

Course notes

Pearson Edexcel · International A-Level · Pure Mathematics

YPM01: course teaching notes

Version: Issue 3, April 2019; first teaching 2018; unit assessment from 2019

These are original teaching notes, and each lesson is labelled with its tier.

P1 · Indices, surds and standard form

How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
- This lesson studies **index** : The power to which a base is raised.

The method

- For the same positive base, multiplication adds indices and division subtracts them.
- A negative index means reciprocal; a fractional index represents a root.
- Standard form has $1 \leq a < 10$.

A worked case

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = \left(\sqrt[n]{a}\right)^m$$

■ Worked example

$0.000072 = 7.2 \times 10^{-5}$. Also $16^{3/4} = (16^{1/4})^3 = 2^3 = 8$. Simplify $\sqrt{72} = 6\sqrt{2}$, then rationalise $1/\sqrt{2} = \sqrt{2}/2$.

$0.000072 = 7.2 \times 10^{-5}$: the point moves 5 places

5 jumps right

0 . 0 0 0 0 7 2

$$7.2 \times 10^{-5}$$

coefficient 7.2 is between 1 and 10; the power is -5

Where it goes wrong

- Index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 .
- Do not round a surd when an exact answer is requested.

■ Watch out

Not true: For every positive a , $a^2 + a^3$ equals a^5 . Index laws only add powers when the terms are multiplied: $2^2 + 2^3 = 12$, but $2^5 = 32$.

Applying it

- Check powers of ten against the original quantity.
- Use surds for exact geometry, and round only the final length when the question asks for a decimal.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Index:** The power to which a base is raised.
- For the same positive base, multiplication adds indices and division subtracts them.
- Watch for: index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 .

P1 · Quadratics and inequalities

Which widths make enough space?

- A rectangular enclosure has area $x(10-x)$. What widths give at least 21 square metres?
- This lesson studies **discriminant** : The quantity $b^2 - 4ac$ that determines the real roots of $ax^2 + bx + c = 0$.

The method

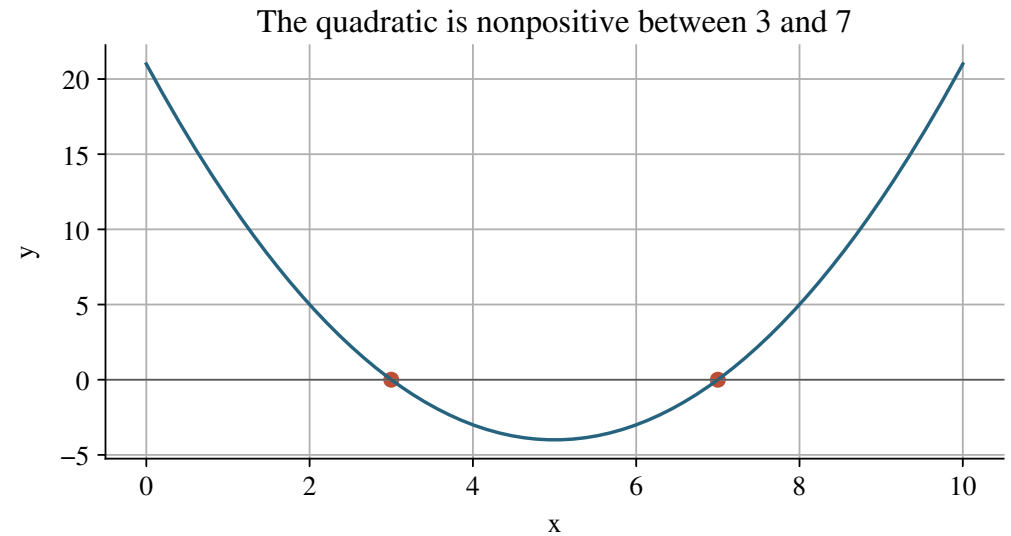
- Factor where possible; otherwise complete the square or use the quadratic formula.
- A quadratic inequality needs the sign on intervals, not only the roots.
- The discriminant identifies repeated or missing real roots.

A worked case

$$ax^2 + bx + c = 0, \quad \Delta = b^2 - 4ac$$

■ Worked example

The condition is $x(10-x) \geq 21$, so $x^2 - 10x + 21 \leq 0$. Factor $(x-3)(x-7) \leq 0$. The upward parabola is nonpositive between its roots, giving $3 \leq x \leq 7$. The maximum area is 25 at $x=5$.



Where it goes wrong

- Multiplying an inequality by a negative number reverses its direction.
- A sketch must show which side of each root satisfies the inequality.
- Geometry may restrict x further.

■ Watch out

Not true: A positive discriminant means that a quadratic has no real roots. A positive discriminant means two distinct real roots; a negative discriminant means no real roots.

Applying it

- Use the vertex to interpret an optimum.
- Check an endpoint and a point between the roots; the algebra and graph should tell the same story.

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- Factor where possible; otherwise complete the square or use the quadratic formula.
- Watch for: multiplying an inequality by a negative number reverses its direction.

P1 · Coordinate geometry and tangents

How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
- This lesson studies **gradient** : The change in y divided by the corresponding change in x.

The method

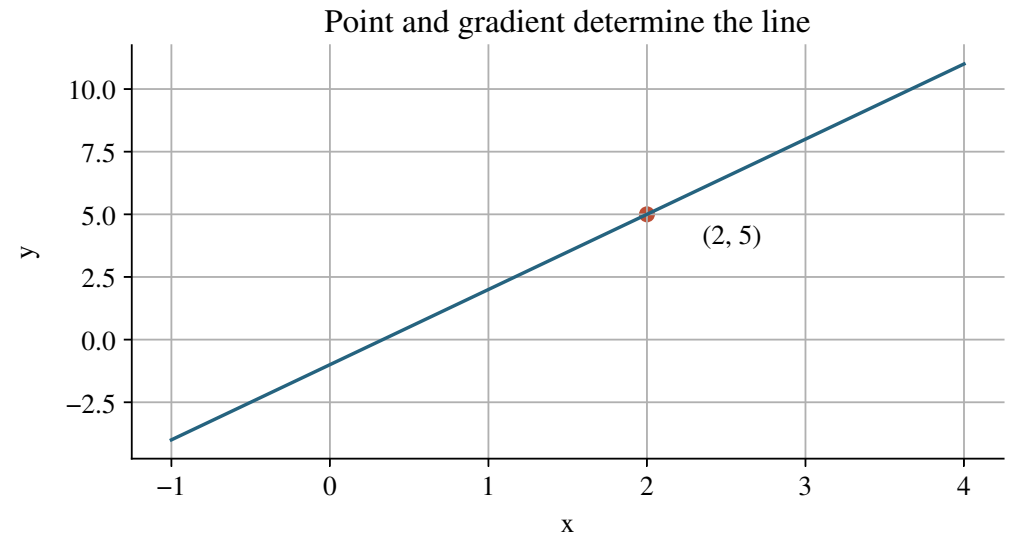
- A line through (x_1, y_1) with gradient m has $y-y_1=m(x-x_1)$.
- Parallel lines have equal gradients.
- Finite perpendicular gradients multiply to -1.
- A circle has $(x-a)^2+(y-b)^2=r^2$.

A worked case

$$y - y_1 = m(x - x_1), \quad (x - a)^2 + (y - b)^2 = r^2$$

■ Worked example

Through (2,5) with gradient 3, $y-5=3(x-2)$, so $y=3x-1$. A perpendicular through the same point has $y-5=-(x-2)/3$. The circle $(x-2)^2+(y+1)^2=25$ has centre (2,-1) and radius 5.



Where it goes wrong

- A vertical line has no finite gradient; do not force it into $y=mx+c$.
- Read the signs of a circle's centre carefully.
- The radius to a tangent is perpendicular to the tangent.

■ Watch out

Not true: Perpendicular nonvertical lines have equal gradients. Perpendicular gradients multiply to -1 : a gradient of 2 needs $-1/2$. Parallel lines have equal gradients.

Applying it

- Before solving a line-circle intersection, predict whether there are zero, one or two intersections.
- Substitution produces a quadratic whose discriminant checks the prediction.

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■ Key idea

- **Gradient:** The change in y divided by the corresponding change in x.
- A line through (x_1, y_1) with gradient m has $y - y_1 = m(x - x_1)$.
- Watch for: A vertical line has no finite gradient; do not force it into $y = mx + c$.

P1 · Right triangles and non-right triangles

Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
 - This lesson studies **hypotenuse** : The side opposite the right angle in a right-angled triangle.
-

The method

- In a right triangle $a^2 + b^2 = c^2$; $\sin \theta = \text{opposite/hypotenuse}$, $\cos \theta = \text{adjacent/hypotenuse}$ and $\tan \theta = \text{opposite/adjacent}$.
 - For other triangles, use the sine or cosine rule, or $\text{area} = ab \sin C/2$.
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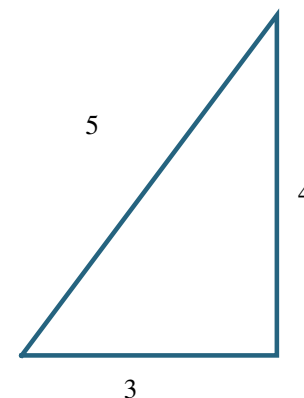
A worked case

$$a^2 + b^2 = c^2, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

■ Worked example

The ladder length is $c = \sqrt{3^2 + 4^2} = 5$ m. Its angle to the ground satisfies $\tan \theta = 4/3$, so $\theta \approx 53.1^\circ$. With two sides 6 and 8 enclosing 60° , $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 60^\circ = 52$.

Label the known sides before choosing a rule



Where it goes wrong

- Label sides relative to the chosen angle.
- Pythagoras needs a right angle.
- A calculator angle mode error can produce a plausible but wrong result.
- Keep unrounded values for later steps.

■ Watch out

Not true: Pythagoras applies to every triangle, including triangles without a right angle. Pythagoras needs a right angle; for other triangles use the cosine rule.

Applying it

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule.
 - Explain why the chosen triangle contains the required length or angle.
-

In your course

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■ Key idea

- **Hypotenuse:** The side opposite the right angle in a right-angled triangle.
- In a right triangle $a^2+b^2=c^2$; $\sin\theta=\text{opposite/hypotenuse}$, $\cos\theta=\text{adjacent/hypotenuse}$ and $\tan\theta=\text{opposite/adjacent}$.
- Watch for: label sides relative to the chosen angle.

P1 · Radians, identities and trigonometric equations

How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
- This lesson studies **radian** : The angle subtended by an arc whose length equals the radius.

The method

- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$.
- Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve.
- List every solution in the stated interval.

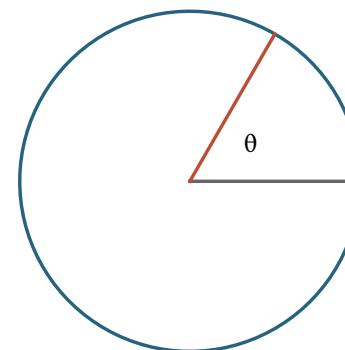
A worked case

$$s = r\theta, \quad A = \frac{1}{2}r^2\theta, \quad \sin^2\theta + \cos^2\theta = 1$$

■ Worked example

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0\leq x<2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0\leq 2x<4\pi$ before dividing by 2.

A radius fixes distance from the centre



radius $\times$ angle gives arc length

Where it goes wrong

- A calculator's inverse sine gives one principal value.
- It does not give every solution.
- Dividing by $\sin x$ can discard solutions where $\sin x=0$; check that case first.

■ Watch out

Not true: The principal inverse sine value is the only solution in a full turn. Sine takes most values twice in a full turn: $\sin x=0.5$ gives 30° and 150° .

Applying it

- Choose an identity that reduces the equation to one trigonometric function.
- Keep exact familiar angles where possible, and check each solution against the original equation.

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■ Key idea

- **Radian:** The angle subtended by an arc whose length equals the radius.
- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$.
- Watch for: A calculator's inverse sine gives one principal value.

P1 · Derivatives and stationary points

What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative**: The instantaneous rate of change, also the gradient of a tangent.

The method

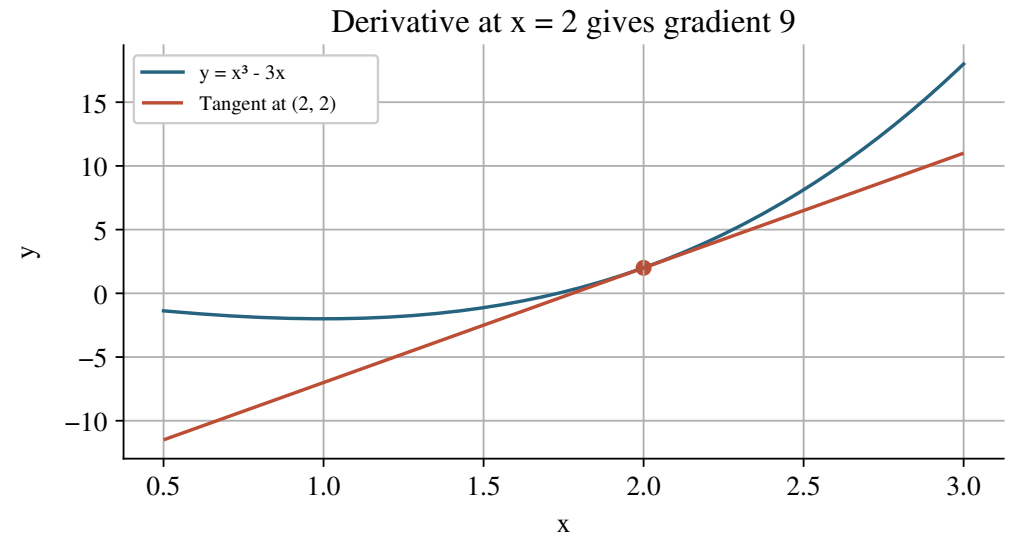
- For $y=ax^n$, $dy/dx=nax^{n-1}$.
- A stationary point satisfies $dy/dx=0$.
- Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.

A worked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad m = f'(x_0)$$

■ Worked example

For $y=x^3-3x$, $dy/dx=3x^2-3$. At $x=2$, the gradient is 9 and $y=2$. The tangent is $y-2=9(x-2)$. The normal gradient is $-1/9$.



Where it goes wrong

- A derivative gives a gradient, not the ordinate.
- Evaluate y and dy/dx separately at the supplied x -coordinate; the normal gradient is the negative reciprocal of a nonzero tangent gradient.

■ Watch out

Not true: A tangent and normal at the same point have the same gradient. A zero derivative can also be a point of inflection: $y=x^3$ at $x=0$ is neither a maximum nor a minimum.

Applying it

- P1 covers polynomial derivatives, gradients, tangents and normals.
- Optimization and stationary-point classification are taught in P2; they are not introduced here.

In your course

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- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Derivative:** The instantaneous rate of change, also the gradient of a tangent.
- For $y=ax^n$, $dy/dx=nax^{n-1}$.
- Watch for: A derivative gives a gradient, not the ordinate.

P1 · Integrals, area and accumulation

How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative** : A function whose derivative equals the given integrand.

The method

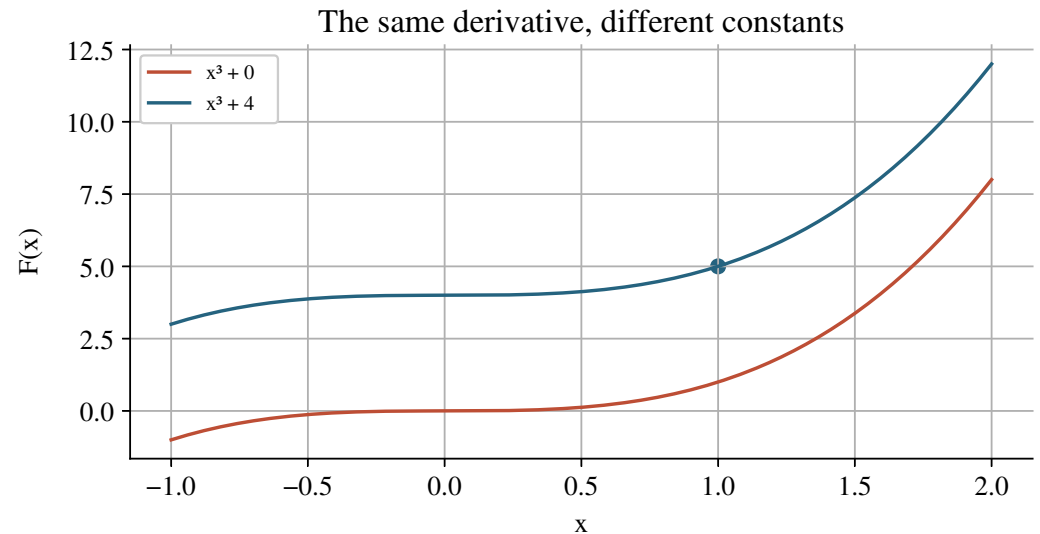
- Integrate polynomial powers by increasing the index by one and dividing by the new index.
- Include an arbitrary constant and use a supplied point to determine it.

