Exercise walk-through

2.2
Factor and remainder theorems

Pearson Edexcel · International A-Level · Pure Mathematics

Algebra: the factor and remainder theorems

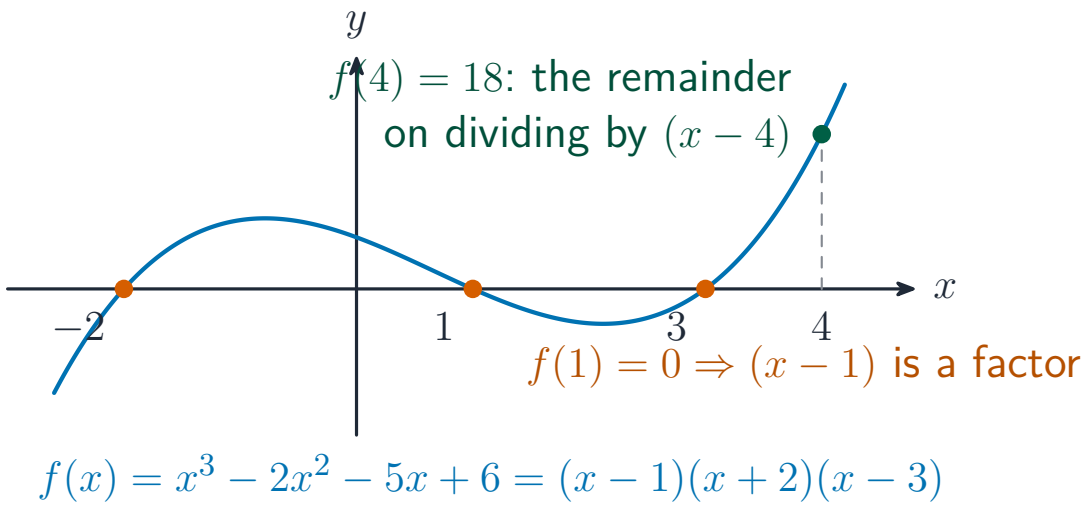
- **Factor theorem** : $(x-b)$ is a factor of $f(x)$ exactly when $f(b)=0$. For divisors of the form $(ax-b)$: if $f\left(\frac{b}{a}\right)=0$ then $(ax-b)$ is a factor.
- **Remainder theorem** : dividing $f(x)$ by $(ax-b)$ leaves remainder $f\left(\frac{b}{a}\right)$.
- **Algebraic division** by $(ax\pm b)$ gives a **quotient** and a **remainder**; the degree drops by one: $f(x)=(ax-b)\times(\text{quotient})+\text{remainder}$.

$$a(-27)+b(9)+5(-3)-3=0 \implies -27a+9b-18=0 \implies b=3a+2$$

Algebra: the factor and remainder theorems (3)

- Check: $f(x) = 8x^3 + 26x^2 + 5x - 3$; $f(-3) = -216 + 234 - 15 - 3 = 0$ ✓ and $f(\frac{1}{2}) = 1 + 6.5 + 2.5 - 3 = 7$ ✓.
- Divide $8x^3 + 26x^2 + 5x - 3$ by $(x - 2)$: **quotient** $8x^2 + 42x + 89$, remainder 175.
- Check by rebuilding: $(x - 2)(8x^2 + 42x + 89) + 175 = 8x^3 + 42x^2 + 89x - 16x^2 - 84x - 178 + 175 = 8x^3 + 26x^2 + 5x - 3$ ✓. The remainder also equals $f(2) = 64 + 104 + 10 - 3 = 175$ ✓.

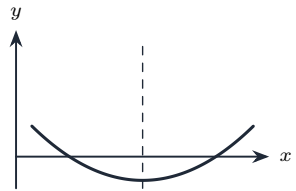
Figure: Algebra: the factor and remainder theorems



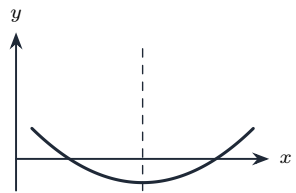
- Roots are factors; the height at x = b is the remainder on dividing by (x − b)
- Read the axes and units first.

Exercises 2.1·2.2

2.1 Show that $(x+1)$ is a factor of $x^3 - 7x - 6$. [2]

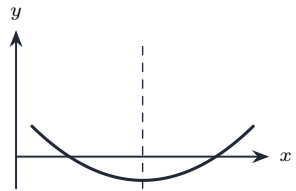


2.2 Hence factorise $x^3 - 7x - 6$ completely. [3]



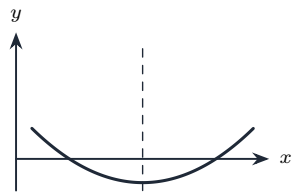
Exercises 2.1·2.2

2.1 Show that $(x+1)$ is a factor of $x^3 - 7x - 6$. [2]



2.1 $f(-1) = -1 + 7 - 6 = 0$, so $(x+1)$ is a factor by the factor theorem.

