Exercise walk-through

3.3
Reciprocal trig and harmonic form

Pearson Edexcel · International A-Level · Pure Mathematics

Further trigonometry: sec, cosec, cot and harmonic form (2)

- Write $3x = 2x + x$ and use the **compound angle** formula:
- Substitute:
- Check at $x = \frac{\pi}{6}$ ($t = \frac{1}{\sqrt{3}}$): numerator $\frac{3}{\sqrt{3}} - \frac{1}{3\sqrt{3}} = \frac{8}{3\sqrt{3}}$; denominator $1 - 1 = 0$ — and $\tan\frac{\pi}{2}$ is undefined ✓ consistent.

$$\alpha = 1.287 \text{ (3 d.p.)} \qquad f(x) = 25\cos(x + 1.287)$$

Exponentials, logarithms and log-graph models

- Gradient: $\dfrac{0-(-2)}{-3.5-0} = -\dfrac{4}{7}$; intercept -2:
- Convert: $y = 10^{-\frac{4}{7}\log_{10} x - 2} = 10^{-2}\, x^{-4/7}$, so
- Check with a point on the line: $\log_{10} x = 0 \Rightarrow x = 1$, $\log_{10} y = -2 \Rightarrow y = \frac{1}{100}$ ✓ and the formula gives $y = \frac{1}{100} \cdot 1 = \frac{1}{100}$ ✓.

$$\log_{10} y = -\tfrac{4}{7}\log_{10} x - 2$$

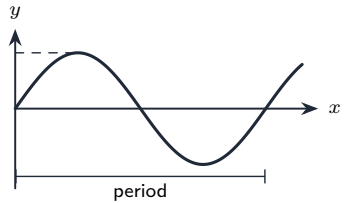
Integration: standard forms and recognition

- **Double angle** with $A = 3x$: $\cos 6x \equiv 1 - 2\sin^2 3x$, so $\sin^2 3x = \frac{1}{2}(1 - \cos 6x)$:
- Check by differentiating: $\frac{1}{2} - \frac{6\cos 6x}{12} = \frac{1}{2}(1 - \cos 6x) = \sin^2 3x$ ✓.
- The numerator is the derivative of the denominator ($f' = 6x$, $f = 3x^2 + 4$):
- Check: $\dfrac{d}{dx}\ln(3x^2 + 4) = \dfrac{6x}{3x^2 + 4}$ ✓. The powered form works the same way: with $u = 3x^2 + 4$ and $du = 6x\,dx$,

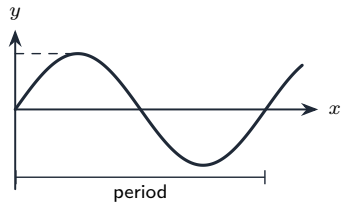
$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c \qquad \int f'(x)\big(f(x)\big)^n\,dx = \frac{\big(f(x)\big)^{n+1}}{n+1} + c \quad (n \neq -1)$$

Exercises 2.1·2.2

2.1 Write $\sec\theta$ in terms of $\cos\theta$ and state the range of $\sec\theta$. [2]

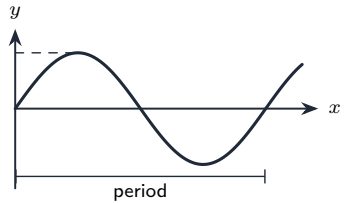


2.2 Simplify $\cot\theta\tan\theta$. [1]



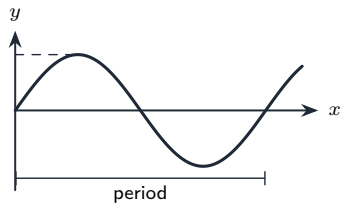
Exercises 2.1·2.2

2.1 Write $\sec\theta$ in terms of $\cos\theta$ and state the range of $\sec\theta$. [2]



2.1 $\sec\theta = \dfrac{1}{\cos\theta}$; since $|\cos\theta| \leq 1$, $\sec\theta \geq 1$ or $\sec\theta \leq -1$.

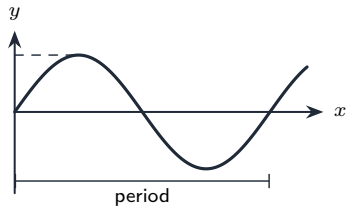
2.2 Simplify $\cot\theta\tan\theta$. [1]



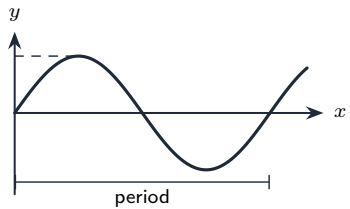
2.2 $\cot\theta\tan\theta = 1$ (where both are defined).

Exercises 2.3·2.4

2.3 State the restricted domain and range of arccos. [2]

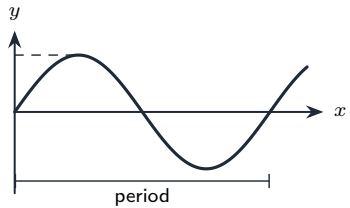


2.4 Use $\sin 2A \equiv 2\sin A\cos A$ to write $\sin A\cos A$ as a single term. [2]



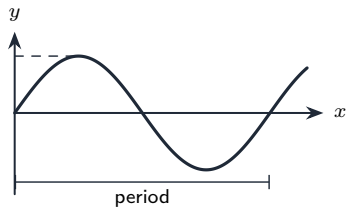
Exercises 2.3·2.4

2.3 State the restricted domain and range of arccos. [2]



2.3 arccos maps $[-1, 1]$ onto $[0, \pi]$.

