Exercise walk-through

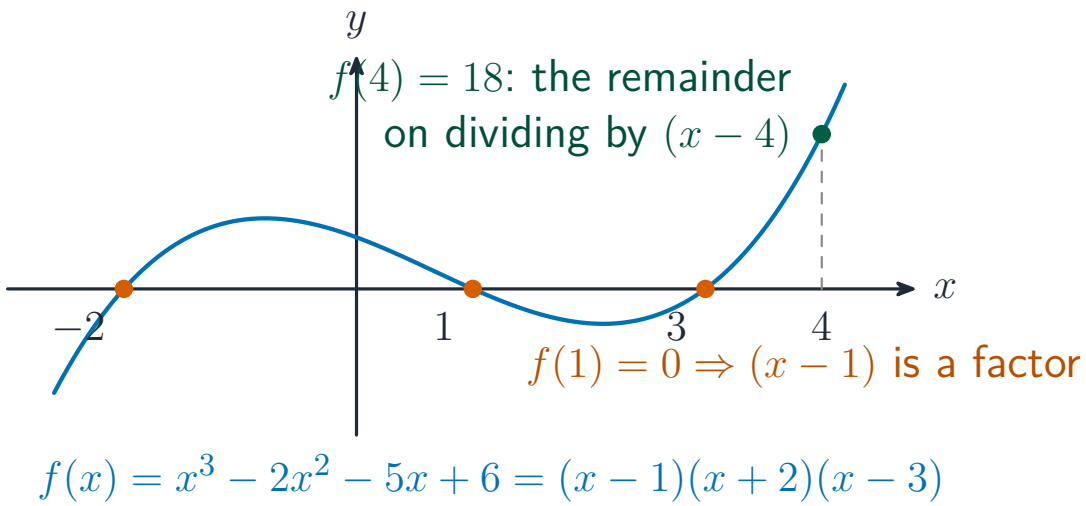
2.7
Trigonometric identities and equations

Pearson Edexcel · International A-Level · Pure Mathematics

Proof

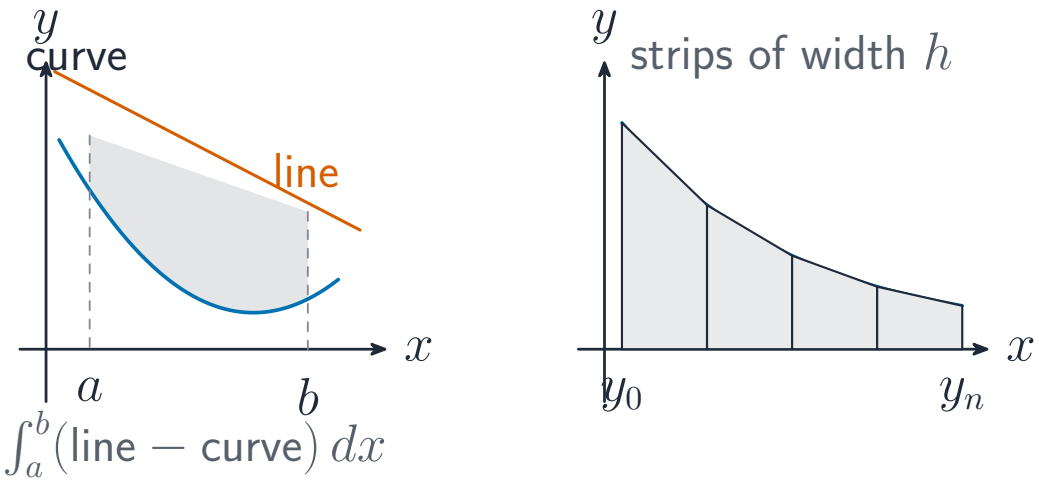
- **Proof by exhaustion** : split the possibilities into cases and check every case.
- **Disproof by counterexample** : one example that breaks the claim disproves it.
- Algebraic **proof**: start from what is given, manipulate identities (not equations to solve), and end with the target statement.

Figure: Algebra: the factor and remainder theorems



- Roots are factors; the height at x = b is the remainder on dividing by (x − b)
- Read the axes and units first.

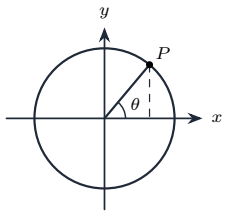
Figure: Integration: areas and the trapezium rule



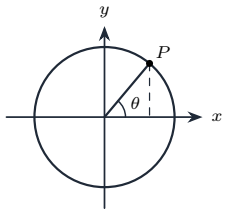
- Area between uses $\int(\text{upper} - \text{lower})$, splitting at crossings; the **trapezium rule** joins neighbouring values with straight strips.
- Read the axes and units first.