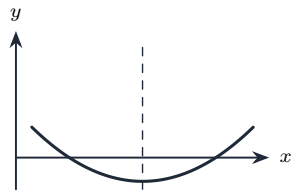
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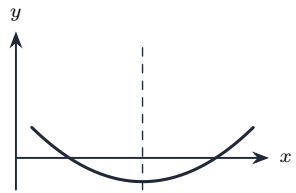
2.2 Divide: $x^3 - 7x - 6 = (x+1)(x^2 - x - 6) = (x+1)(x-3)(x+2)$. Check: $f(0) = -6$ and $(1)(-3)(2) = -6$ ✓.

Exercises 2.3·2.4

2.3 Find the remainder when $x^3 + x^2 - 2$ is divided by $(x - 1)$, and say what your answer implies. [2]

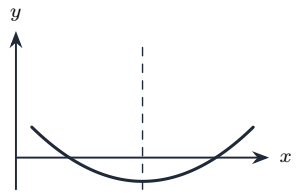


2.4 Find the remainder when $3x^3 - 5x^2 + 7$ is divided by $(2x - 1)$. [3]



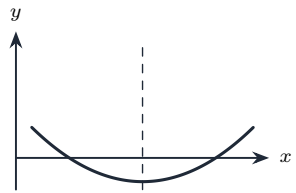
Exercises 2.3·2.4

2.3 Find the remainder when $x^3 + x^2 - 2$ is divided by $(x - 1)$, and say what your answer implies. [2]



2.3 $f(1) = 1 + 1 - 2 = 0$: the remainder is 0, so $(x - 1)$ is itself a factor.

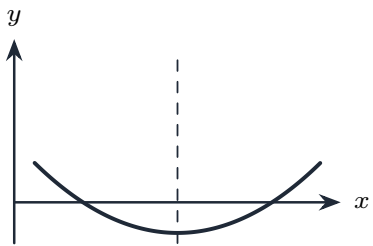
2.4 Find the remainder when $3x^3 - 5x^2 + 7$ is divided by $(2x - 1)$. [3]



2.4 $f\left(\frac{1}{2}\right) = 3\left(\frac{1}{8}\right) - 5\left(\frac{1}{4}\right) + 7 = \frac{3}{8} - \frac{5}{4} + 7 = \frac{49}{8}$.

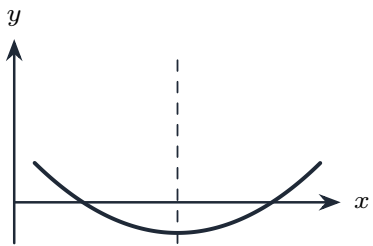
Exercise 2.5

2.5 $f(x) = x^3 + ax + b$ has $(x - 2)$ as a factor and leaves remainder 15 when divided by $(x - 3)$. Find a and b. [5]



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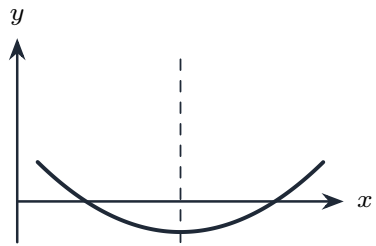
2.5 $f(x) = x^3 + ax + b$ has $(x - 2)$ as a factor and leaves remainder 15 when divided by $(x - 3)$. Find a and b. [5]



2.5 $f(2) = 8 + 2a + b = 0$ and $f(3) = 27 + 3a + b = 15$. Subtracting: $19 + a = 15$, so $a = -4$; then $8 - 8 + b = 0$ gives $b = 0$. $f(x) = x^3 - 4x$. Check: $f(2) = 0$ ✓ and $f(3) = 27 - 12 = 15$ ✓.

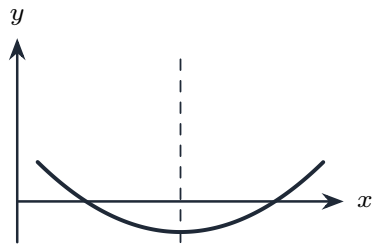
Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA12 June 2025 Q4.)* $f(x) = 2x^3 + px^2 + 11x + q$, where $(x-1)$ is a factor of $f(x)$ and $f(x)$ leaves remainder -36 when divided by $(x+2)$. Find p and q, and verify both conditions. [5]



Exercise 3.1

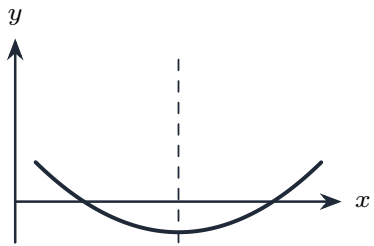
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3.1 $f(1) = 2+p+11+q = 0 \implies p+q = -13$. $f(-2) = -16+4p-22+q = -36 \implies 4p+q = 2$. Subtracting: $3p = 15$, so $p = 5$ and $q = -18$. Verify: $f(1) = 2+5+11-18 = 0$ ✓; $f(-2) = -16+20-22-18 = -36$ ✓.

