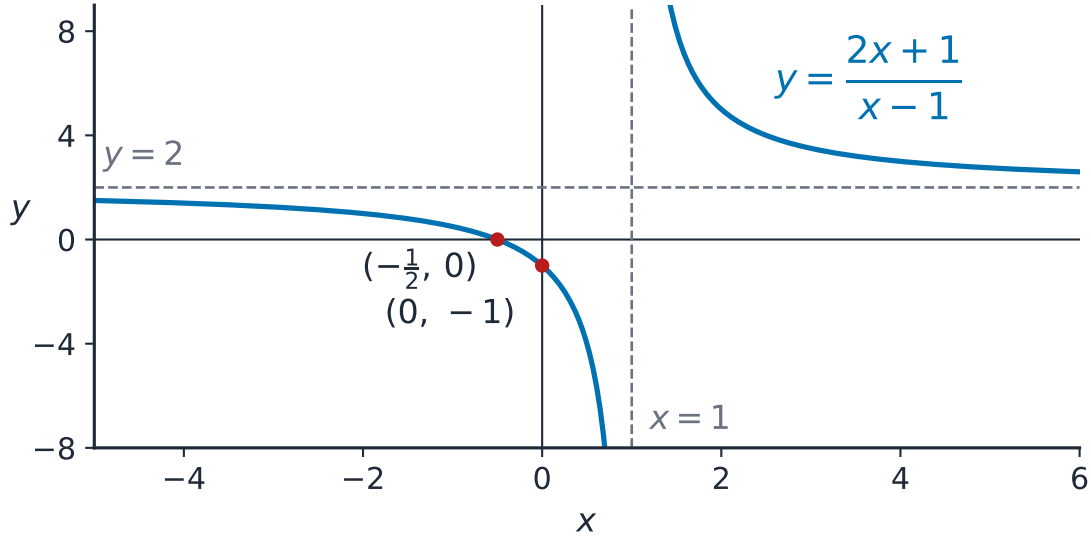
Exercise walk-through

3.5
Differentiation rules and implicit work

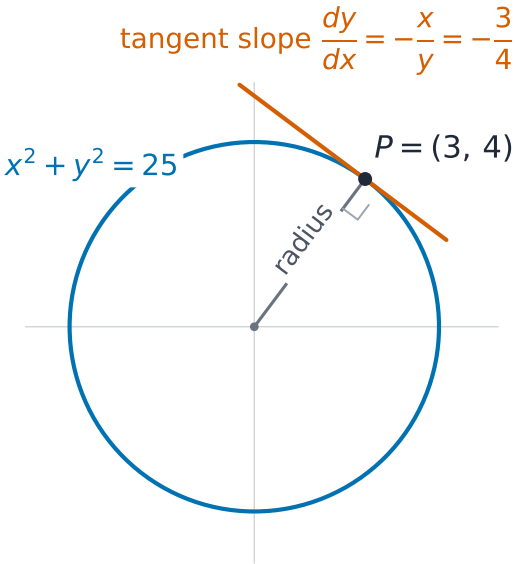
Pearson Edexcel · International A-Level · Pure Mathematics

Figure: Algebra: rational expressions and functions



- Asymptotes first, then intercepts, then the two branches
- Read the axes and units first.

Figure: Differentiation: rules, implicit and models



- **Implicit differentiation** gives the tangent to a circle, perpendicular to the radius
- Match each part of the diagram to the text.