A worked case

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

■ Worked example

An antiderivative of $3x^2$ is x^3+C because differentiating x^3 gives $3x^2$. If $F(1)=5$, then $1+C=5$, so $C=4$ and $F(x)=x^3+4$.



Where it goes wrong

- An indefinite integral needs a constant.
- Integrating each term changes its power; copying the derivative rule gives the wrong result.

■ Watch out

Not true: Every indefinite integral has only one possible antiderivative. Area below the axis counts as negative, so a definite integral can be smaller than the total area, or even zero.

Applying it

- P1 uses indefinite polynomial integration and a point to determine the constant.
- Definite integrals, areas and the trapezium rule are P2 content.

In your course

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- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Antiderivative:** A function whose derivative equals the given integrand.
- Integrate polynomial powers by increasing the index by one and dividing by the new index.
- Watch for: an indefinite integral needs a constant.

P2 · Deduction, contradiction and induction

When does a pattern become a proof?

- Checking several integers can suggest a pattern. What turns the pattern into a proof for every integer?
 - This lesson studies **counterexample** : A single valid case that disproves a universal claim.
-

The method

- A deductive proof starts from stated definitions or assumptions.
 - A counterexample refutes an all-values claim.
 - Induction requires a base case and an implication from $n=k$ to $n=k+1$; checking successive cases alone is not induction.
-

A worked case

$$(2a + 1) + (2b + 1) = 2(a + b + 1)$$

■ Worked example

For odd integers $2a+1$ and $2b+1$, their sum is $2(a+b+1)$, hence even. The claim n^2+n+41 is always prime fails at $n=41$: the value is $41 \times 43 = 1763$. For $1+\dots+n=n(n+1)/2$, the induction step adds $k+1$ to the assumed sum.

$$\text{Odd + odd: } (2a + 1) + (2b + 1)$$

$$\text{Factor 2: } 2(a + b + 1)$$

The bracket is an integer $\Rightarrow$ the sum is even

Where it goes wrong

- Do not assume the conclusion while proving it.
- In contradiction, identify the impossible consequence and reject the initial contrary assumption.
- Induction is not compulsory in AQA 7357; it belongs in its own appropriate further-mathematics scope.

■ Watch out

Not true: Checking the first ten integers proves a claim for every positive integer. A pattern can hold for the first ten cases and then fail; a proof must cover every case.

Applying it

- AQA proof includes deduction, exhaustion and contradiction.
 - IAL P2 introduces exhaustion and counterexample, while P4 introduces contradiction.
 - Keep the method matched to the named unit.
-

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Counterexample:** A single valid case that disproves a universal claim.
- A deductive proof starts from stated definitions or assumptions.
- Watch for: do not assume the conclusion while proving it.

P2 · Quadratics and inequalities

Which widths make enough space?

- A rectangular enclosure has area $x(10-x)$. What widths give at least 21 square metres?
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The method

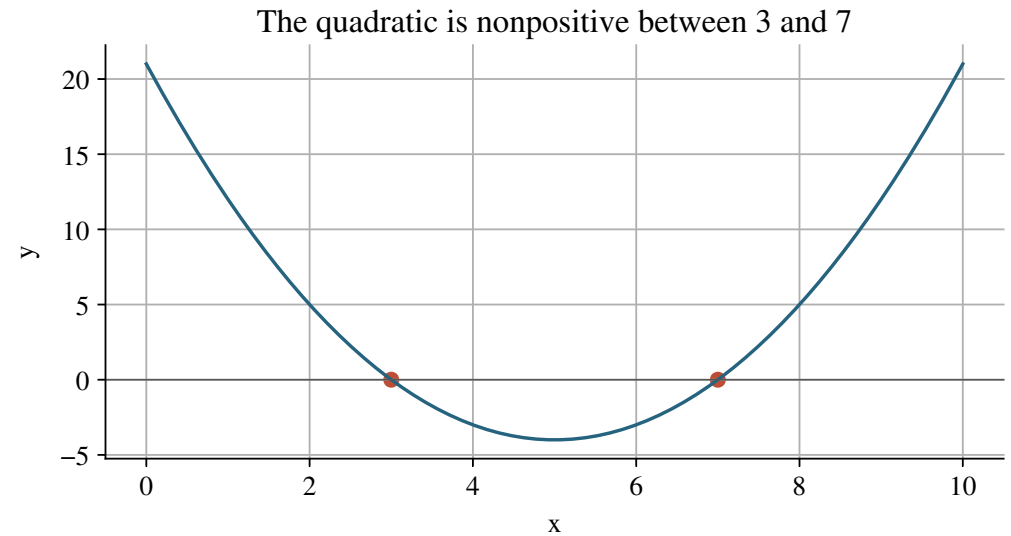
- Factor where possible; otherwise complete the square or use the quadratic formula.
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A worked case

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The condition is $x(10-x) \geq 21$, so $x^2-10x+21 \leq 0$. Factor $(x-3)(x-7) \leq 0$. The upward parabola is nonpositive between its roots, giving $3 \leq x \leq 7$. The maximum area is 25 at $x=5$.



Where it goes wrong

- Multiplying an inequality by a negative number reverses its direction.
- A sketch must show which side of each root satisfies the inequality.
- Geometry may restrict x further.

■ Watch out

Not true: A positive discriminant means that a quadratic has no real roots. A positive discriminant means two distinct real roots; a negative discriminant means no real roots.

Applying it

- Use the vertex to interpret an optimum.
- Check an endpoint and a point between the roots; the algebra and graph should tell the same story.

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P2 · Coordinate geometry and tangents

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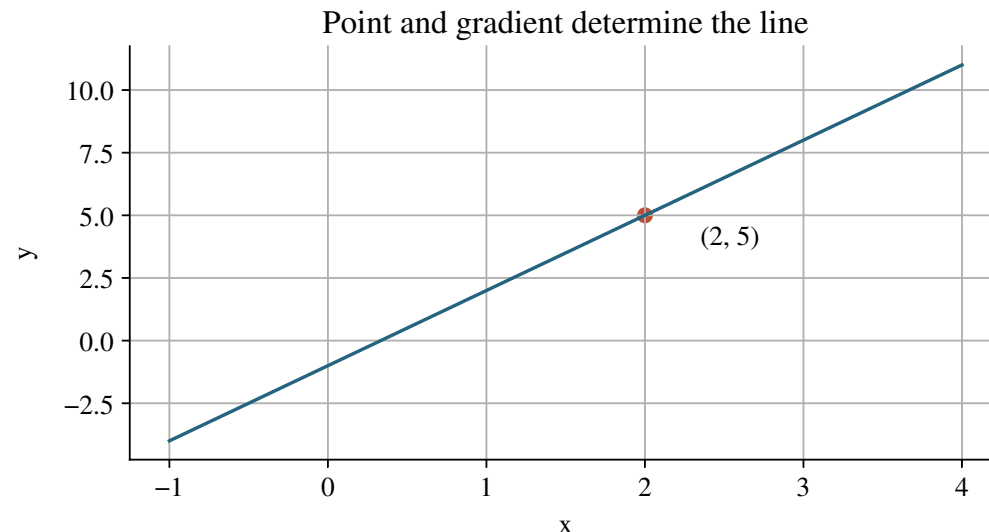
- A line through (x_1, y_1) with gradient m has $y-y_1=m(x-x_1)$.
- Parallel lines have equal gradients.
- Finite perpendicular gradients multiply to -1.
- A circle has $(x-a)^2+(y-b)^2=r^2$.

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Through (2,5) with gradient 3, $y-5=3(x-2)$, so $y=3x-1$. A perpendicular through the same point has $y-5=-(x-2)/3$. The circle $(x-2)^2+(y+1)^2=25$ has centre (2,-1) and radius 5.



Where it goes wrong

- A vertical line has no finite gradient; do not force it into $y=mx+c$.
- Read the signs of a circle's centre carefully.
- The radius to a tangent is perpendicular to the tangent.

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Not true: Perpendicular nonvertical lines have equal gradients. Perpendicular gradients multiply to -1 : a gradient of 2 needs $-1/2$. Parallel lines have equal gradients.

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- **Gradient:** The change in y divided by the corresponding change in x .
- A line through (x_1, y_1) with gradient m has $y - y_1 = m(x - x_1)$.
- Watch for: A vertical line has no finite gradient; do not force it into $y = mx + c$.

P2 · Sequences, series and recurrence

Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common ratio**: The constant multiplier between consecutive terms of a geometric sequence.

The method

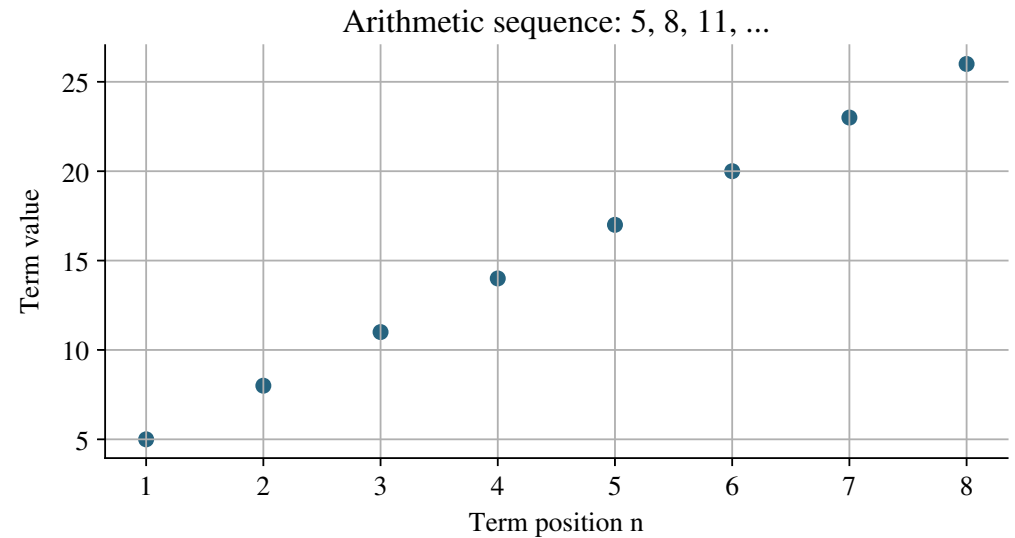
- For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(2a + (n-1)d)/2$.
- For a geometric progression, $u_n = ar^{(n-1)}$ and $S_n = a(1 - r^n)/(1 - r)$.
- An infinite geometric sum exists only if $|r| < 1$.

A worked case

$$u_n = a + (n-1)d, \quad S_n = \frac{n}{2} [2a + (n-1)d]$$

■ Worked example

For $a=5, d=3, n=8$, $u_8 = 5 + 7 \times 3 = 26$ and $S_8 = 8(10 + 21)/2 = 124$. For $a=12, r=1/2$, $S_{\infty} = 12/(1 - 1/2) = 24$. For $u_{(n+1)} = 2u_n + 1$ with $u_1 = 1$, the next terms are 3, 7, 15.



Where it goes wrong

- The first term has index 1, so the exponent is $n-1$.
- A sequence is a list; a series is a sum.
- A geometric sequence can alternate in sign and still converge.

■ Watch out

Not true: Every geometric series has a finite sum to infinity. A sum to infinity exists only when $-1 < r < 1$; for $r=2$ the terms grow without limit.

Applying it

- Explain whether the context justifies additive or multiplicative change.
- In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.

In your course

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■ Key idea

- **Common ratio:** The constant multiplier between consecutive terms of a geometric sequence.
- For an arithmetic progression, $u_n=a+(n-1)d$ and $S_n=n(2a+(n-1)d)/2$.
- Watch for: the first term has index 1, so the exponent is $n-1$.

P2 · Binomial expansion and valid approximations

How does a small change affect a power?

- A small measurement change affects a power of a quantity. An expansion can show the size of first and second effects.
- This lesson studies **binomial coefficient** : The number of ways to choose a specified number of objects from a set.

The method

- For positive integer n , $(a+b)^n$ is a finite binomial expansion.
- For noninteger n , expand $(1+x)^n$ as $1+nx+n(n-1)x^2/2+\dots$ with $|x|<1$.
- Factor out constants before using this form.

A worked case

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \dots$$

■ Worked example

$(1+2x)^5=1+10x+40x^2+80x^3+80x^4+32x^5$. For $(1+x)^{-1}$, the first three terms are $1-x+x^2$; at $x=0.1$ this gives 0.91 versus the exact $1/1.1\approx0.909091$.

$(1+2x)^5$: Pascal row $\times$ powers of 2

Pascal row 5	1	5	10	10	5	1
$\times 2^k$	1	2	4	8	16	32
coefficient	1	10	40	80	80	32
	1	x	x ²	x ³	x ⁴	x ⁵

Where it goes wrong

- An expansion in $2x$ requires $|2x|<1$ for the infinite series, not merely $|x|<1$.
- Positive-integer expansions are finite and do not have that convergence restriction.

■ Watch out

Not true: Every binomial expansion is a finite polynomial, including powers that are not positive integers. For a negative or fractional power the expansion is an infinite series, valid only for $|x|<1$.

Applying it

- State the range of validity alongside an approximation.
- Retain enough terms to support the requested accuracy, and distinguish a coefficient from the whole term containing x .

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■ Key idea

- **Binomial coefficient:** The number of ways to choose a specified number of objects from a set.
- For positive integer n , $(a+b)^n$ is a finite binomial expansion.
- Watch for: an expansion in $2x$ requires $|2x| < 1$ for the infinite series, not merely $|x| < 1$.

P2 · Exponentials, logarithms and modelling

Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.
- This lesson studies **half-life** : The time for a decaying quantity to fall to half its initial value.

The method

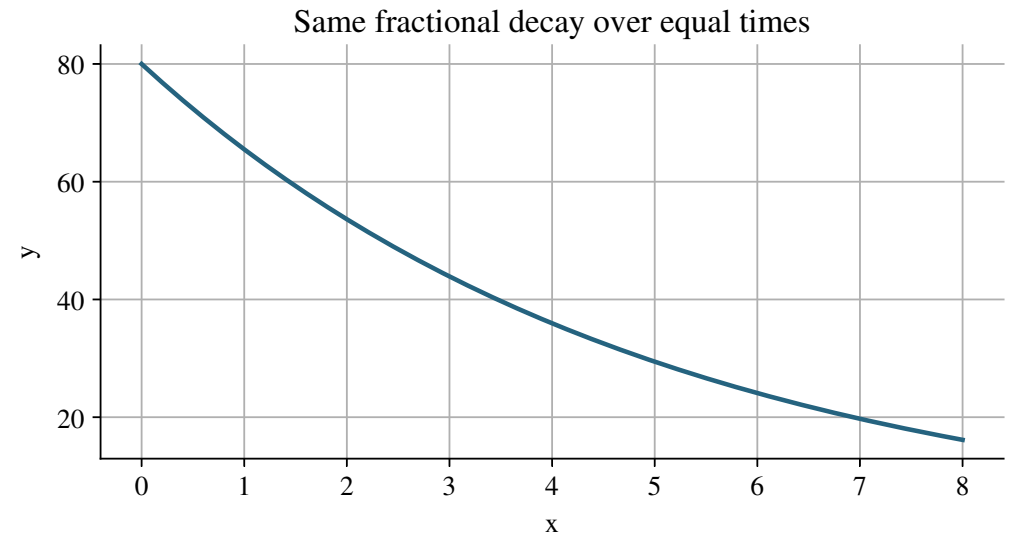
- For $y = Ae^{kt}$, k is a proportional rate.
- Taking logs gives $\ln y = \ln A + kt$.
- Logarithms require positive arguments, and $\log(x+y)$ is not $\log x + \log y$.

