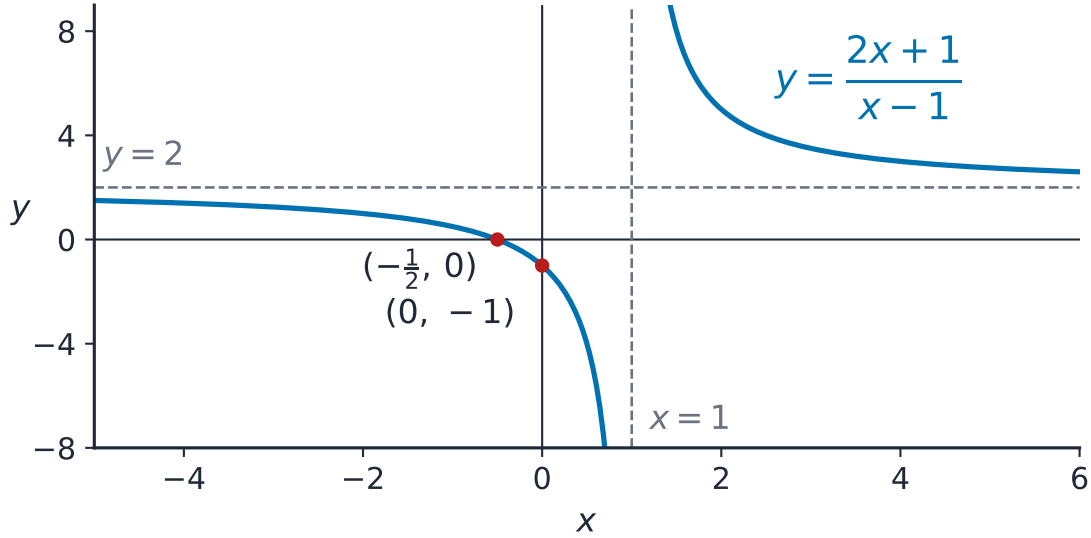
Exercise walk-through

3.2
Modulus and combined transformations

Pearson Edexcel · International A-Level · Pure Mathematics

Figure: Algebra: rational expressions and functions



- Asymptotes first, then intercepts, then the two branches
- Read the axes and units first.

Integration: standard forms and recognition

- **Double angle** with $A = 3x$: $\cos 6x \equiv 1 - 2\sin^2 3x$, so $\sin^2 3x = \frac{1}{2}(1 - \cos 6x)$:
- Check by differentiating: $\frac{1}{2} - \frac{6\cos 6x}{12} = \frac{1}{2}(1 - \cos 6x) = \sin^2 3x$ ✓.
- The numerator is the derivative of the denominator ($f' = 6x$, $f = 3x^2 + 4$):
- Check: $\dfrac{d}{dx}\ln(3x^2 + 4) = \dfrac{6x}{3x^2 + 4}$ ✓. The powered form works the same way: with $u = 3x^2 + 4$ and $du = 6x\,dx$,

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c \qquad \int f'(x)\big(f(x)\big)^n\,dx = \frac{\big(f(x)\big)^{n+1}}{n+1} + c \quad (n \neq -1)$$

Numerical methods: sign changes and iteration

- **Sign change** : if f is continuous on $[a, b]$ and $f(a)f(b) < 0$, a root lies between a and b. Halve the interval to pin a root to any accuracy.
- **Iteration** : a rearrangement $x_{n+1} = f(x_n)$ from a start value converges to a **fixed point** (or not — always check against a sign-change bound).
- Check the **iteration** converges: $|x_3 - x_2|$ small and the values settle toward 0.43 ✓.
- g is continuous, and the **sign change** between 0.4295 and 0.4305 brackets a root, so the root is 0.430 to 3 d.p. ✓.

$$x_2 = \tfrac{1}{3}\arctan\left(\tfrac{3}{0.8}\right) = \tfrac{1}{3}\arctan(3.75) = 0.4367 \qquad x_4 = 0.4307 \text{ (4 d.p.)}$$

Exercises 2.1·2.3

2.1 Sketch $y = |x - 2|$, marking the vertex. [2]

2.2 Solve $|x - 3| = 5$. [2]



2.3 Solve $|2x + 1| = 7$. [2]



Exercises 2.1·2.3

2.1 Sketch $y = |x - 2|$, marking the vertex. [2]

2.1 A V with vertex at $(2, 0)$, arms of gradient ± 1.

2.2 Solve $|x - 3| = 5$. [2]



2.2 $x - 3 = \pm 5$: $x = 8$ or $x = -2$.