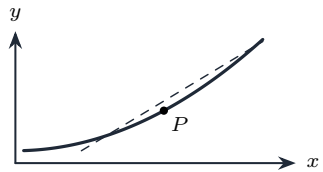
Integration: standard forms and recognition

- **Double angle** with $A = 3x$: $\cos 6x \equiv 1 - 2\sin^2 3x$, so $\sin^2 3x = \frac{1}{2}(1 - \cos 6x)$:
- Check by differentiating: $\frac{1}{2} - \frac{6\cos 6x}{12} = \frac{1}{2}(1 - \cos 6x) = \sin^2 3x$ ✓.
- The numerator is the derivative of the denominator ($f' = 6x$, $f = 3x^2 + 4$):
- Check: $\dfrac{d}{dx}\ln(3x^2 + 4) = \dfrac{6x}{3x^2 + 4}$ ✓. The powered form works the same way: with $u = 3x^2 + 4$ and $du = 6x\,dx$,

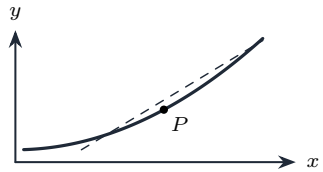
$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c \qquad \int f'(x)\big(f(x)\big)^n\,dx = \frac{\big(f(x)\big)^{n+1}}{n+1} + c \quad (n \neq -1)$$

Exercises 2.1·2.2

2.1 Differentiate $y = \sin 5x$. [1]

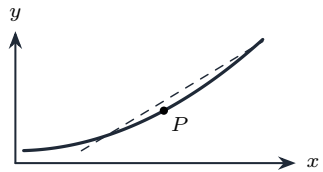


2.2 Differentiate $y = \mathrm{e}^{-3x}$. [1]



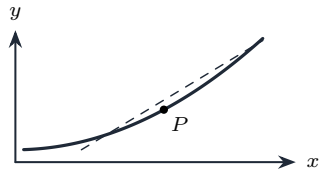
Exercises 2.1·2.2

2.1 Differentiate $y = \sin 5x$. [1]



2.1 $\dfrac{dy}{dx} = 5\cos 5x$.

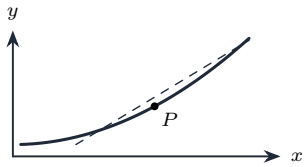
2.2 Differentiate $y = \mathrm{e}^{-3x}$. [1]



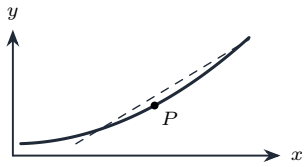
2.2 $\dfrac{dy}{dx} = -3\mathrm{e}^{-3x}$.

Exercises 2.3·2.4

2.3 Differentiate $y = x^2 \cos x$ using the product rule. [3]

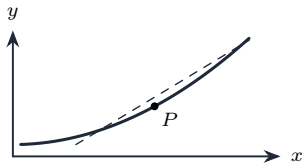


2.4 Differentiate $y = \dfrac{e^x}{x}$ using the quotient rule. [3]



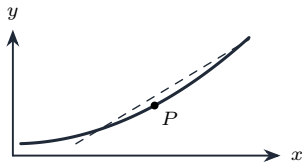
Exercises 2.3·2.4

2.3 Differentiate $y = x^2 \cos x$ using the product rule. [3]



2.3 $\dfrac{dy}{dx} = 2x\cos x - x^2 \sin x.$

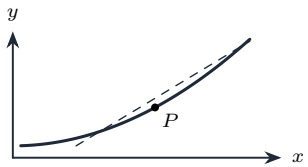
2.4 Differentiate $y = \dfrac{\mathrm{e}^x}{x}$ using the quotient rule. [3]



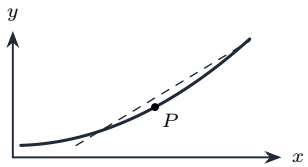
2.4 $\dfrac{dy}{dx} = \dfrac{\mathrm{e}^x \cdot x - \mathrm{e}^x \cdot 1}{x^2} = \dfrac{\mathrm{e}^x(x-1)}{x^2}.$