A worked case

$$y = Ae^{kt}, \quad \ln y = \ln A + kt$$

■ Worked example

If $y = 80e^{(-0.2t)}$, $y = 40$ gives $e^{(-0.2t)} = 1/2$. Hence $t = \ln 2 / 0.2 \approx 3.466$. For $3^x = 20$, $x = \ln 20 / \ln 3$. Plotting $\ln y$ against t linearises this exponential model.



Where it goes wrong

- A fitted exponential is a model, not a guarantee.
- Specify the time units and range of use.
- A negative k describes decay; a negative starting amount usually contradicts the context.

■ Watch out

Not true: For positive x and y , $\ln(x+y) = \ln x + \ln y$. The log of a sum has no simple law; the product rule is $\ln(xy) = \ln x + \ln y$.

Applying it

- Compare actual observations with the model.
- Systematic departures may indicate changing conditions.
- In a report, explain what the rate and initial value mean, rather than giving a bare equation.

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The method

- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$.
- Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve.
- List every solution in the stated interval.

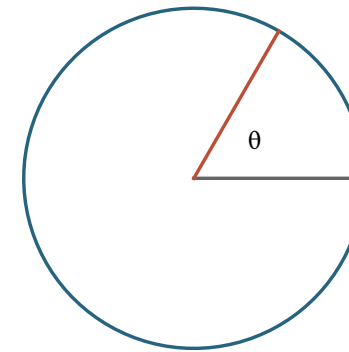
A worked case

$$s = r\theta, \quad A = \frac{1}{2}r^2\theta, \quad \sin^2\theta + \cos^2\theta = 1$$

■ Worked example

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0\leq x<2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0\leq 2x<4\pi$ before dividing by 2.

A radius fixes distance from the centre



radius $\times$ angle gives arc length

Where it goes wrong

- A calculator's inverse sine gives one principal value.
- It does not give every solution.
- Dividing by $\sin x$ can discard solutions where $\sin x=0$; check that case first.

■ Watch out

Not true: The principal inverse sine value is the only solution in a full turn. Sine takes most values twice in a full turn: $\sin x=0.5$ gives 30° and 150° .

Applying it

- Choose an identity that reduces the equation to one trigonometric function.
- Keep exact familiar angles where possible, and check each solution against the original equation.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Radian:** The angle subtended by an arc whose length equals the radius.
- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$.
- Watch for: A calculator's inverse sine gives one principal value.

P2 · Derivatives and stationary points

What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative**: The instantaneous rate of change, also the gradient of a tangent.

The method

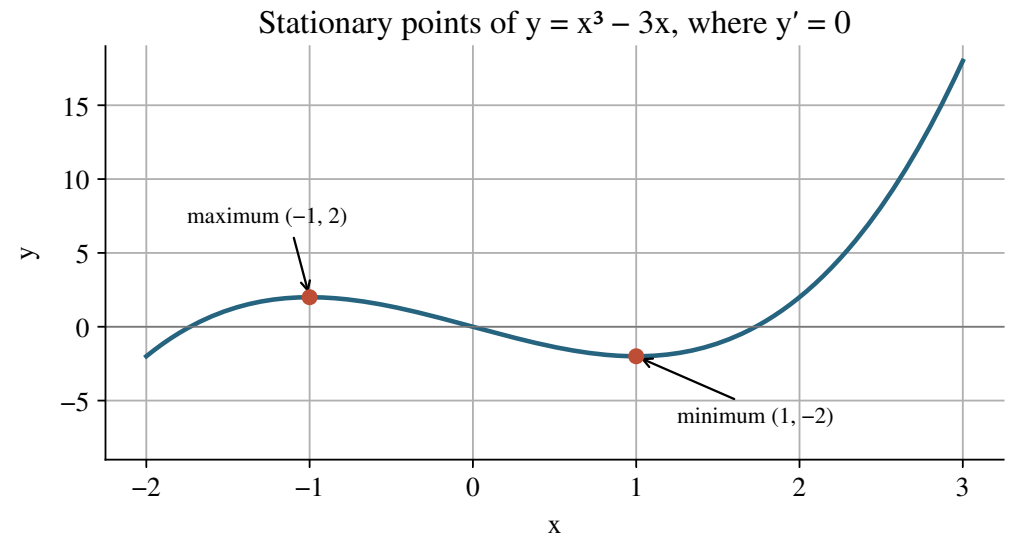
- For $y=ax^n$, $dy/dx=nax^{n-1}$.
- A stationary point satisfies $dy/dx=0$.
- Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.

A worked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad f'(x) = 0$$

■ Worked example

For $y=x^3-3x$, $dy/dx=3x^2-3$. At $x=1$, the gradient is 0 and $y=-2$. The second derivative is $6x$, positive at $x=1$, so this is a local minimum. At $x=-1$, $y=2$ and the second derivative is negative, giving a local maximum.



Where it goes wrong

- A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.
- An endpoint can also produce an extreme value on a restricted domain.

■ Watch out

Not true: Every point with zero derivative is a local maximum or minimum. A zero derivative can also be a point of inflection: $y=x^3$ at $x=0$ is neither a maximum nor a minimum.

Applying it

- At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there.
- At advanced level, connect the derivative to rates and optimization with a valid domain.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Derivative:** The instantaneous rate of change, also the gradient of a tangent.
- For $y = ax^n$, $dy/dx = anx^{n-1}$.
- Watch for: A zero derivative does not always mean a maximum or minimum: $y = x^3$ is stationary at 0 but continues increasing.

P2 · Integrals, area and accumulation

How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative**: A function whose derivative equals the given integrand.

The method

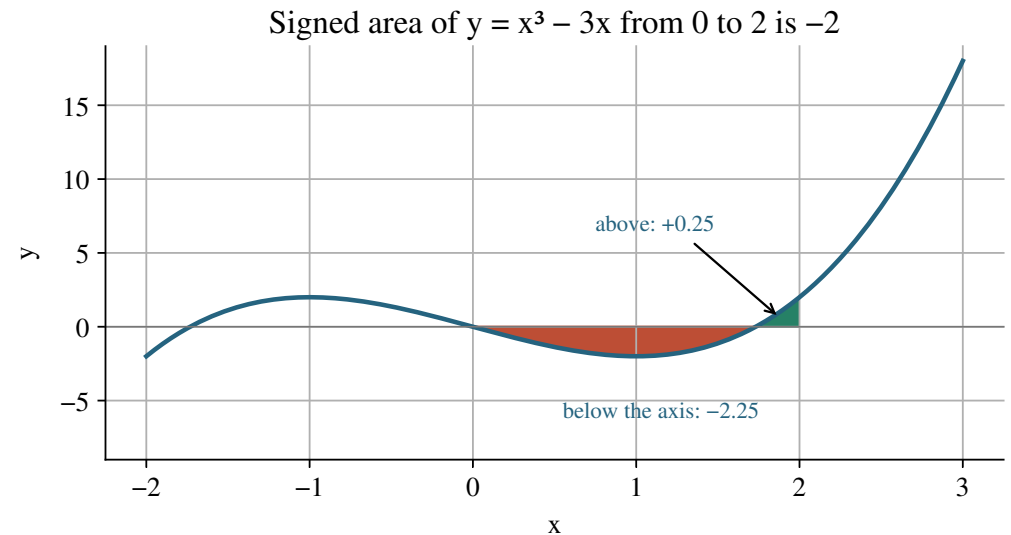
- For $n \neq -1$, the integral of ax^n is $ax^{n+1}/(n+1) + C$.
- A definite integral is signed accumulation.
- Split at sign changes when total area or distance is required.

A worked case

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

■ Worked example

For $v(t) = 3t^2 - 3$ over $0 \leq t \leq 2$, displacement = $[t^3 - 3t]_0^2 = 2$. Since v changes sign at $t = 1$, distance = $-[t^3 - 3t]_0^1 + [t^3 - 3t]_1^2 = 2 + 4 = 6$. The constants cancel only for a definite integral.



Where it goes wrong

- The integral of $1/x$ is $\ln|x| + C$, not the power rule with $n = -1$.
- Signed area can be zero even when the total enclosed area is positive.

■ Watch out

Not true: A definite integral equals the total positive area, even when the graph crosses the axis. Area below the axis counts as negative, so a definite integral can be smaller than the total area, or even zero.

Applying it

- Identify what the integral means and include the correct units.
- A rate measured per second integrates to the underlying quantity, not to another rate.
- Differentiate an antiderivative to check it.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Antiderivative:** A function whose derivative equals the given integrand.
- For $n \neq -1$, the integral of ax^n is $ax^{(n+1)}/(n+1)+C$.
- Watch for: the integral of $1/x$ is $\ln|x|+C$, not the power rule with $n=-1$.

P2 · Root finding and numerical integration

How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
- This lesson studies **iteration**: A repeated update in which each new approximation is calculated from the previous one.

The method

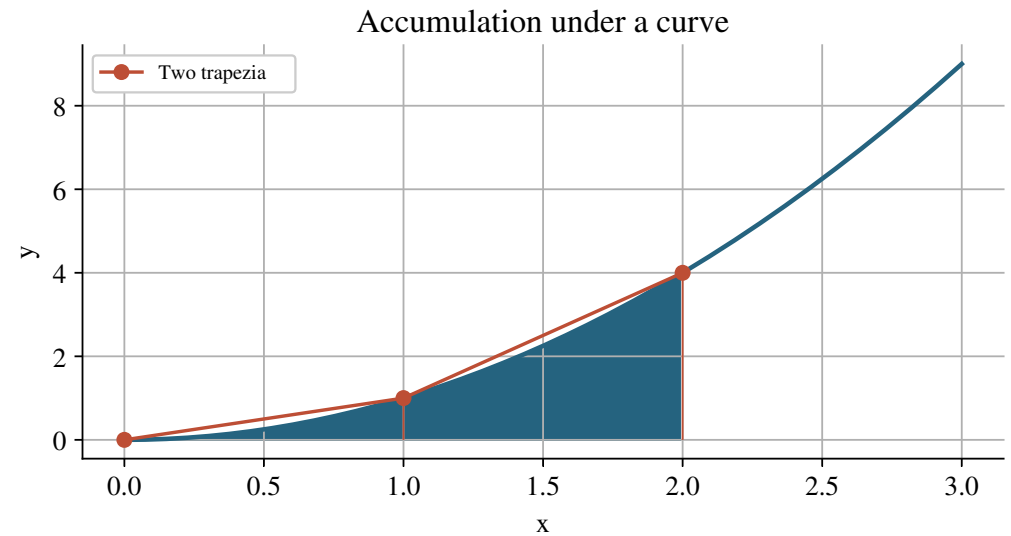
- For equal strip width h , the trapezium estimate is $h/2$ times the sum of the two endpoint heights plus twice the internal heights.

A worked case

$$A \approx \frac{h}{2}(y_0 + 2y_1 + \cdots + 2y_{n-1} + y_n)$$

■ Worked example

For $y=x^2$ on $[0,2]$ with two equal strips, $h=1$ and the heights are 0,1,4. The estimate is $(1/2)(0+2 \times 1+4)=3$. The exact area is $8/3$, so this convex curve gives an overestimate.



Where it goes wrong

- Use equal strip widths and include internal heights twice.
- Curvature decides whether the estimate is above or below the exact area; being increasing alone does not.

■ Watch out

Not true: An increasing function gives a trapezium overestimate. A sign change shows a root only if the function is continuous there: $1/x$ changes sign across $x=0$ but has no root.

Applying it

- P2 introduces the trapezium rule.
- Newton–Raphson and other root-finding methods belong to other named units; they are not part of this P2 lesson.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Iteration:** A repeated update in which each new approximation is calculated from the previous one.
- For equal strip width h , the trapezium estimate is $h/2$ times the sum of the two endpoint heights plus twice the internal heights.
- Watch for: use equal strip widths and include internal heights twice.

P3 · Domains, inverses and composition

Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
- This lesson studies **domain** : The set of allowed inputs to a function.

The method

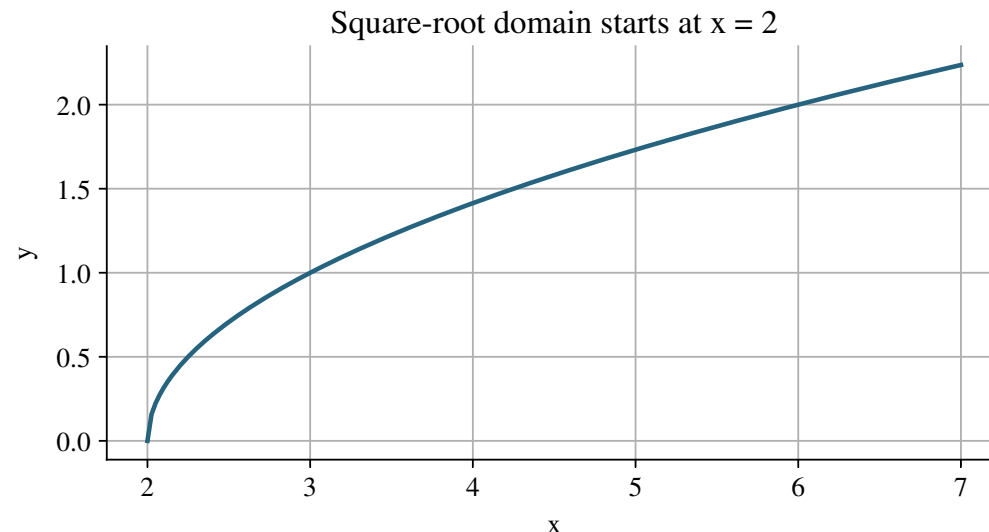
- State the domain and range.
- For an inverse, first ensure the function is one-to-one on its domain.
- Composition fg means apply g first, then f ; the intermediate output must be an allowed input to f .

A worked case

$$f(g(x)) = (f \circ g)(x), \quad f^{-1}(f(x)) = x$$

■ Worked example

For $f(x)=\sqrt{x-2}$, $x \geq 2$ and the range is $y \geq 0$. From $y=\sqrt{x-2}$, $x=y^2+2$. Thus $f^{-1}(y)=y^2+2$ with $y \geq 0$. For $g(x)=x+3$, $fg(1)=f(4)=\sqrt{2}$.



Where it goes wrong

- Squaring can introduce extraneous solutions.
- Restricting a parabola's domain is essential before claiming an inverse.
- A horizontal translation inside f has the opposite sign to the graph's movement.

■ Watch out

Not true: Every quadratic function on all real numbers has an inverse function. A quadratic on all real numbers is many-to-one ($f(1)=f(-1)$ for x^2), so its domain must be restricted first.

Applying it

- Check $f(f^{-1}(x))=x$ on the inverse domain.
- Use a sketch to test whether a horizontal line meets the original graph more than once.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Domain:** The set of allowed inputs to a function.
- State the domain and range.
- Watch for: squaring can introduce extraneous solutions.

P3 · Radians, identities and trigonometric equations

How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
- This lesson studies **radian** : The angle subtended by an arc whose length equals the radius.

The method

- For θ in radians, arc length $s=r\theta$ and sector area $A=\frac{1}{2}r^2\theta$.
- Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve.
- List every solution in the stated interval.

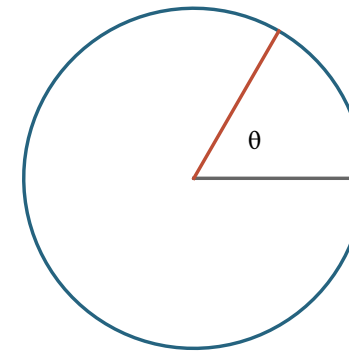
A worked case

$$s = r\theta, \quad A = \frac{1}{2}r^2\theta, \quad \sin^2\theta + \cos^2\theta = 1$$

■ Worked example

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0\leq x<2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0\leq 2x<4\pi$ before dividing by 2.

A radius fixes distance from the centre



radius $\times$ angle gives arc length

Where it goes wrong

- A calculator's inverse sine gives one principal value.
- It does not give every solution.
- Dividing by $\sin x$ can discard solutions where $\sin x=0$; check that case first.

■ Watch out

Not true: The principal inverse sine value is the only solution in a full turn. Sine takes most values twice in a full turn: $\sin x=0.5$ gives 30° and 150° .