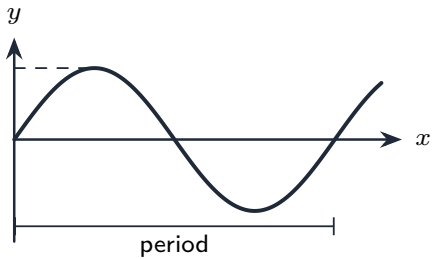
2.4 Use $\sin 2A \equiv 2 \sin A \cos A$ to write $\sin A \cos A$ as a single term. [2]



2.4 $\sin A \cos A = \frac{1}{2} \sin 2A$.

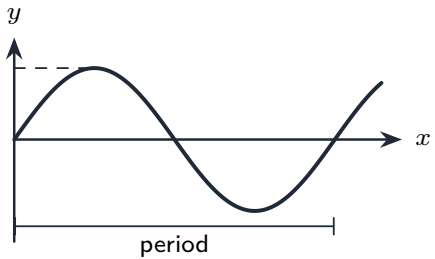
Exercise 2.5

2.5 Express $12\cos\theta + 5\sin\theta$ in the form $R\cos(\theta - \alpha)$ with exact R and $\alpha = \arctan\frac{5}{12}$. [3]



Exercise 2.5

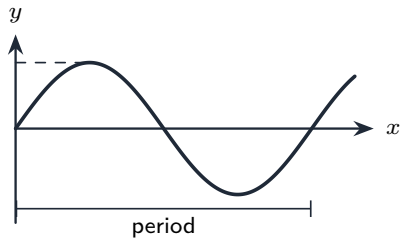
2.5 Express $12\cos\theta + 5\sin\theta$ in the form $R\cos(\theta - \alpha)$ with exact R and $\alpha = \arctan\frac{5}{12}$. [3]



2.5 $R = \sqrt{144 + 25} = 13$, $\alpha = \arctan\frac{5}{12}$: $12\cos\theta + 5\sin\theta = 13\cos(\theta - \alpha)$. Check: $13\cos\alpha = 12$ ✓ and $13\sin\alpha = 5$ ✓ (5-12-13 triangle).

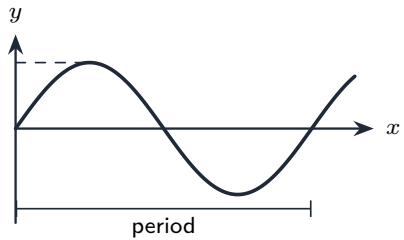
Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA13 June 2025 Q2.)* Express $8\cos x - 15\sin x$ in the form $R\cos(x+\alpha)$, $R > 0$, $0 < \alpha < \frac{\pi}{2}$, giving R exactly and α to 3 decimal places. Hence write down the maximum value of $8\cos x - 15\sin x$ and the smallest positive x at which it occurs. [6]



Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA13 June 2025 Q2.)* Express $8\cos x - 15\sin x$ in the form $R\cos(x+\alpha)$, $R > 0$, $0 < \alpha < \frac{\pi}{2}$, giving R exactly and α to 3 decimal places. Hence write down the maximum value of $8\cos x - 15\sin x$ and the smallest positive x at which it occurs. [6]



3.1 $R = \sqrt{64+225} = 17$; $17\cos\alpha = 8$, $17\sin\alpha = 15$ so $\tan\alpha = \frac{15}{8}$, $\alpha = 1.080$ (3 d.p.):

$$8\cos x - 15\sin x = 17\cos(x+1.080)$$

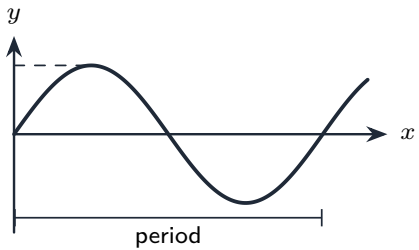
Maximum $= 17$ when $x + 1.080 = 0 \pmod{2\pi}$; the smallest positive x is $2\pi - 1.080 = 5.203$ (3 d.p.). Check: $\cos(5.203 + 1.080) = \cos(6.283) = \cos 2\pi = 1$ ✓.

Exercise 3.2

3.2 *(Adapted from Edexcel IAL WMA13 June 2025 Q8a.)* Prove, using the compound angle formulae, that

$$\tan 3x \equiv \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

[4]

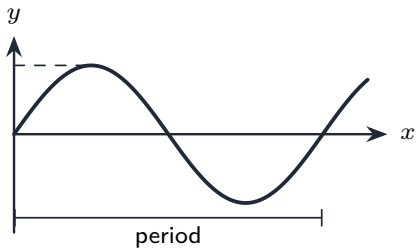


Exercise 3.2

3.2 *(Adapted from Edexcel IAL WMA13 June 2025 Q8a.)* Prove, using the compound angle formulae, that

$$\tan 3x \equiv \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

[4]



3.2 Write $3x = 2x + x$:

$$\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}, \qquad \tan 2x = \frac{2t}{1 - t^2} \quad (t = \tan x)$$

Full solution on next slide

Full solution 3.2

3.2 Write $3x = 2x + x$:

$$\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}, \qquad \tan 2x = \frac{2t}{1 - t^2} \quad (t = \tan x)$$

Substitute and multiply through by $(1 - t^2)$:

$$\tan 3x = \frac{\frac{2t}{1-t^2} + t}{1 - \frac{2t^2}{1-t^2}} = \frac{2t + t - t^3}{(1 - t^2) - 2t^2} = \frac{3t - t^3}{1 - 3t^2} \qquad \blacksquare$$

Local review guidance: correct R and matching of both components; exact or correctly rounded values; maximum read from the amplitude with its location. This is authored classroom guidance.