Exercises 2.5·2.6

2.5 Differentiate $y = \ln(4x^3 + 1)$ using the chain rule. [2]

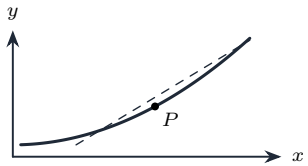


2.6 Find $\dfrac{dy}{dx}$ for $x = \mathrm{e}^{2y}$. [3]



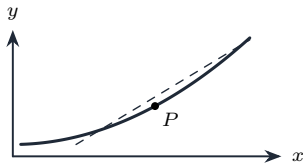
Exercises 2.5·2.6

2.5 Differentiate $y = \ln(4x^3 + 1)$ using the chain rule. [2]



2.5 $\dfrac{dy}{dx} = \dfrac{12x^2}{4x^3 + 1}$ (chain: outer ln, inner $4x^3 + 1$).

2.6 Find $\dfrac{dy}{dx}$ for $x = \mathrm{e}^{2y}$. [3]



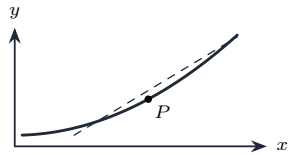
2.6 $\dfrac{dx}{dy} = 2\mathrm{e}^{2y} = 2x$, so $\dfrac{dy}{dx} = \dfrac{1}{2x}$.

Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA13 June 2025 Q7.)* The curve C has equation $y = \mathrm{e}^{-x^2}\sin 3x$ for $0 \leq x \leq 3$. Show, using calculus, that the x-coordinates of the stationary points of C satisfy

$$x = \tfrac{1}{3}\arctan\frac{3}{2x}$$

[4]

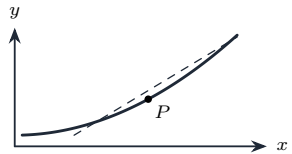


Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA13 June 2025 Q7.)* The curve C has equation $y = \mathrm{e}^{-x^2}\sin 3x$ for $0 \leq x \leq 3$. Show, using calculus, that the x-coordinates of the stationary points of C satisfy

$$x = \tfrac{1}{3}\arctan\frac{3}{2x}$$

[4]



3.1 From the worked derivative: stationary points have

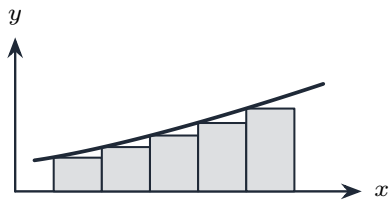
$$\mathrm{e}^{-x^2}\left(3\cos 3x - 2x\sin 3x\right) = 0$$

$\mathrm{e}^{-x^2} > 0$ always, so $3\cos 3x = 2x\sin 3x$. Dividing by $\cos 3x$ (at a stationary point $\cos 3x \neq 0$, since that would force $\sin 3x = \pm 1 \neq 0$ and then $0 = \pm 2x$ fails for $x > 0$):

$$3 = 2x\tan 3x \implies \tan 3x = \frac{3}{2x} \implies 3x = \arctan\frac{3}{2x} \implies x = \tfrac{1}{3}\arctan\frac{3}{2x} \qquad \blacksquare$$

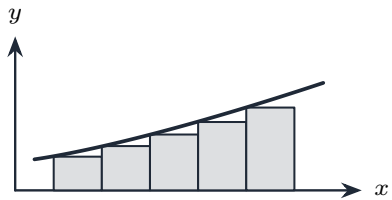
Exercise 3.2

3.2 A cylindrical tank drains so that the volume of water is $V = 500\mathrm{e}^{-0.1t}$ litres after t minutes. Find the rate at which water leaves at $t = 10$, and the time at which the rate is 5 litres per minute. [5]



Exercise 3.2

3.2 A cylindrical tank drains so that the volume of water is $V = 500\mathrm{e}^{-0.1t}$ litres after t minutes. Find the rate at which water leaves at $t = 10$, and the time at which the rate is 5 litres per minute. [5]



3.2 $\dfrac{dV}{dt} = -50\mathrm{e}^{-0.1t}$; at $t = 10$: $-50\mathrm{e}^{-1} = -18.4$ litres per minute (leaving at 18.4 L/min). Rate $= 5$: $50\mathrm{e}^{-0.1t} = 5 \Rightarrow \mathrm{e}^{-0.1t} = 0.1 \Rightarrow t = 10\ln 10 = 23.0$ minutes (3 s.f.). Check: $V(23.0) = 500\mathrm{e}^{-2.303} = 50.0$ litres, one tenth of the initial – matching the rate being one tenth of the initial 50 L/min ✓.