Applying it

- Choose an identity that reduces the equation to one trigonometric function.
 - Keep exact familiar angles where possible, and check each solution against the original equation.
-

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Radian:** The angle subtended by an arc whose length equals the radius.
- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$.
- Watch for: A calculator's inverse sine gives one principal value.

P3 · Exponentials, logarithms and modelling

Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.
- This lesson studies **half-life** : The time for a decaying quantity to fall to half its initial value.

The method

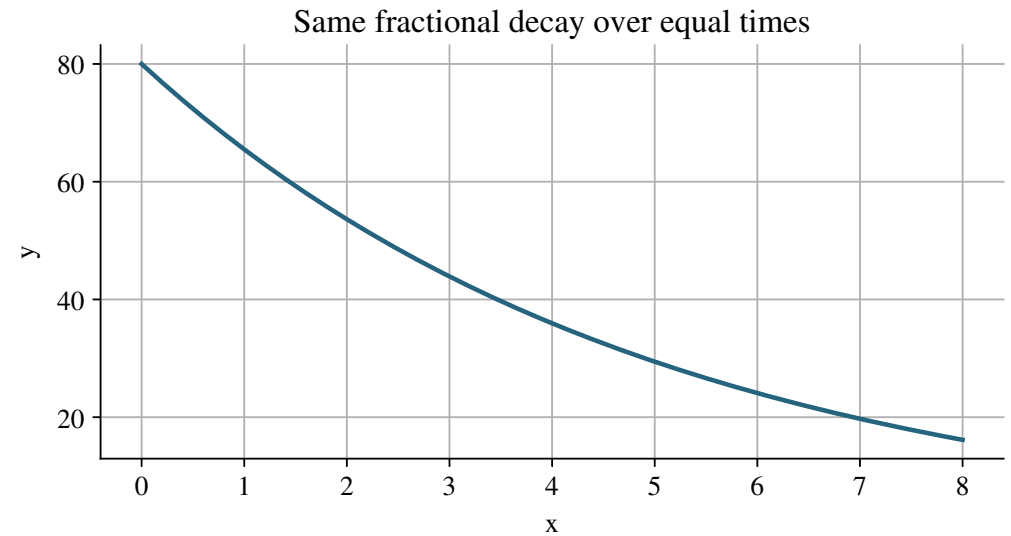
- For $y=Ae^{kt}$, k is a proportional rate.
- Taking logs gives $\ln y = \ln A + kt$.
- Logarithms require positive arguments, and $\log(x+y)$ is not $\log x + \log y$.

A worked case

$$y = Ae^{kt}, \quad \ln y = \ln A + kt$$

■ Worked example

If $y=80e^{(-0.2t)}$, $y=40$ gives $e^{(-0.2t)}=1/2$. Hence $t=\ln 2/0.2 \approx 3.466$. For $3^x=20$, $x=\ln 20/\ln 3$. Plotting $\ln y$ against t linearises this exponential model.



Where it goes wrong

- A fitted exponential is a model, not a guarantee.
- Specify the time units and range of use.
- A negative k describes decay; a negative starting amount usually contradicts the context.

■ Watch out

Not true: For positive x and y , $\ln(x+y) \neq \ln x + \ln y$. The log of a sum has no simple law; the product rule is $\ln(xy) = \ln x + \ln y$.

Applying it

- Compare actual observations with the model.
- Systematic departures may indicate changing conditions.
- In a report, explain what the rate and initial value mean, rather than giving a bare equation.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Half-life:** The time for a decaying quantity to fall to half its initial value.
- For $y=Ae^{kt}$, k is a proportional rate.
- Watch for: A fitted exponential is a model, not a guarantee.

P3 · Chain, product, quotient and implicit differentiation

Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
- This lesson studies **chain rule** : The rule that multiplies the outer derivative by the inner derivative for a composite function.

The method

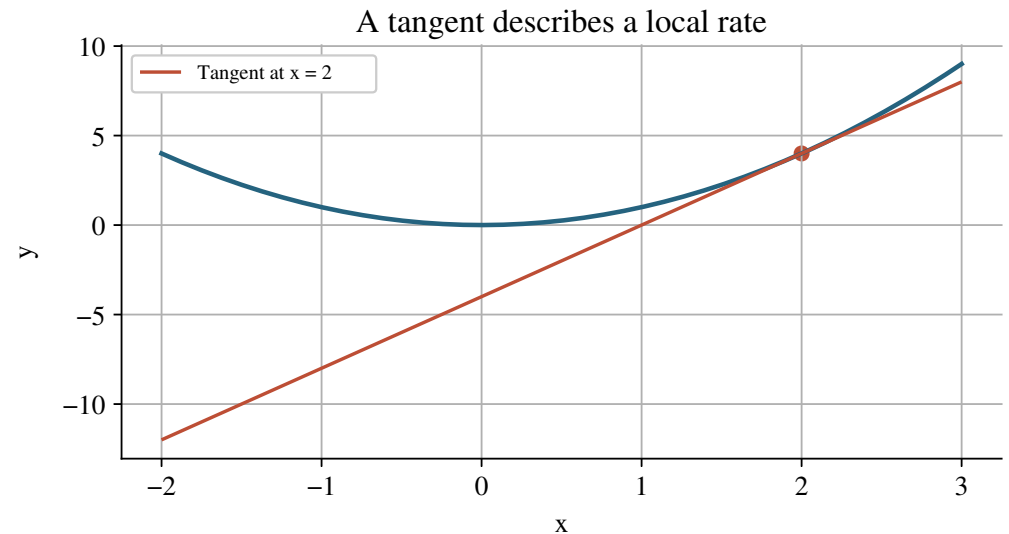
- For $y=f(g(x))$, $y' = f'(g(x))g'(x)$.
- For uv , differentiate to $u'v + uv'$.
- For u/v , use $(u'v - uv')/v^2$, where $v \neq 0$.

A worked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

■ Worked example

For $y=(3x+1)^4$, $y' = 4(3x+1)^3 \times 3 = 12(3x+1)^3$. At $x=0$, the gradient is 12.
For $x^2+y^2=25$, differentiate implicitly: $2x+2y y' = 0$, so $y' = -x/y$ when $y \neq 0$.



Where it goes wrong

- A derivative of a product is not the product of the derivatives.
- In implicit differentiation, every differentiated function of y brings a dy/dx factor.
- A quotient's denominator is squared.

■ Watch out

Not true: The derivative of $u(x)v(x)$ is u' times v prime. The product rule is $(uv)' = u'v + uv'$: for $x \cdot x$ the derivative is $2x$, not $1 \times 1 = 1$.

Applying it

- Choose a useful form before differentiating: expanding a short polynomial may be simpler.
- For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Chain rule:** The rule that multiplies the outer derivative by the inner derivative for a composite function.
- For $y=f(g(x))$, $y' = f'(g(x))g'(x)$.
- Watch for: A derivative of a product is not the product of the derivatives.

P3 · Integrals, area and accumulation

How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative** : A function whose derivative equals the given integrand.

The method

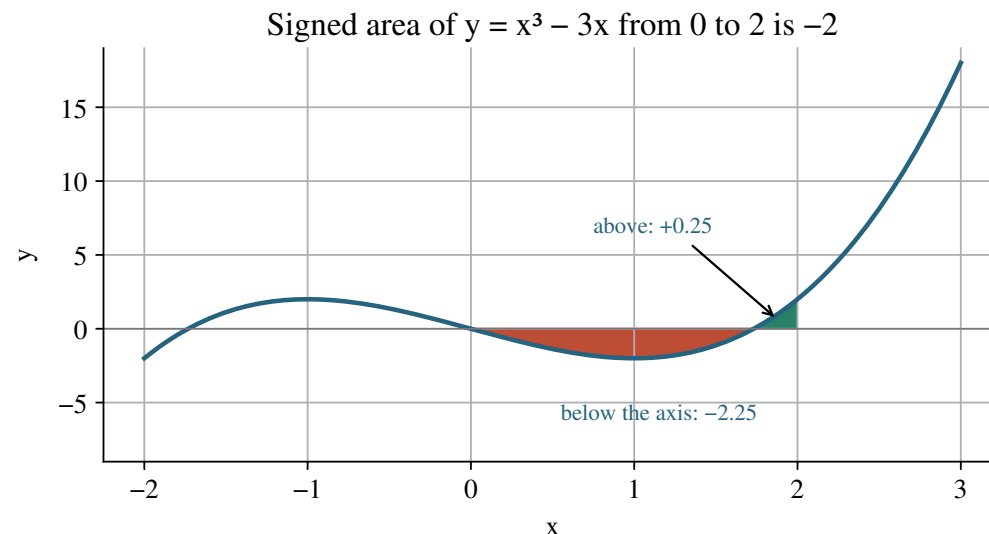
- For $n \neq -1$, the integral of ax^n is $ax^{n+1}/(n+1) + C$.
- A definite integral is signed accumulation.
- Split at sign changes when total area or distance is required.

A worked case

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

■ Worked example

For $v(t)=3t^2-3$ over $0 \leq t \leq 2$, displacement= $[t^3-3t]_0^2=2$. Since v changes sign at $t=1$, distance= $-[t^3-3t]_0^1 + [t^3-3t]_1^2 = 2+4=6$. The constants cancel only for a definite integral.



Where it goes wrong

- The integral of $1/x$ is $\ln|x| + C$, not the power rule with $n=-1$.
- Signed area can be zero even when the total enclosed area is positive.

■ Watch out

Not true: A definite integral equals the total positive area, even when the graph crosses the axis. Area below the axis counts as negative, so a definite integral can be smaller than the total area, or even zero.

Applying it

- Identify what the integral means and include the correct units.
- A rate measured per second integrates to the underlying quantity, not to another rate.
- Differentiate an antiderivative to check it.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Antiderivative:** A function whose derivative equals the given integrand.
- For $n \neq -1$, the integral of ax^n is $ax^{(n+1)}/(n+1) + C$.
- Watch for: the integral of $1/x$ is $\ln|x| + C$, not the power rule with $n = -1$.

P3 · Fixed-point iteration

How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
- This lesson studies **iteration**: A repeated update in which each new approximation is calculated from the previous one.

The method

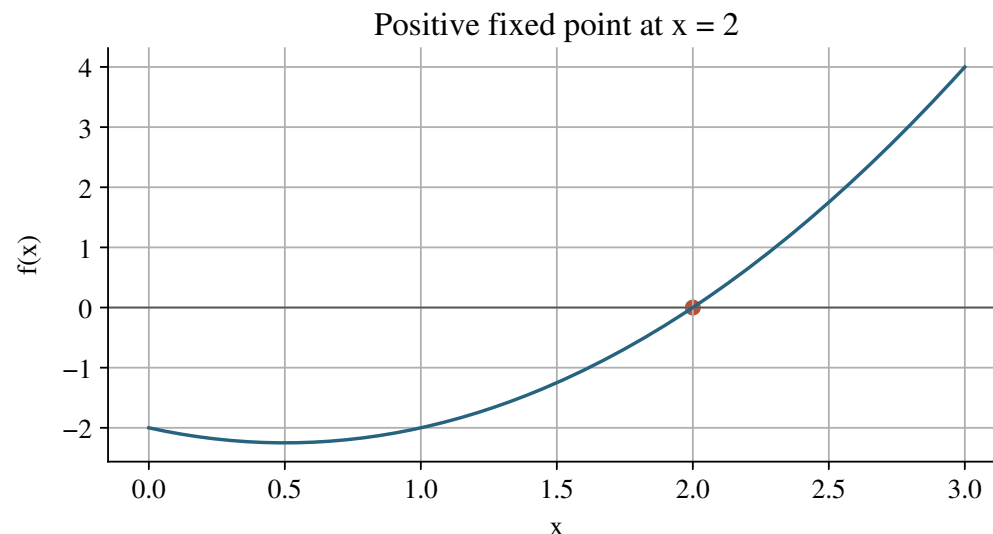
- Rearrange the equation as $x = g(x)$, choose a starting value and iterate.
- A limit L must satisfy $L = g(L)$.
- Near a fixed point, $|g'(L)| < 1$ provides a local convergence check; a sign bracket can validate the reported rounded root.

A worked case

$$x_{n+1} = \sqrt{2 + x_n}$$

■ Worked example

For $x^2 - x - 2 = 0$ choose $x_{\text{next}} = \sqrt{2 + x}$, starting at $x_0 = 1$. Then $x_1 = \sqrt{3} \approx 1.73205$, $x_2 \approx 1.93185$ and the positive fixed point is 2. At 2, $g'(x) = 1/(2\sqrt{2}) = 1/4$, so small local errors shrink. The square-root rearrangement seeks the positive root; it does not give the negative root -1.



Where it goes wrong

- Different rearrangements can have different convergence behaviour.
- Check the iterates, domain and original equation; a square-root update cannot reach a negative fixed point.

■ Watch out

Not true: Every rearrangement of an equation converges to the same root from every starting value. A sign change shows a root only if the function is continuous there: $1/x$ changes sign across $x = 0$ but has no root.

Applying it

- P3 uses equation rearrangement and numerical iteration.
- This lesson excludes Newton and numerical integration, which belong to other units.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Iteration:** A repeated update in which each new approximation is calculated from the previous one.
- Rearrange the equation as $x=g(x)$, choose a starting value and iterate.
- Watch for: different rearrangements can have different convergence behaviour.

P4 · Deduction, contradiction and induction

When does a pattern become a proof?

- Checking several integers can suggest a pattern. What turns the pattern into a proof for every integer?
 - This lesson studies **counterexample** : A single valid case that disproves a universal claim.
-

The method

- A deductive proof starts from stated definitions or assumptions.
 - A counterexample refutes an all-values claim.
 - Induction requires a base case and an implication from $n=k$ to $n=k+1$; checking successive cases alone is not induction.
-

A worked case

$$(2a + 1) + (2b + 1) = 2(a + b + 1)$$

■ Worked example

For odd integers $2a+1$ and $2b+1$, their sum is $2(a+b+1)$, hence even. The claim n^2+n+41 is always prime fails at $n=41$: the value is $41 \times 43 = 1763$. For $1+\dots+n=n(n+1)/2$, the induction step adds $k+1$ to the assumed sum.

$$\text{Odd + odd: } (2a + 1) + (2b + 1)$$

$$\text{Factor 2: } 2(a + b + 1)$$

The bracket is an integer $\Rightarrow$ the sum is even

Where it goes wrong

- Do not assume the conclusion while proving it.
- In contradiction, identify the impossible consequence and reject the initial contrary assumption.
- Induction is not compulsory in AQA 7357; it belongs in its own appropriate further-mathematics scope.

■ Watch out

Not true: Checking the first ten integers proves a claim for every positive integer. A pattern can hold for the first ten cases and then fail; a proof must cover every case.

Applying it

- AQA proof includes deduction, exhaustion and contradiction.
 - IAL P2 introduces exhaustion and counterexample, while P4 introduces contradiction.
 - Keep the method matched to the named unit.
-

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Counterexample:** A single valid case that disproves a universal claim.
- A deductive proof starts from stated definitions or assumptions.
- Watch for: do not assume the conclusion while proving it.

P4 · Equations, identities and rearrangement

When do two plans cost the same?

- Two mobile plans cost $20+3x$ and $44+x$ yuan for x GB. When do they cost the same?
- This lesson studies **identity** : An equality that holds for every allowed value of its variable.