Exercises 2.1·2.2

2.1 Simplify $\dfrac{\sin\theta\cos\theta}{\tan\theta}$. [2]

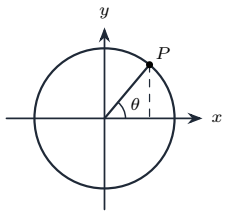


2.2 Solve $\sin\theta = -\frac{1}{2}$ for $0 \leq \theta < 360°$. [3]



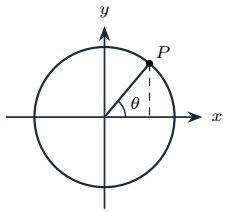
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2.1 $\dfrac{\sin\theta\cos\theta}{\sin\theta/\cos\theta} = \cos^2\theta.$

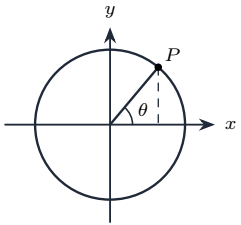
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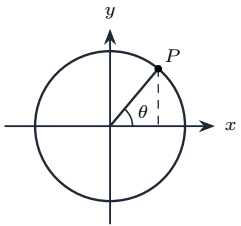
2.2 Third and fourth quadrants: $\theta = 180° + 30° = 210°$ and $360° - 30° = 330°$.

Exercises 2.3·2.4

2.3 Solve $\tan 2\theta = 1$ for $0 \leq \theta < 180°$. [4]

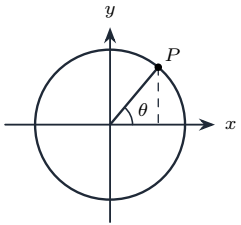


2.4 Given $\sin\theta = 0.6$ with θ acute, find $\cos\theta$ and $\tan\theta$. [3]



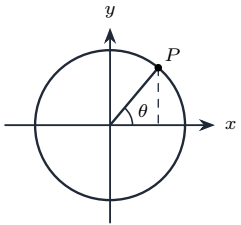
Exercises 2.3·2.4

2.3 Solve $\tan 2\theta = 1$ for $0 \le \theta < 180°$. [4]



2.3 2θ runs over $[0°, 360°)$: $2\theta = 45°$ or $225°$, so $\theta = 22.5°$ or $112.5°$.

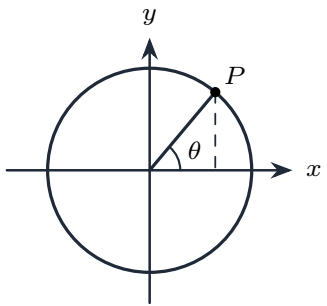
2.4 Given $\sin\theta = 0.6$ with θ acute, find $\cos\theta$ and $\tan\theta$. [3]



2.4 $\cos\theta = \sqrt{1 - 0.36} = 0.8$ (acute, so positive); $\tan\theta = 0.6/0.8 = 0.75$.

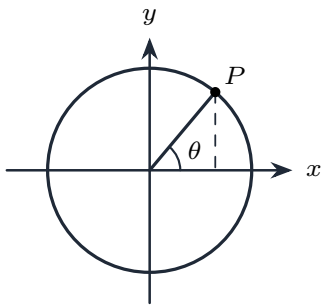
Exercise 2.5

2.5 Solve $2\cos^2\theta + \cos\theta - 1 = 0$ for $0 \leq \theta < 360°$. [4]



Exercise 2.5

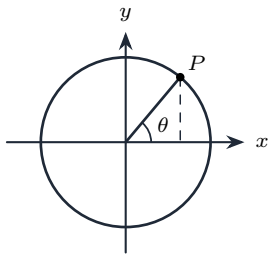
2.5 Solve $2\cos^2\theta + \cos\theta - 1 = 0$ for $0 \leq \theta < 360°$. [4]



2.5 $(2\cos\theta - 1)(\cos\theta + 1) = 0$: $\cos\theta = \frac{1}{2}$ gives $60°, 300°$; $\cos\theta = -1$ gives $180°$. Solutions: $60°, 180°, 300°$.

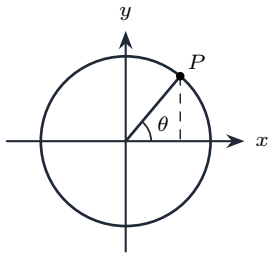
Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA12 June 2025 Q8(i).)* Solve, for $0 \leq x < \pi$, the equation $\tan\left(2x - \frac{\pi}{4}\right) = \sqrt{3}$, giving the answers in radians in the form $k\pi$ where k is a rational constant to be found. [4]



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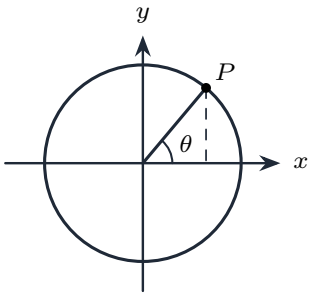
3.1 $\tan\left(2x - \frac{\pi}{4}\right) = \sqrt{3} = \tan\frac{\pi}{3}$:

$$2x - \frac{\pi}{4} = \frac{\pi}{3} + k\pi \implies 2x = \frac{7\pi}{12} + k\pi \implies x = \frac{7\pi}{24} + \frac{k\pi}{2}$$

In $0 \leq x < \pi$: $x = \dfrac{7\pi}{24}$ $(k = 0)$ and $x = \dfrac{19\pi}{24}$ $(k = 1)$. Check: at $x = \frac{7\pi}{24}$, $2x - \frac{\pi}{4} = \frac{\pi}{3}$ and $\tan\frac{\pi}{3} = \sqrt{3}$ ✓.