Local review guidance: correct rule with both parts differentiated; simplified derivative; exponential-rate equation solved with logs. This is authored classroom guidance.

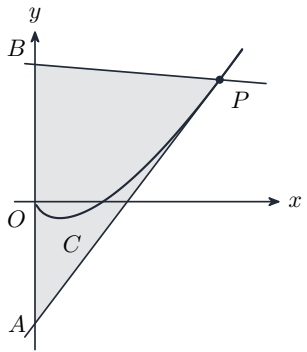
Exercise 3.3

3.3 The curve C has equation $y = 2x \ln x$, $x > 0$. The point P on C has x-coordinate e. The tangent to C at P meets the y-axis at A, and the normal to C at P meets the y-axis at B.

(a) Show that $\dfrac{dy}{dx} = 2 \ln x + 2$. [2]

(b) Find an equation of the tangent to C at P. [3]

(c) Find the exact area of the triangle APB. [4]



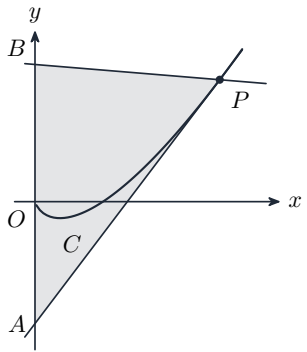
Exercise 3.3

3.3 The curve C has equation $y = 2x \ln x$, $x > 0$. The point P on C has x-coordinate e. The tangent to C at P meets the y-axis at A, and the normal to C at P meets the y-axis at B.

(a) Show that $\dfrac{dy}{dx} = 2 \ln x + 2$. [2]

(b) Find an equation of the tangent to C at P. [3]

(c) Find the exact area of the triangle APB. [4]



3.3

(a)

Full solution on next slide

exercises.lol exercises.com.cn

Full solution 3.3

3.3

(a)

- product rule, $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$, with $u = 2x$, $v = \ln x$:

$$\frac{dy}{dx} = 2x \cdot \frac{1}{x} + 2\ln x = 2\ln x + 2$$

(b)

- at $x = \mathrm{e}$: $y = 2\mathrm{e}\ln \mathrm{e} = 2\mathrm{e}$, gradient $m = 2\ln \mathrm{e} + 2 = 4$
- $y - y_1 = m(x - x_1)$: $y - 2\mathrm{e} = 4(x - \mathrm{e})$, so $\boldsymbol{y = 4x - 2\mathrm{e}}$

(c)

- at $x = 0$ the tangent gives $A(0,\ -2\mathrm{e})$
- the normal has gradient $-\frac{1}{4}$: $y - 2\mathrm{e} = -\frac{1}{4}(x - \mathrm{e})$; at $x = 0$, $y = 2\mathrm{e} + \frac{1}{4}\mathrm{e} = \frac{9}{4}\mathrm{e}$, so $B(0,\ \frac{9}{4}\mathrm{e})$
- base AB on the y-axis, height the x-coordinate of P, area $= \frac{1}{2} \times AB \times \mathrm{e}$:

$$\text{area} = \tfrac{1}{2}\left(\tfrac{9}{4}\mathrm{e} + 2\mathrm{e}\right)\mathrm{e} = \tfrac{1}{2} \cdot \tfrac{17}{4}\mathrm{e} \cdot \mathrm{e} = \boldsymbol{\tfrac{17}{8}\mathrm{e}^2}$$