The method

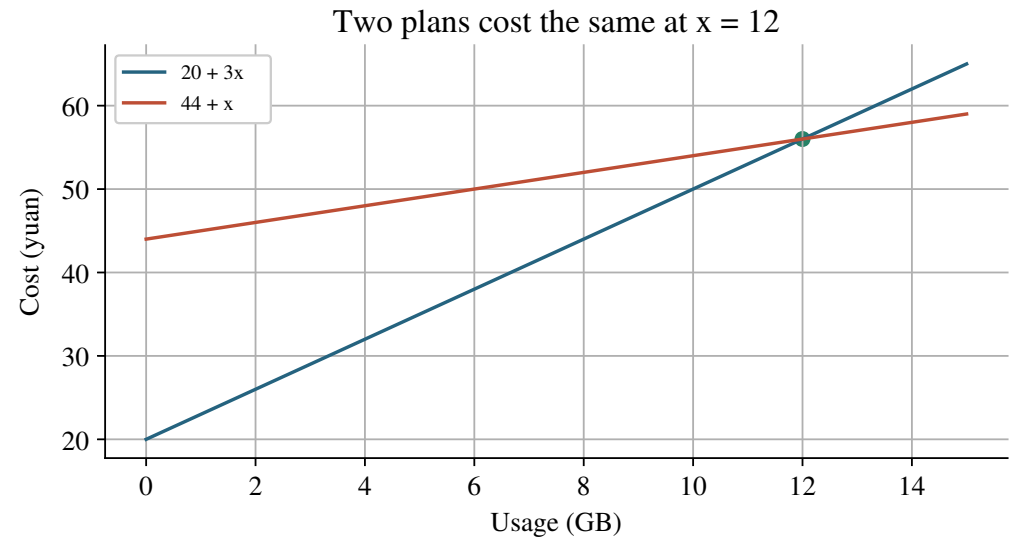
- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs.
- Preserve equality by applying the same operation to both sides.
- State restrictions before dividing by a variable.

A worked case

$$C_1 = 20 + 3x, \quad C_2 = 44 + x, \quad C_1 = C_2$$

■ Worked example

$20+3x=44+x$ gives $2x=24$ and $x=12$. Both plans then cost 56. In $A=\pi r^2$, divide by π and take the positive square root to obtain $r=\sqrt{(A/\pi)}$, because r is a length.



Where it goes wrong

- Cancelling a term is not the same as cancelling a factor.
- In $(x^2+2x)/x$, factor the numerator and retain $x \neq 0$.
- Check a rearrangement by substitution.

■ Watch out

Not true: Cancelling x from $(x+3)/x$ leaves 3 for every nonzero x . x is a term of the numerator, not a factor, so it cannot be cancelled: at $x=1$, $(1+3)/1=4$, not 3.

Applying it

- Set up the equation from units and the meaning of the unknown.
- A negative or fractional solution may be algebraically correct but impossible for a count.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Identity:** An equality that holds for every allowed value of its variable.
- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs.
- Watch for: cancelling a term is not the same as cancelling a factor.

P4 · Binomial expansion and valid approximations

How does a small change affect a power?

- A small measurement change affects a power of a quantity. An expansion can show the size of first and second effects.
- This lesson studies **binomial coefficient** : The number of ways to choose a specified number of objects from a set.

The method

- For positive integer n , $(a+b)^n$ is a finite binomial expansion.
- For noninteger n , expand $(1+x)^n$ as $1+nx+n(n-1)x^2/2+\dots$ with $|x|<1$.
- Factor out constants before using this form.

A worked case

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \dots$$

■ Worked example

$(1+2x)^5=1+10x+40x^2+80x^3+80x^4+32x^5$. For $(1+x)^{-1}$, the first three terms are $1-x+x^2$; at $x=0.1$ this gives 0.91 versus the exact $1/1.1\approx0.909091$.

$(1+2x)^5$: Pascal row $\times$ powers of 2

Pascal row 5	1	5	10	10	5	1
$\times 2^k$	1	2	4	8	16	32
coefficient	1	10	40	80	80	32
	1	x	x ²	x ³	x ⁴	x ⁵

Where it goes wrong

- An expansion in $2x$ requires $|2x|<1$ for the infinite series, not merely $|x|<1$.
- Positive-integer expansions are finite and do not have that convergence restriction.

■ Watch out

Not true: Every binomial expansion is a finite polynomial, including powers that are not positive integers. For a negative or fractional power the expansion is an infinite series, valid only for $|x|<1$.

Applying it

- State the range of validity alongside an approximation.
- Retain enough terms to support the requested accuracy, and distinguish a coefficient from the whole term containing x .

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Binomial coefficient:** The number of ways to choose a specified number of objects from a set.
- For positive integer n , $(a+b)^n$ is a finite binomial expansion.
- Watch for: an expansion in $2x$ requires $|2x| < 1$ for the infinite series, not merely $|x| < 1$.

P4 · Conics, parametric curves and tangent reasoning

How can a rotating parameter trace an ellipse?

- An ellipse can be traced by a rotating parameter even when it is awkward to describe as a single $y=f(x)$.
- This lesson studies **parametric equation** : A representation in which coordinates are expressed using a common parameter.

The method

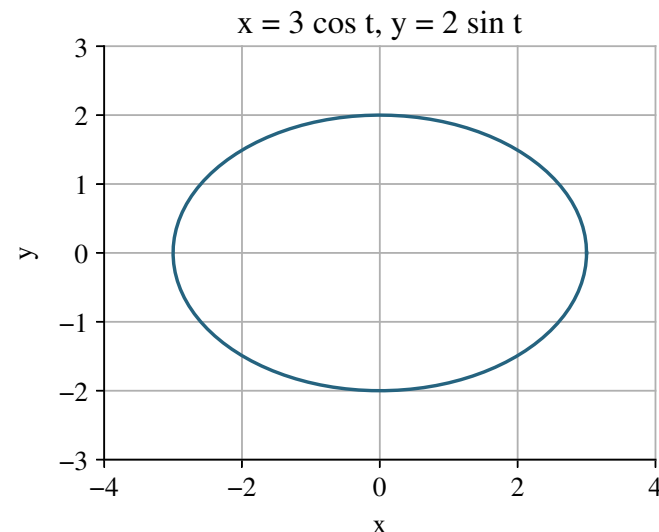
- For $x=a \cos t, y=b \sin t$, eliminating t gives $x^2/a^2 + y^2/b^2 = 1$.
- For parametric curves, $dy/dx = (dy/dt)/(dx/dt)$, where $dx/dt \neq 0$.
- A zero denominator may signal a vertical tangent.

A worked case

$$x = a \cos t, \quad y = b \sin t, \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

■ Worked example

For $x=3\cos t, y=2\sin t$ at $t=\pi/4$, the gradient is $(2\cos t)/(-3\sin t)=-2/3$. The point is $(3/\sqrt{2}, \sqrt{2})$. The area inside the ellipse is $\pi ab=6\pi$; its semiaxes are 3 and 2.



Where it goes wrong

- Eliminating a parameter may lose a domain restriction or direction of travel.
- A vertical tangent cannot be assigned a finite dy/dx .
- Distinguish semiaxes from full widths.

■ Watch out

Not true: Eliminating a parameter preserves every restriction and direction automatically. Elimination can lose restrictions: $x=t^2, y=t$ gives $x=y^2$, but $x \geq 0$ and the direction of motion must be stated separately.

Applying it

- For advanced coordinate geometry, use the defining equation and a consistent parameter.
- For IAL P4 parametric integration, use the specification's restricted requirements rather than importing all Further Pure conics.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Parametric equation:** A representation in which coordinates are expressed using a common parameter.
- For $x=a \cos t, y=b \sin t$, eliminating t gives $x^2/a^2+y^2/b^2=1$.
- Watch for: eliminating a parameter may lose a domain restriction or direction of travel.

P4 · Chain, product, quotient and implicit differentiation

Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
- This lesson studies **chain rule** : The rule that multiplies the outer derivative by the inner derivative for a composite function.

The method

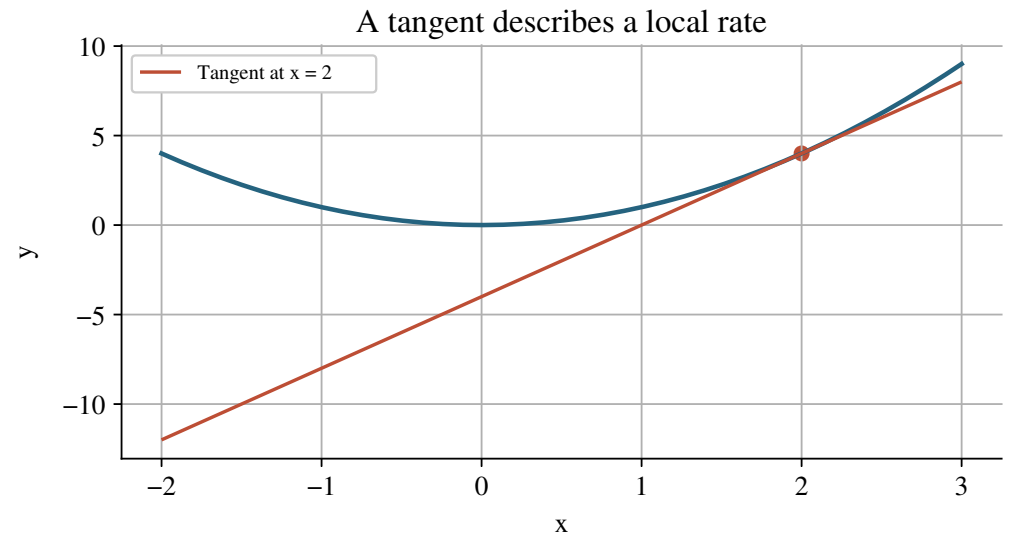
- For $y=f(g(x))$, $y' = f'(g(x))g'(x)$.
- For uv , differentiate to $u'v+uv'$.
- For u/v , use $(u'v-uv')/v^2$, where $v \neq 0$.

A worked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

■ Worked example

For $y=(3x+1)^4$, $y' = 4(3x+1)^3 \times 3 = 12(3x+1)^3$. At $x=0$, the gradient is 12.
For $x^2+y^2=25$, differentiate implicitly: $2x+2y y' = 0$, so $y' = -x/y$ when $y \neq 0$.



Where it goes wrong

- A derivative of a product is not the product of the derivatives.
- In implicit differentiation, every differentiated function of y brings a dy/dx factor.
- A quotient's denominator is squared.

■ Watch out

Not true: The derivative of $u(x)v(x)$ is u' times v prime. The product rule is $(uv)' = u'v + uv'$: for $x \cdot x$ the derivative is $2x$, not $1 \times 1 = 1$.

Applying it

- Choose a useful form before differentiating: expanding a short polynomial may be simpler.
- For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Chain rule:** The rule that multiplies the outer derivative by the inner derivative for a composite function.
- For $y=f(g(x))$, $y' = f'(g(x))g'(x)$.
- Watch for: A derivative of a product is not the product of the derivatives.

P4 · Substitution, parts and partial fractions

Can we integrate the two factors separately?

- A simple-looking product such as $x e^x$ cannot be integrated by separately integrating each factor.
- This lesson studies **integration by parts** : An integration method based on the derivative of a product.

The method

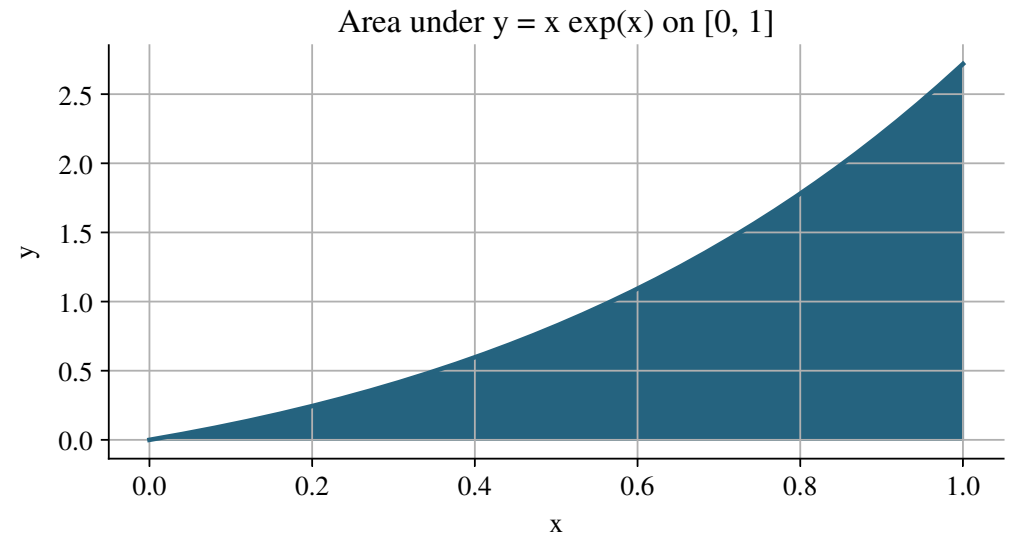
- Use substitution when an inner derivative appears as a factor.
- By parts, $\int u v' = uv - \int u' v$.
- For a rational function, divide first if needed, then decompose into partial fractions.

A worked case

$$\int u v' dx = uv - \int u' v dx$$

■ Worked example

For integral $\int x e^x dx$, take $u=x$ and $v' = e^x$. Then the integral is $x e^x - e^x + C$.
For integral $\int 2x/(x^2+1) dx$, substitute $w=x^2+1$, $dw=2x dx$; the answer is $\ln(x^2+1)+C$.



Where it goes wrong

- Choosing u and v' well matters: the remaining integral should become simpler.
- In a definite substitution, either change the limits or return to x before applying the original limits.

■ Watch out

Not true: The integral of a product equals the product of its separate integrals. A product needs a method such as integration by parts or substitution: $\int x \cdot x dx = x^3/3$, not $(x^2/2)(x^2/2)$.

Applying it

- Check by differentiating.
- For volumes of revolution around the x -axis use $V = \pi \int y^2 dx$; do not confuse the square of a function with the integral of the function.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Integration by parts:** An integration method based on the derivative of a product.
- Use substitution when an inner derivative appears as a factor.
- Watch for: choosing u and v prime well matters: the remaining integral should become simpler.

P4 · Differential equations and numerical solutions

What does the starting value decide?

- A growing population's rate is proportional to its current size. The rate equation describes a whole family until an initial population is supplied.
- This lesson studies **initial condition** : A specified value that selects a particular solution from a family.

The method

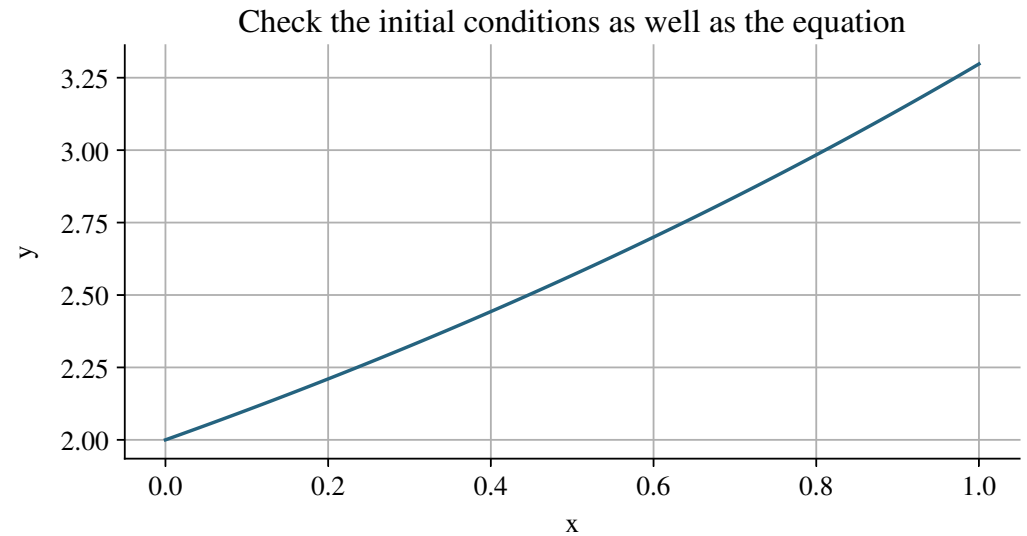
- Separate the variables in $dy/dx=ky$, integrate both sides and apply an initial condition.
- Include any equilibrium solution excluded when dividing by y .

A worked case

$$\frac{dy}{dx} = ky, \quad y = Ae^{kx}$$

■ Worked example

For $dy/dx=0.5y$ and $y(0)=2$, separation gives $\ln|y|=0.5x+C$, hence $y=Ae^{(0.5x)}$. The initial value gives $A=2$. Differentiate the result to check the equation and substitute $x=0$ to check the initial condition.



Where it goes wrong

- An initial condition determines the integration constant; it is not a replacement for integrating.
- Dividing by y can omit an equilibrium solution $y=0$.
- Euler accuracy depends on step size and the equation.

■ Watch out

Not true: An initial condition never changes the constant in a differential-equation solution. Euler's method depends on the step size; smaller steps usually give a more accurate value.