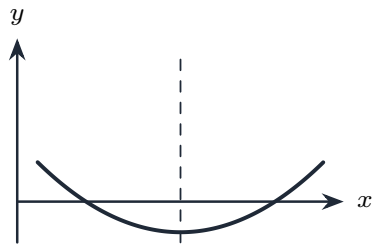
Exercise 3.2

3.2 Using the factor theorem, fully factorise $x^3 + 4x^2 + x - 6$. [4]



Exercise 3.2

3.2 Using the factor theorem, fully factorise $x^3 + 4x^2 + x - 6$. [4]



3.2 Test $f(-2) = -8 + 16 - 2 - 6 = 0$: $(x + 2)$ is a factor. Divide: $x^3 + 4x^2 + x - 6 = (x + 2)(x^2 + 2x - 3) = (x + 2)(x + 3)(x - 1)$. Check: $f(0) = -6$ and $(2)(3)(-1) = -6$ ✓.

Local review guidance: correct theorem applied at the right root; exact quotient and remainder; verification by rebuilding or by f evaluated. This is authored classroom guidance.

Exercise 3.3 (a)

3.3 The figure shows part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2x^3 + ax^2 + bx + 6$$

and a and b are constants. The curve crosses the x-axis at $(-2,\ 0)$ and $(3,\ 0)$ and the y-axis at $(0,\ 6)$.

(a) Use the factor theorem to find the values of a and b. [4]

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(a) Use the factor theorem to find the values of a and b. [4]

(a)

- factor theorem: $(x + 2)$ and $(x - 3)$ are factors, so $\mathrm{f}(-2) = 0$ and $\mathrm{f}(3) = 0$

$$\mathrm{f}(-2) = -16 + 4a - 2b + 6 = 0 \ \Rightarrow\ 2a - b = 5$$

$$\mathrm{f}(3) = 54 + 9a + 3b + 6 = 0 \ \Rightarrow\ 3a + b = -20$$

- adding: $5a = -15$, so $a = \mathbf{-3}$ and $b = 2(-3) - 5 = \mathbf{-11}$

Exercise 3.3 (b)

3.3 The figure shows part of the curve with equation $y = \mathrm{f}(x)$, where

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(b) Factorise $\mathrm{f}(x)$ completely, and write down the x-coordinate of the third point where the curve crosses the x-axis. [3]

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(b)

- $(x+2)(x-3) = x^2 - x - 6$; divide, or compare the x^3 and constant terms:

$$2x^3 - 3x^2 - 11x + 6 = (x^2 - x - 6)(2x - 1) = \boldsymbol{(x+2)(x-3)(2x-1)}$$

- the third crossing is at $x = \boldsymbol{\frac{1}{2}}$

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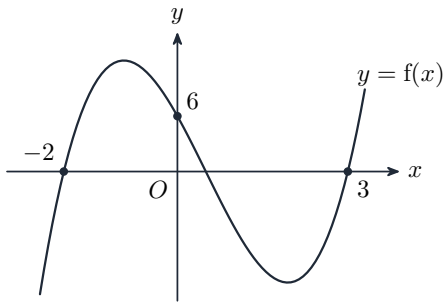
Exercise 3.3 (c)

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(c) Find the remainder when $\mathrm{f}(x)$ is divided by $(2x + 1)$. [2]



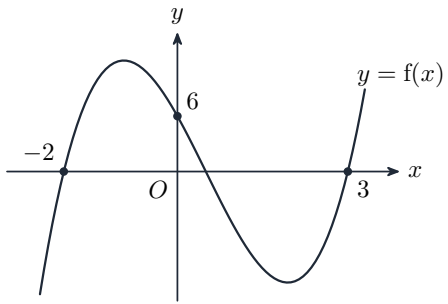
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(c) Find the remainder when $\mathrm{f}(x)$ is divided by $(2x + 1)$. [2]



(c)

- remainder theorem: the remainder on division by $(2x + 1)$ is $\mathrm{f}(-\frac{1}{2})$:

$$\mathrm{f}(-\tfrac{1}{2}) = 2(-\tfrac{1}{8}) - 3(\tfrac{1}{4}) - 11(-\tfrac{1}{2}) + 6 = -0.25 - 0.75 + 5.5 + 6 = \mathbf{10.5}$$

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