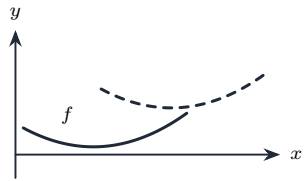
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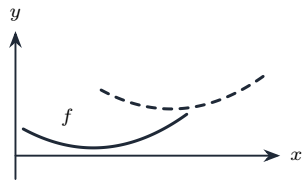
2.3 $2x + 1 = \pm 7$: $x = 3$ or $x = -4$.

Exercises 2.4·2.5

2.4 Describe the transformations mapping $y = f(x)$ onto $y = 3f(x) - 2$. [2]

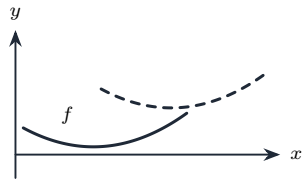


2.5 Given $y = f(x)$ passes through $(2, 5)$, state the image of this point on $y = f(2x) + 3$. [3]



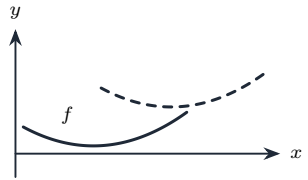
Exercises 2.4·2.5

2.4 Describe the transformations mapping $y = f(x)$ onto $y = 3f(x) - 2$. [2]



2.4 Stretch parallel to the y-axis, factor 3, then translation $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ (down 2).

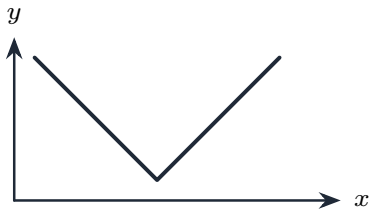
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2.5 $y = f(2x)$ halves x-coordinates: $(2, 5) \to (1, 5)$; adding 3 moves it up: $(1, 8)$.

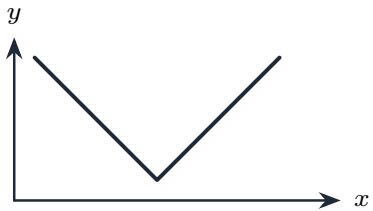
Exercise 3.1

3.1 Solve $|x^2 - 4| = 3x + 2$, giving exact answers and checking each one. [5]



Exercise 3.1

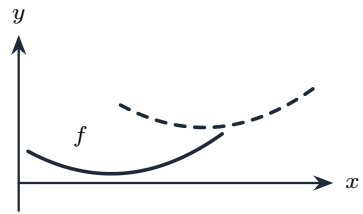
3.1 Solve $|x^2 - 4| = 3x + 2$, giving exact answers and checking each one. [5]



3.1 Case $x^2 - 4 \geq 0$ ($|x| \geq 2$): $x^2 - 4 = 3x + 2 \implies x^2 - 3x - 6 = 0 \implies x = \frac{3 \pm \sqrt{33}}{2}$. The + root $= 4.37 \geq 2$ ✓; the − root $= -1.37$ fails $|x| \geq 2$ ×. Case $x^2 - 4 < 0$ ($|x| < 2$): $4 - x^2 = 3x + 2 \implies x^2 + 3x - 2 = 0 \implies x = \frac{-3 \pm \sqrt{17}}{2}$. The + root $= 0.56$, inside $(-2, 2)$ ✓; the − root $= -3.56$ fails ×. Solutions: $x = \dfrac{3 + \sqrt{33}}{2}$ and $x = \dfrac{-3 + \sqrt{17}}{2}$. Check the second: $|0.31 - 4| = 3.69$ and $3(0.56) + 2 = 3.69$ ✓.

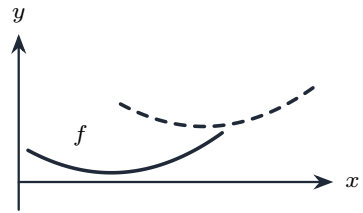
Exercise 3.2

3.2 The graph of $y = f(x)$ has a maximum at $(1, 4)$ and crosses the x-axis at $x = -1$ and $x = 3$. Sketch, on separate axes, $y = |f(x)|$ and $y = f(|x|)$, labelling key points. [5]



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3.2 $y = |f(x)|$: for $-1 \leq x \leq 3$ the original sits below zero between the roots, so reflect that dip above the axis – maxima at $(1, 4)$ and the reflected dip peaking at $(1, 4)$ only if the minimum was exactly -4; otherwise label the reflected minimum as $|\text{min}|$, keeping the roots $x = -1, 3$ on the axis. $y = f(|x|)$: keep the $x \geq 0$ half (crossing at $x = 3$, falling from the maximum at $x = 1$) and mirror it into $x < 0$, so the maximum appears at both $x = \pm 1$ and the crossing at $x = \pm 3$.

Local review guidance: correct case split with its condition; exact valid roots; each root checked against its case. This is authored classroom guidance.