Applying it

- This unit uses first-order separation and initial conditions.
- Euler numerical integration and second-order complementary functions are excluded from this lesson.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Initial condition:** A specified value that selects a particular solution from a family.
- Separate the variables in $dy/dx=ky$, integrate both sides and apply an initial condition.
- Watch for: an initial condition determines the integration constant; it is not a replacement for integrating.

P4 · Spatial vectors, lines and angles

Can nonparallel paths still miss each other?

- Two paths in space can be neither parallel nor intersecting. A two-dimensional sketch may conceal their separation.
- This lesson studies **direction vector** : A nonzero vector giving the direction of a line.

The method

- A line has $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$ with position vector $\mathbf{a}$ and nonzero direction $\mathbf{b}$.
- To find an intersection, equate all components and solve for both line parameters.
- For an angle use $\mathbf{a} \cdot \mathbf{b}=|\mathbf{a}||\mathbf{b}|\cos\theta$ where scalar products are in the named syllabus.

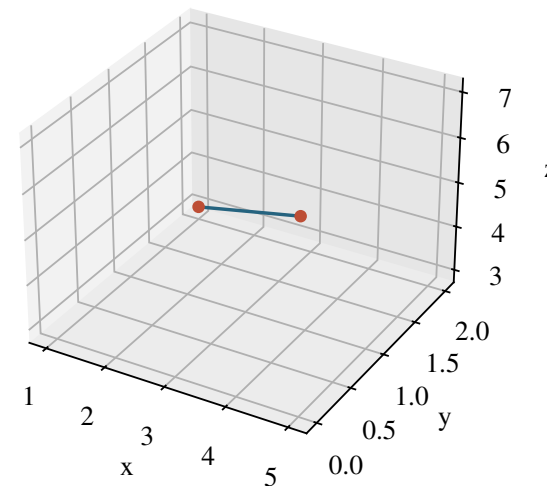
A worked case

$$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}, \quad \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$$

■ Worked example

For $\mathbf{a}=(1,2,3), \mathbf{b}=(2,-1,2)$, $\lambda=2$ gives $\mathbf{r}=(5,0,7)$. The magnitude of $\mathbf{b}$ is $\sqrt{(4+1+4)}=3$. With $\mathbf{c}=(1,0,-1)$, $\mathbf{b} \cdot \mathbf{c}=2-2=0$, so $\mathbf{b}$ and $\mathbf{c}$ are perpendicular.

$$\mathbf{r} = (1, 2, 3) + \lambda(2, -1, 2)$$



Where it goes wrong

- Satisfying two component equations does not guarantee the third.
- Nonparallel lines may be skew.
- A line's direction vector is not its position vector, and a scalar parameter has no coordinate units.

■ Watch out

Not true: Any two nonparallel lines in three dimensions must intersect. In three dimensions nonparallel lines can be skew: they do not meet.

Applying it

- State whether a requested angle is acute, directed or an angle between lines.
- Use the syllabus-specific plane or distance method only when it is actually required for that unit.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Direction vector:** A nonzero vector giving the direction of a line.
- A line has $r=a+\lambda b$ with position vector a and nonzero direction b .
- Watch for: satisfying two component equations does not guarantee the third.

FP1 · Complex arithmetic and quadratic roots

What if the root is not real?

- The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots.
- This lesson studies **complex conjugate** : The number obtained by reversing the sign of the imaginary part.

The method

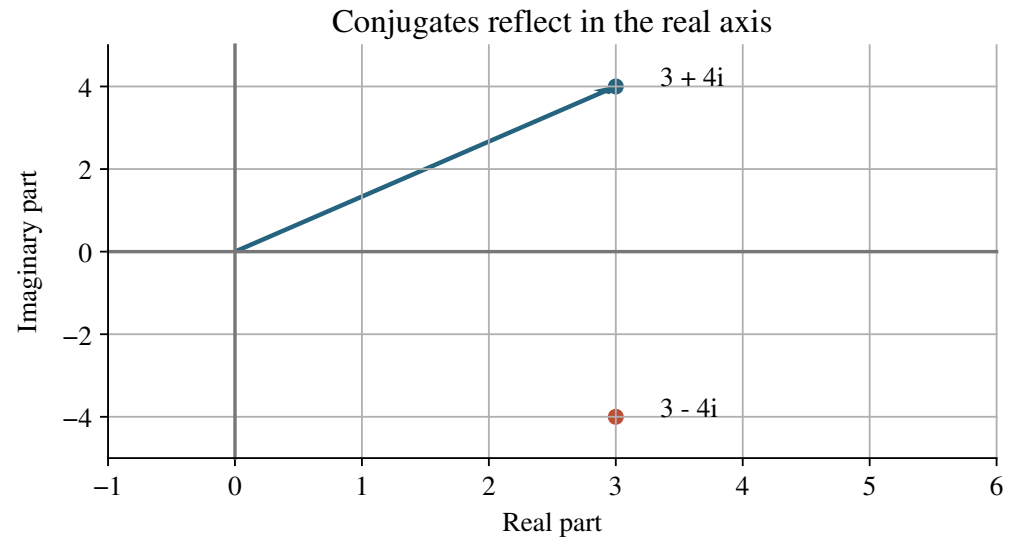
- Write $z=a+bi$ with $i^2=-1$.
- Add components, multiply brackets and use a conjugate to make a division denominator real.
- Non-real roots of a real-coefficient quadratic occur in conjugate pairs.

A worked case

$$z = a + bi, \quad |z| = \sqrt{a^2 + b^2}, \quad z\bar{z} = a^2 + b^2$$

■ Worked example

For $z=3+4i$, $|z|=5$ and z conjugate= $3-4i$. The reciprocal is $(3-4i)/25$. For $x^2-6x+13=0$, the roots are $3\pm 2i$. Multiplying by the conjugate makes a complex denominator real.



Where it goes wrong

- The square root of a sum is not the sum of the square roots.
- An inverse tangent alone may return the wrong quadrant.
- Do not confuse modulus with squared modulus.

■ Watch out

Not true: For $a+bi$, its modulus is $a+b$. The modulus is $\sqrt{a^2+b^2}$: for $3+4i$ it is 5, not 7.

Applying it

- FP1 uses complex arithmetic, modulus, Argand representation and quadratic roots.
- De Moivre powers and roots of unity are reserved for FP2.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Complex conjugate:** The number obtained by reversing the sign of the imaginary part.
- Write $z=a+bi$ with $i^2=-1$.
- Watch for: the square root of a sum is not the sum of the square roots.

FP1 · Parabolas and rectangular hyperbolas

How can a rotating parameter trace an ellipse?

- An ellipse can be traced by a rotating parameter even when it is awkward to describe as a single $y=f(x)$.
- This lesson studies **parabola** : The locus of points equally distant from a fixed focus and a fixed directrix.

The method

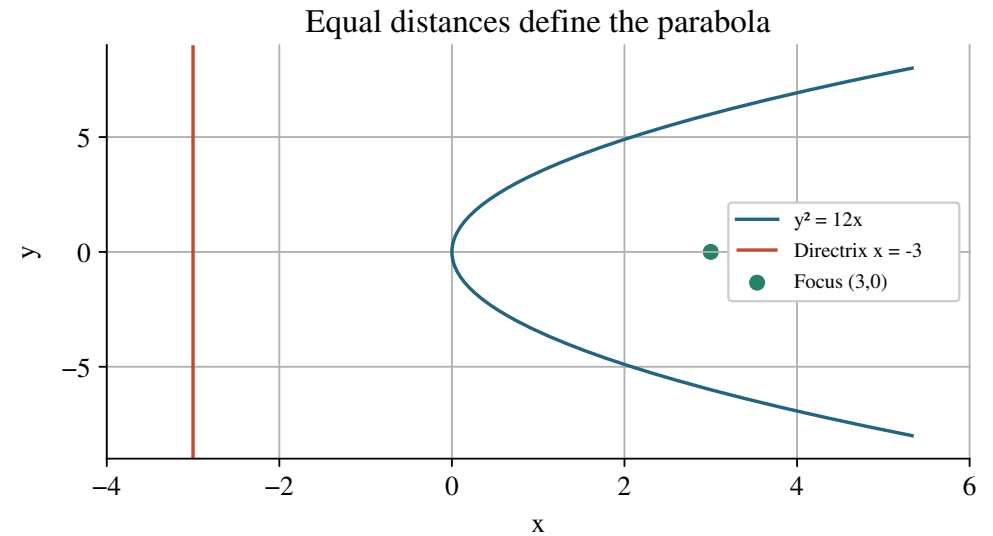
- For $y^2=4ax$, the focus is $(a,0)$, directrix $x=-a$ and vertex $(0,0)$.
- For $xy=c^2$, the coordinate axes are asymptotes.
- Read the parameter from the coefficient rather than assuming it equals $4a$.

A worked case

$$y^2 = 12x, \quad xy = 9$$

■ Worked example

For $y^2=12x$, $a=3$: the focus is $(3,0)$ and directrix $x=-3$. At $x=3$, $y=\pm 6$; at $(3,6)$, distance to the focus and directrix is 6. For $xy=9$, the point $(3,3)$ lies on the hyperbola and $x=0, y=0$ are its asymptotes.



Where it goes wrong

- The coefficient of x is $4a$, not a .
- Include both signs when solving y^2 .
- An asymptote is approached; it is not a finite intercept of $xy=c^2$.

■ Watch out

Not true: For $y^2=4ax$, the focus x -coordinate is $4a$. Elimination can lose restrictions: $x=t^2$, $y=t$ gives $x=y^2$, but $x \geq 0$ and the direction of motion must be stated separately.

Applying it

- FP1 coordinate-geometry support for parabolas and rectangular hyperbolas.
- Ellipse area and ellipse parametrisation are excluded.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Parabola:** The locus of points equally distant from a fixed focus and a fixed directrix.
- For $y^2=4ax$, the focus is $(a,0)$, directrix $x=-a$ and vertex $(0,0)$.
- Watch for: the coefficient of x is $4a$, not a .

FP1 · Matrix operations, inverses and plane transformations

Where do the basis vectors go?

- A map stretches one direction and shears another. A matrix records the transformation of each basis vector.
- This lesson studies **determinant** : A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.

The method

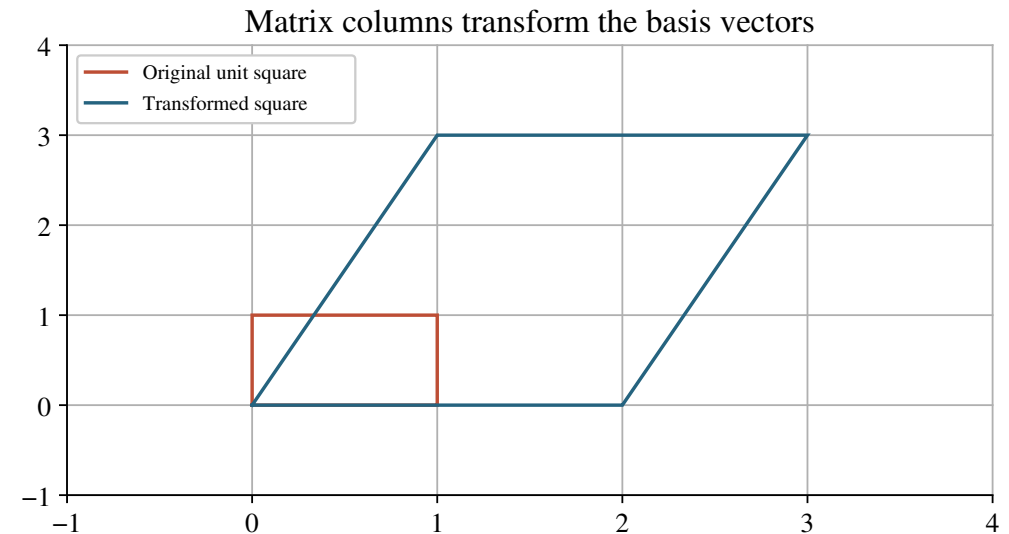
- A 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has determinant $ad-bc$ and inverse $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} / (ad-bc)$ if the determinant is nonzero.
- Transform column vectors by left multiplication.
- The rightmost matrix acts first in a product.

A worked case

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

■ Worked example

For $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $\det A = 6$. $A(1,2) = (4,6)$. Its inverse is $\begin{bmatrix} 1/2 & -1/6 \\ 0 & 1/3 \end{bmatrix}$. Check by multiplying A by its inverse to obtain the identity matrix.



Where it goes wrong

- Matrix multiplication is usually not commutative.
- A determinant of zero means the transformation loses a dimension and has no inverse.
- Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.

■ Watch out

Not true: Every square matrix has an inverse. A square matrix with determinant 0 is singular and has no inverse.

Applying it

- FP1 uses 2×2 matrix operations, inverses and plane transformations.
- Eigenvalues and 3×3 diagonalisation belong to FP3, not this unit.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Determinant:** A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
- A 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has determinant $ad-bc$ and inverse $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} / (ad-bc)$ if the determinant is nonzero.
- Watch for: matrix multiplication is usually not commutative.

FP1 · Bisection and Newton root finding

How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
- This lesson studies **iteration** : A repeated update in which each new approximation is calculated from the previous one.

The method

- For a continuous function with a sign change, bisect a root bracket and keep the half with opposite endpoint signs.
- Newton updates x to $x - f(x)/f'(x)$, provided the derivative is nonzero.

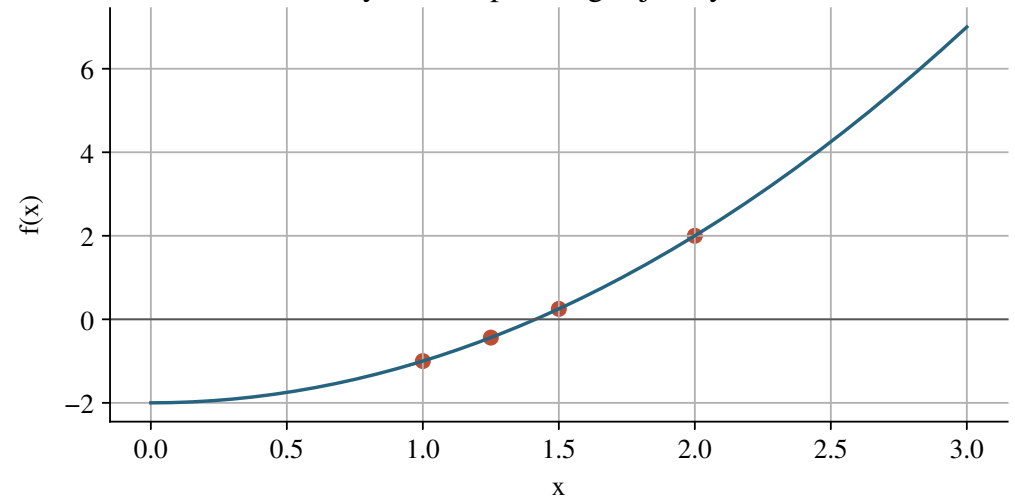
A worked case

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

■ Worked example

For $f(x) = x^2 - 2$, $f(1) = -1$ and $f(2) = 2$. At midpoint 1.5, $f = 0.25$, so the new bracket is $[1, 1.5]$. At midpoint 1.25, $f = -0.4375$, so the next bracket is $[1.25, 1.5]$. Newton from 1.5 gives 1.4166667.

Continuity and endpoint signs justify the bracket



Where it goes wrong

- A sign change must occur on an interval where the function is continuous.
- Newton can fail if its derivative is zero, or if its iterates leave the useful domain.

■ Watch out

Not true: Every sign change proves a root, including one across a discontinuity. A sign change shows a root only if the function is continuous there: $1/x$ changes sign across $x=0$ but has no root.

Applying it

- FP1 numerical methods concern roots: bisection, interpolation and Newton methods.
- Numerical integration is not included in this lesson.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Iteration:** A repeated update in which each new approximation is calculated from the previous one.
- For a continuous function with a sign change, bisect a root bracket and keep the half with opposite endpoint signs.
- Watch for: A sign change must occur on an interval where the function is continuous.

FP1 · Finite sums and induction

Can a polynomial replace an exponential nearby?

- Near a known point, a complicated function can be replaced by a polynomial approximation. The approximation needs a centre and a valid scale.
- This lesson studies **sigma notation** : Notation that adds indexed terms over a stated finite range.