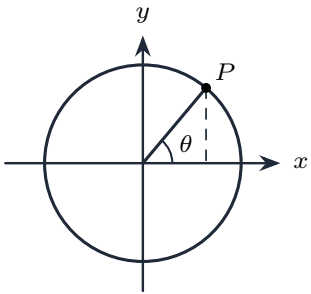
Exercise 3.2

3.2 *(Adapted from the same question.)* Solve, for $0 \leq \theta < 360°$, the equation $4\sin\theta\tan\theta = 3\cos\theta$, giving your answers to one decimal place. [6]



Exercise 3.2

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3.2 Replace $\tan\theta$ and multiply through by $\cos\theta$:

$$\frac{4\sin^2\theta}{\cos\theta} = 3\cos\theta \implies 4\sin^2\theta = 3\cos^2\theta \implies 4(1-\cos^2\theta) = 3\cos^2\theta \implies \cos^2\theta = \frac{4}{7}$$

$$\cos\theta = \pm\frac{2}{\sqrt{7}} = \pm 0.7559$$

Full solution on next slide

Full solution 3.2

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$$\cos\theta = \pm\frac{2}{\sqrt{7}} = \pm 0.7559$$

- $\cos\theta = 0.7559$: $\theta = 40.9°$ and $360° - 40.9° = 319.1°$.
- $\cos\theta = -0.7559$: $\theta = 180° - 40.9° = 139.1°$ and $180° + 40.9° = 220.9°$. Solutions: $40.9°, 139.1°, 220.9°, 319.1°$ (1 d.p.). Check $40.9°$: $\sin = 0.654$, $\tan = 0.865$: $4(0.654)(0.865) = 2.26$ and $3(0.756) = 2.27$ ✓ (rounding).

Local review guidance: identity applied to reach one function; correct root values; all interval solutions listed with quadrant reasoning. This is authored classroom guidance.

Exercise 3.3 (a)

3.3 The figure shows the curves with equations $y = 3\cos x$ and $y = 2\sin^2 x$ for $0 \leq x \leq 360°$. The curves meet at the points A and B.

(a) Show that the x-coordinates of A and B satisfy $2\cos^2 x + 3\cos x - 2 = 0$. [3]

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(a)

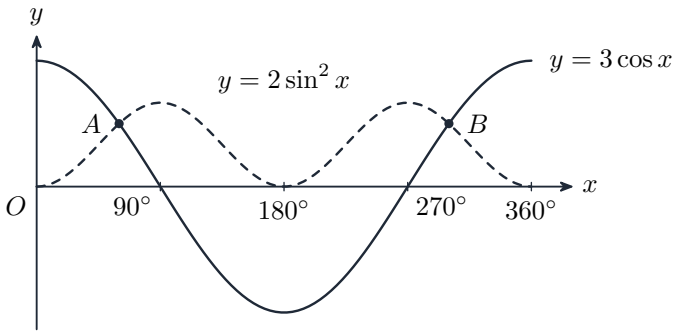
- at the points of intersection $3\cos x = 2\sin^2 x$
- use $\sin^2 x + \cos^2 x = 1$, so $\sin^2 x = 1 - \cos^2 x$:

$$3\cos x = 2(1 - \cos^2 x) \;\Rightarrow\; 2\cos^2 x + 3\cos x - 2 = 0$$

Exercise 3.3 (b)

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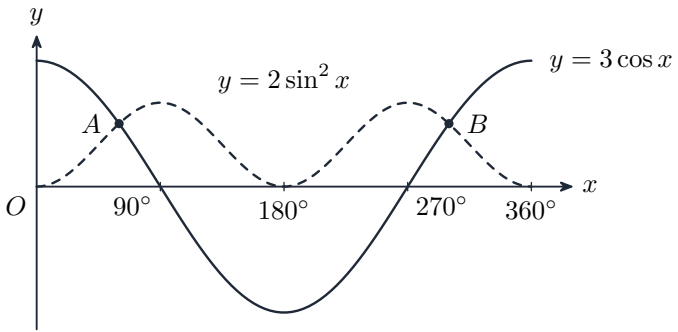
(b) Hence find the coordinates of A and B. [4]



Exercise 3.3 (b)

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(b) Hence find the coordinates of A and B. [4]



(b)

- factorise the quadratic in $\cos x$: $(2\cos x - 1)(\cos x + 2) = 0$
- $\cos x = -2$ has no solutions, since $-1 \leq \cos x \leq 1$; so $\cos x = \frac{1}{2}$
- $x = 60°$ or $x = 360° - 60° = 300°$, and $y = 3\cos x = 3(\frac{1}{2}) = 1.5$
- $A(\mathbf{60°},\ \mathbf{1.5})$ and $B(\mathbf{300°},\ \mathbf{1.5})$