The method

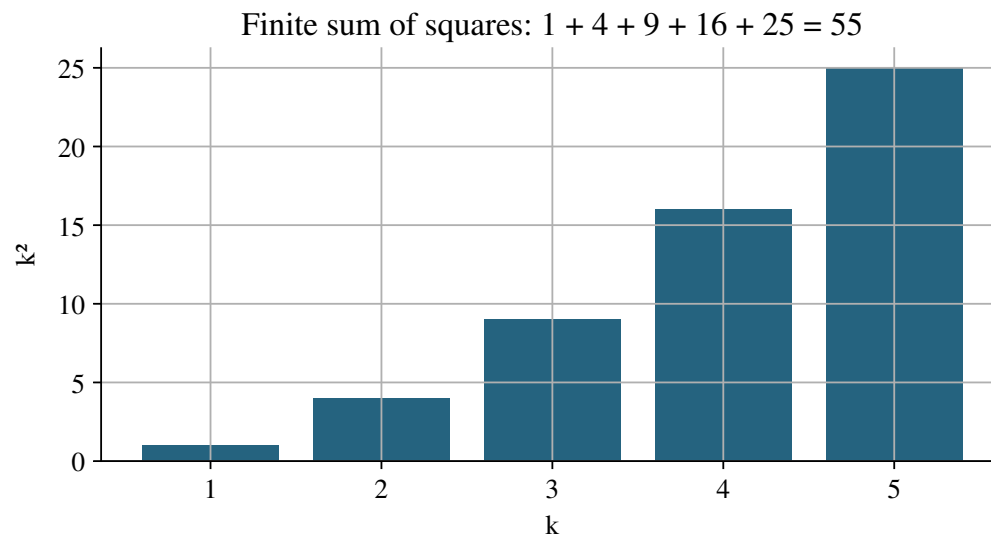
- For $k=1$ to n , $\text{sum } k = n(n+1)/2$, $\text{sum } k^2 = n(n+1)(2n+1)/6$ and $\text{sum } k^3 = [n(n+1)/2]^2$.
- Differences between consecutive terms can telescope.
- In induction, verify the starting case and show the next case follows from the assumed case.

A worked case

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

■ Worked example

For $n=5$, $\text{sum } k=15$, $\text{sum } k^2=1+4+9+16+25=55$ and $\text{sum } k^3=1+8+27+64+125=225$. In proving a sum formula, adding the $(n+1)$ th term to the assumed expression must produce the formula with n replaced by $n+1$.



Where it goes wrong

- The sum of squares is not the square of the sum.
- State the index range and check the first case; an inductive step by itself is not a complete induction proof.

■ Watch out

Not true: The sum of squared terms equals the square of their sum. A Taylor polynomial is an approximation; it is usually accurate only near its centre.

Applying it

- FP1 finite series and induction support only.
- Maclaurin and Taylor approximation belong to later further-pure units and are excluded.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Sigma notation:** Notation that adds indexed terms over a stated finite range.
- For $k=1$ to n , $\sum k=n(n+1)/2$, $\sum k^2=n(n+1)(2n+1)/6$ and $\sum k^3=[n(n+1)/2]^2$.
- Watch for: the sum of squares is not the square of the sum.

FP1 · Deduction, contradiction and induction

When does a pattern become a proof?

- Checking several integers can suggest a pattern. What turns the pattern into a proof for every integer?
 - This lesson studies **counterexample** : A single valid case that disproves a universal claim.
-

The method

- A deductive proof starts from stated definitions or assumptions.
 - A counterexample refutes an all-values claim.
 - Induction requires a base case and an implication from $n=k$ to $n=k+1$; checking successive cases alone is not induction.
-

A worked case

$$(2a + 1) + (2b + 1) = 2(a + b + 1)$$

■ Worked example

For odd integers $2a+1$ and $2b+1$, their sum is $2(a+b+1)$, hence even. The claim n^2+n+41 is always prime fails at $n=41$: the value is $41 \times 43 = 1763$. For $1+\dots+n=n(n+1)/2$, the induction step adds $k+1$ to the assumed sum.

$$\text{Odd + odd: } (2a + 1) + (2b + 1)$$

$$\text{Factor 2: } 2(a + b + 1)$$

The bracket is an integer $\rightarrow$ the sum is even

Where it goes wrong

- Do not assume the conclusion while proving it.
- In contradiction, identify the impossible consequence and reject the initial contrary assumption.
- Induction is not compulsory in AQA 7357; it belongs in its own appropriate further-mathematics scope.

■ Watch out

Not true: Checking the first ten integers proves a claim for every positive integer. A pattern can hold for the first ten cases and then fail; a proof must cover every case.

Applying it

- AQA proof includes deduction, exhaustion and contradiction.
 - IAL P2 introduces exhaustion and counterexample, while P4 introduces contradiction.
 - Keep the method matched to the named unit.
-

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Counterexample:** A single valid case that disproves a universal claim.
- A deductive proof starts from stated definitions or assumptions.
- Watch for: do not assume the conclusion while proving it.

FP2 · Quadratics and inequalities

Which widths make enough space?

- A rectangular enclosure has area $x(10-x)$. What widths give at least 21 square metres?
- This lesson studies **discriminant** : The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.

The method

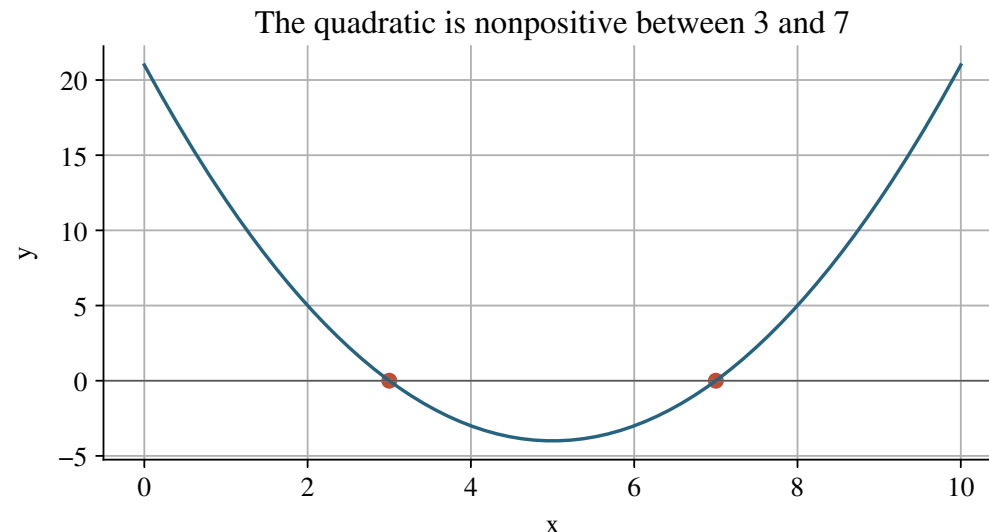
- Factor where possible; otherwise complete the square or use the quadratic formula.
- A quadratic inequality needs the sign on intervals, not only the roots.
- The discriminant identifies repeated or missing real roots.

A worked case

$$ax^2 + bx + c = 0, \quad \Delta = b^2 - 4ac$$

■ Worked example

The condition is $x(10-x) \geq 21$, so $x^2-10x+21 \leq 0$. Factor $(x-3)(x-7) \leq 0$. The upward parabola is nonpositive between its roots, giving $3 \leq x \leq 7$. The maximum area is 25 at $x=5$.



Where it goes wrong

- Multiplying an inequality by a negative number reverses its direction.
- A sketch must show which side of each root satisfies the inequality.
- Geometry may restrict x further.

■ Watch out

Not true: A positive discriminant means that a quadratic has no real roots. A positive discriminant means two distinct real roots; a negative discriminant means no real roots.

Applying it

- Use the vertex to interpret an optimum.
- Check an endpoint and a point between the roots; the algebra and graph should tell the same story.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Discriminant:** The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.
- Factor where possible; otherwise complete the square or use the quadratic formula.
- Watch for: multiplying an inequality by a negative number reverses its direction.

FP2 · Series, finite differences and Taylor expansions

Can a polynomial replace an exponential nearby?

- Near a known point, a complicated function can be replaced by a polynomial approximation. The approximation needs a centre and a valid scale.
- This lesson studies **Taylor polynomial** : A polynomial formed from a function's derivatives at a chosen centre.

The method

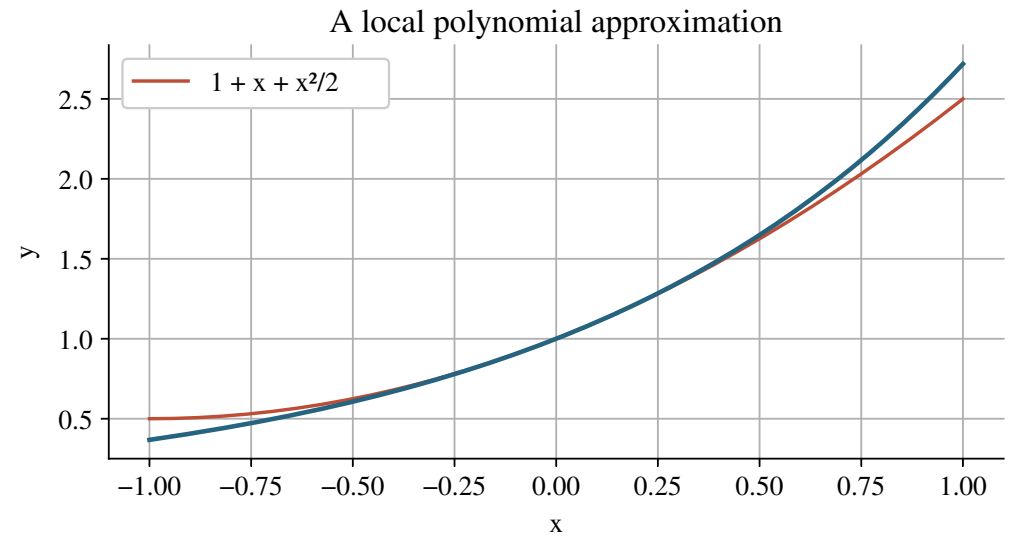
- About $x=a$, $f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\dots$ when the expansion is valid.
- At $a=0$ it is a Maclaurin expansion.
- Finite sums may also simplify by cancellation or standard sum formulae.

A worked case

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots$$

■ Worked example

The first three terms of e^x are $1+x+x^2/2$. At $x=0.1$ this gives 1.105, close to $e^{0.1}\approx 1.105170$. For the sum of k^2 from $k=1$ to 5, $n(n+1)(2n+1)/6$ gives $5\times 6\times 11/6=55$.



Where it goes wrong

- The factorial belongs in the denominator of every Taylor coefficient.
- The expansion centre is not always zero.
- An approximation close to its centre can be poor far away.

■ Watch out

Not true: A Taylor polynomial is exact for the original function at every input. A Taylor polynomial is an approximation; it is usually accurate only near its centre.

Applying it

- For a telescoping series, write several terms and identify both surviving ends.
- For a Taylor approximation, state its order and compare with a known value or a remainder estimate where required.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Taylor polynomial:** A polynomial formed from a function's derivatives at a chosen centre.
- About $x=a$, $f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\dots$ when the expansion is valid.
- Watch for: the factorial belongs in the denominator of every Taylor coefficient.

FP2 · Complex numbers and roots

What if the root is not real?

- The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots.
- This lesson studies **complex conjugate** : The number obtained by reversing the sign of the imaginary part.

The method

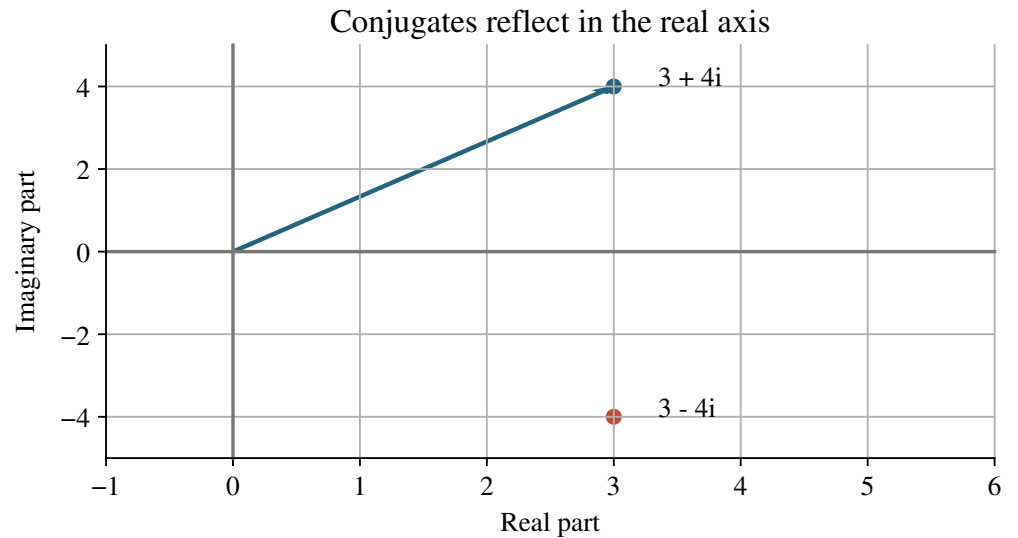
- Write $z=a+bi$ with $i^2=-1$.
- The conjugate is $a-bi$ and z times its conjugate= a^2+b^2 .
- The modulus is $\sqrt{a^2+b^2}$; the argument needs the correct quadrant.
- Conjugate roots occur for polynomials with real coefficients.

A worked case

$$z = a + bi, \quad |z| = \sqrt{a^2 + b^2}, \quad z\bar{z} = a^2 + b^2$$

■ Worked example

For $z=3+4i$, $|z|=5$ and z conjugate= $3-4i$. The reciprocal is $(3-4i)/25$. For $x^2-6x+13=0$, the roots are $3\pm 2i$. Multiplying by the conjugate makes a complex denominator real.



Where it goes wrong

- The square root of a sum is not the sum of the square roots.
- An inverse tangent alone may return the wrong quadrant.
- Do not confuse modulus with squared modulus.

■ Watch out

Not true: For $a+bi$, its modulus is $a+b$. The modulus is $\sqrt{a^2+b^2}$: for $3+4i$ it is 5, not 7.

Applying it

- Draw the real axis horizontally and imaginary axis vertically.
- For products, moduli multiply and arguments add; powers extend this geometric pattern through de Moivre's theorem.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Complex conjugate:** The number obtained by reversing the sign of the imaginary part.
- Write $z=a+bi$ with $i^2=-1$.
- Watch for: the square root of a sum is not the sum of the square roots.

FP2 · Differential equations and numerical solutions

What does the starting value decide?

- A growing population's rate is proportional to its current size. The rate equation describes a whole family until an initial population is supplied.
- This lesson studies **initial condition** : A specified value that selects a particular solution from a family.

The method

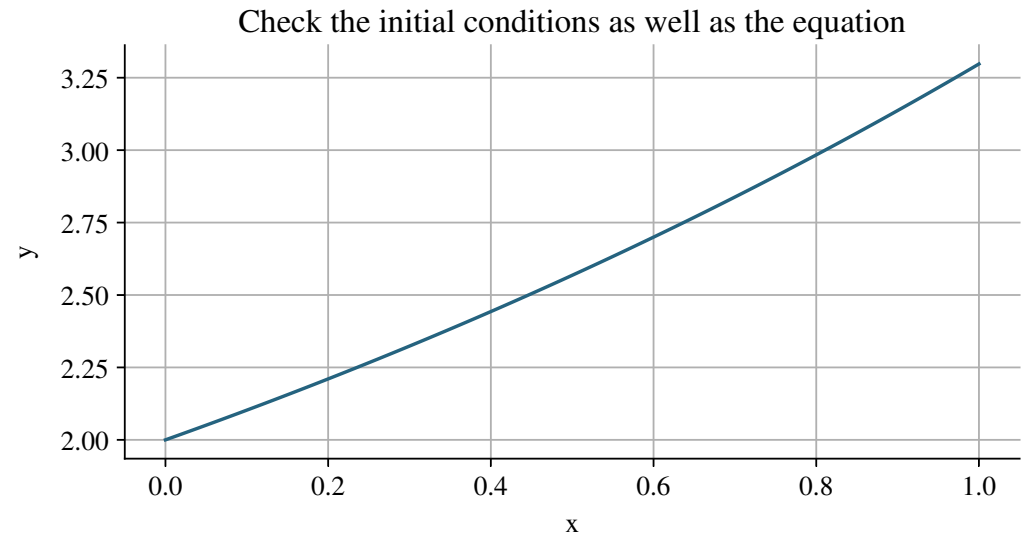
- For $dy/dx=ky$, separate variables: $dy/y=k \, dx$, giving $y=Ae^{(kx)}$.
- Use an initial condition to find A.
- Euler's method takes $y_{next}=y+h \, f(x,y)$, with a chosen step h .

A worked case

$$\frac{dy}{dx} = ky, \quad y = Ae^{kx}$$

■ Worked example

With $dy/dx=0.5y$ and $y(0)=2$, $y=2e^{(0.5x)}$. Euler with $h=0.2$ gives $y(0.2) \approx 2+0.2 \times 1 = 2.2$ and $y(0.4) \approx 2.2+0.2 \times 1.1 = 2.42$. The exact second value is about 2.4428.



Where it goes wrong

- An initial condition determines the integration constant; it is not a replacement for integrating.
- Dividing by y can omit an equilibrium solution $y=0$.
- Euler accuracy depends on step size and the equation.

■ Watch out

Not true: Every step size gives exactly the same Euler approximation. Euler's method depends on the step size; smaller steps usually give a more accurate value.

Applying it

- For second-order linear equations, combine the complementary function with an appropriate particular integral, then apply the required initial conditions.
- This is Further Pure scope; check the named unit.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Initial condition:** A specified value that selects a particular solution from a family.
- For $dy/dx=ky$, separate variables: $dy/y=k dx$, giving $y=Ae^{(kx)}$.
- Watch for: an initial condition determines the integration constant; it is not a replacement for integrating.

FP2 · Second-order linear differential equations

Why do we need two initial conditions?

- A displacement model involves both velocity and acceleration. Solving only a first-order rate equation cannot capture both initial conditions.
- This lesson studies **complementary function** : The general solution of the associated homogeneous linear differential equation.

The method

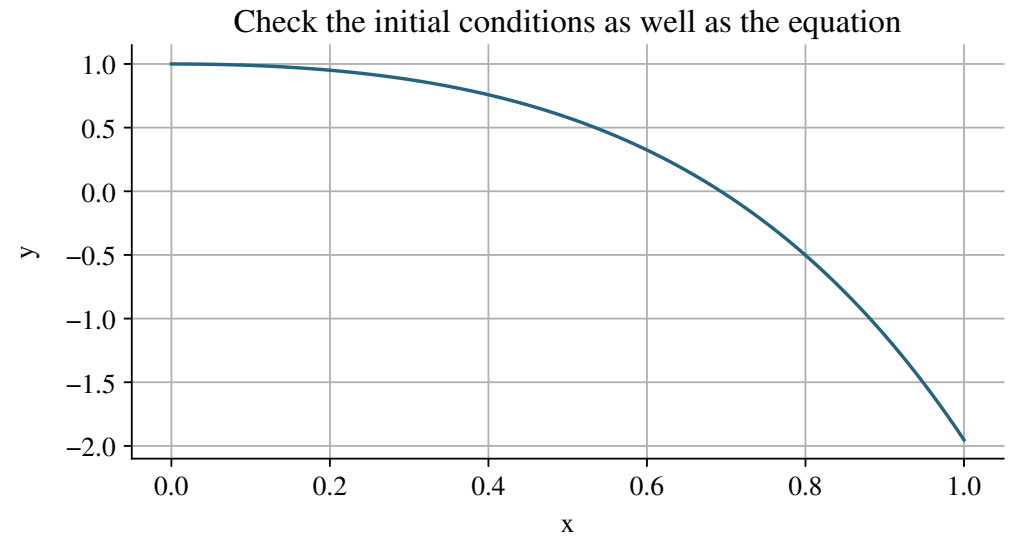
- For $y''+ay'+by=f(x)$, solve the auxiliary quadratic for the homogeneous part.
- Add a suitable particular integral for the forcing term.
- Repeated and complex roots require their correct forms.

A worked case

$$y'' + ay' + by = f(x), \quad m^2 + am + b = 0$$

■ Worked example

For $y''-3y'+2y=0$, the auxiliary equation is $m^2-3m+2=0$ with roots 1,2. Hence $y=Ae^x+Be^{2x}$. With $y(0)=1$ and $y'(0)=0$, $A+B=1$ and $A+2B=0$, so $A=2, B=-1$.



Where it goes wrong

- Two arbitrary constants need two independent conditions.
- If the trial particular integral duplicates a complementary-function term, multiply the trial by x as required.
- Do not discard a valid oscillatory solution because the auxiliary roots are complex.

■ Watch out

Not true: One initial value is sufficient to determine both constants in a second-order general solution. A second-order solution has two constants, so it needs two conditions.

Applying it

- Differentiate the final expression and substitute into the original differential equation.
- Check both initial conditions separately, and interpret the permitted solution interval.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Complementary function:** The general solution of the associated homogeneous linear differential equation.
- For $y'' + ay' + by = f(x)$, solve the auxiliary quadratic for the homogeneous part.
- Watch for: two arbitrary constants need two independent conditions.

FP2 · Polar curves and area

Why describe a curve by direction?

- A petal-shaped curve is simpler when its radius depends on direction. Cartesian coordinates can hide this structure.
- This lesson studies **polar coordinate** : A position described by distance r and angle θ from a chosen origin and reference ray.

The method

- Use $x = r \cos \theta$ and $y = r \sin \theta$.
- A polar area is one half the integral of r^2 with respect to θ over a correctly chosen interval.
- Identify symmetry and repeated tracing before selecting limits.

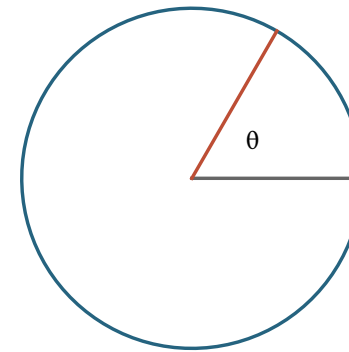
A worked case

$$x = r \cos \theta, \quad y = r \sin \theta, \quad A = \frac{1}{2} \int r^2 d\theta$$

■ Worked example

For $r=2$ and $\theta=\pi/3$, $x=1$ and $y=\sqrt{3}$. For the circle $r=2$ over a complete turn, $A=(1/2) \int_0^{2\pi} 4 d\theta=4\pi$. A half-turn gives 2π , exactly half the disk.

A radius fixes distance from the centre



radius $\times$ angle gives arc length

Where it goes wrong

- Negative r places a point in the opposite direction; it is not an ordinary negative distance along the same ray.
- A parametrisation may trace the same region more than once, so a full parameter interval can overcount area.

■ Watch out

Not true: Using any full parameter interval counts each polar region exactly once. The interval must match the curve's period; too long an interval counts a region twice.

Applying it

- Sketch enough points to establish orientation and bounds.
- For an enclosed region between two curves, determine intersections and which radial square contributes the outer boundary on each interval.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Polar coordinate:** A position described by distance r and angle θ from a chosen origin and reference ray.
- Use $x=r \cos\theta$ and $y=r \sin\theta$.
- Watch for: negative r places a point in the opposite direction; it is not an ordinary negative distance along the same ray.

FP3 · Hyperbolic functions and inverse relations

How can growth and decay make a symmetric curve?

- A hanging cable has a curved profile related to exponentials. Hyperbolic functions combine growth and decay symmetrically.
- This lesson studies **hyperbolic cosine** : The function $\cosh x = (\hat{e^x} + \hat{e^{-x}})/2$.

The method

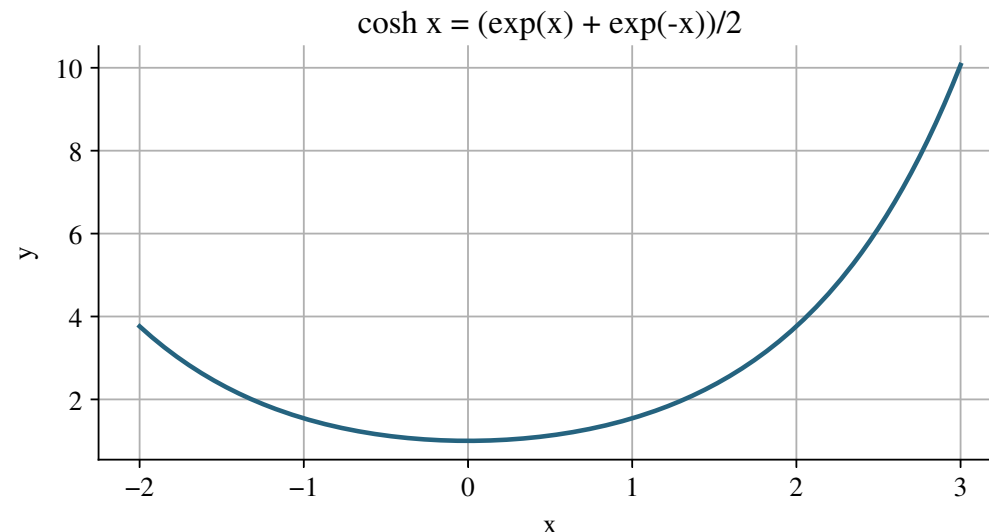
- Define $\sinh x = (\hat{e^x} - \hat{e^{-x}})/2$ and $\cosh x = (\hat{e^x} + \hat{e^{-x}})/2$.
- Their identity is $\cosh^2 x - \sinh^2 x = 1$.
- Derivatives are $\sinh \text{ prime} = \cosh$ and $\cosh \text{ prime} = \sinh$.

A worked case

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \cosh^2 x - \sinh^2 x = 1$$

■ Worked example

At $x=\ln 2$, $\hat{e^x}=2$ and $\hat{e^{-x}}=1/2$. Thus $\cosh x=1.25$ and $\sinh x=0.75$. Their squared difference is $1.5625-0.5625=1$. To invert $y=\sinh x$, solve a quadratic in $\hat{e^x}$ and choose the positive root.



Where it goes wrong

- The hyperbolic identity has a minus sign. $\cosh$ is not one-to-one on all real inputs; its usual inverse uses $x \geq 0$.
- Ordinary circular-trigonometric identities cannot be substituted unchanged.

■ Watch out

Not true: The identity for hyperbolic functions is $\cosh^2 x + \sinh^2 x = 1$. The hyperbolic identity is $\cosh^2 x - \sinh^2 x = 1$.

Applying it

- Use exponential definitions to prove identities and solve equations.
- State domain restrictions for inverse functions before differentiating or integrating them.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Hyperbolic cosine:** The function $\cosh x = (\hat{e^x} + \hat{e^{-x}})/2$.
- Define $\sinh x = (\hat{e^x} - \hat{e^{-x}})/2$ and $\cosh x = (\hat{e^x} + \hat{e^{-x}})/2$.
- Watch for: the hyperbolic identity has a minus sign. $\cosh$ is not one-to-one on all real inputs; its usual inverse uses $x \geq 0$.

FP3 · Conics, parametric curves and tangent reasoning

How can a rotating parameter trace an ellipse?

- An ellipse can be traced by a rotating parameter even when it is awkward to describe as a single $y=f(x)$.
- This lesson studies **parametric equation** : A representation in which coordinates are expressed using a common parameter.

The method

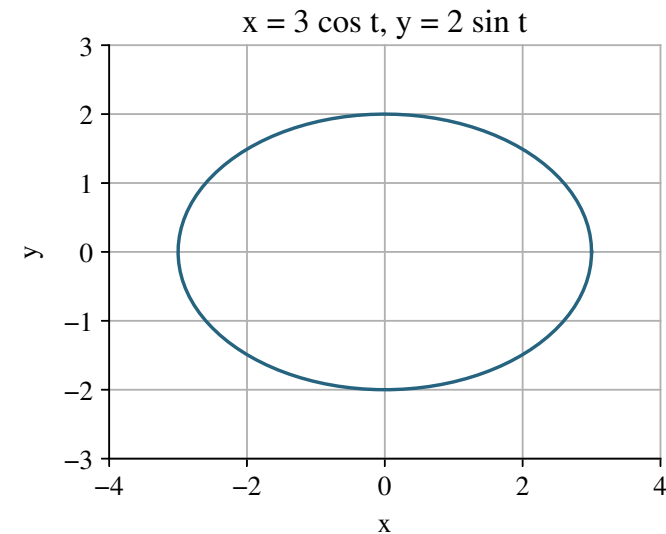
- For $x=a \cos t, y=b \sin t$, eliminating t gives $x^2/a^2 + y^2/b^2 = 1$.
- For parametric curves, $dy/dx = (dy/dt)/(dx/dt)$, where $dx/dt \neq 0$.
- A zero denominator may signal a vertical tangent.

A worked case

$$x = a \cos t, \quad y = b \sin t, \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

■ Worked example

For $x=3\cos t, y=2\sin t$ at $t=\pi/4$, the gradient is $(2\cos t)/(-3\sin t)=-2/3$. The point is $(3/\sqrt{2}, \sqrt{2})$. The area inside the ellipse is $\pi ab=6\pi$; its semiaxes are 3 and 2.



Where it goes wrong

- Eliminating a parameter may lose a domain restriction or direction of travel.
- A vertical tangent cannot be assigned a finite dy/dx .
- Distinguish semiaxes from full widths.

■ Watch out

Not true: Eliminating a parameter preserves every restriction and direction automatically. Elimination can lose restrictions: $x=t^2, y=t$ gives $x=y^2$, but $x \geq 0$ and the direction of motion must be stated separately.

Applying it

- For advanced coordinate geometry, use the defining equation and a consistent parameter.
- For IAL P4 parametric integration, use the specification's restricted requirements rather than importing all Further Pure conics.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Parametric equation:** A representation in which coordinates are expressed using a common parameter.
- For $x=a \cos t, y=b \sin t$, eliminating t gives $x^2/a^2 + y^2/b^2 = 1$.
- Watch for: eliminating a parameter may lose a domain restriction or direction of travel.

FP3 · Chain, product, quotient and implicit differentiation

Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
- This lesson studies **chain rule** : The rule that multiplies the outer derivative by the inner derivative for a composite function.

The method

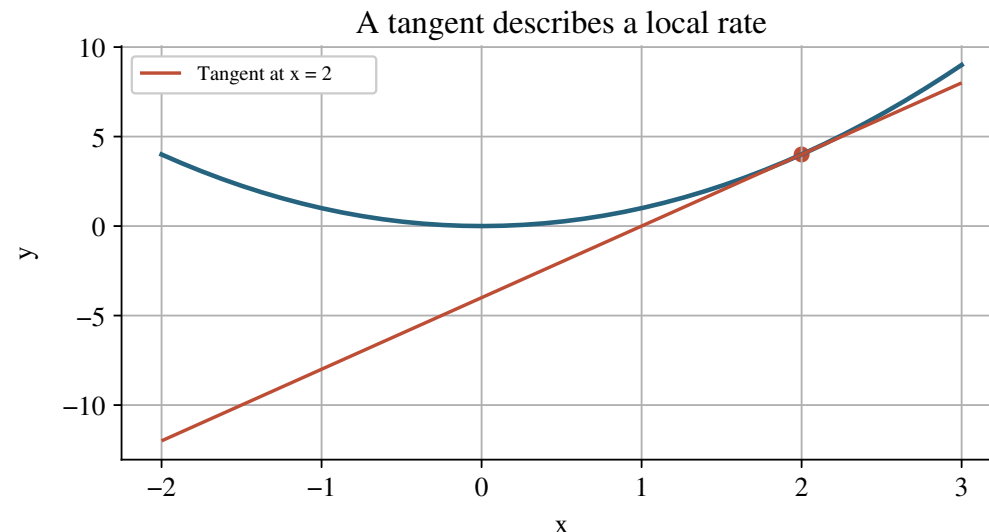
- For $y=f(g(x))$, $y' = f'(g(x))g'(x)$.
- For uv , differentiate to $u'v + uv'$.
- For u/v , use $(u'v - uv')/v^2$, where $v \neq 0$.

A worked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

■ Worked example

For $y=(3x+1)^4$, $y' = 4(3x+1)^3 \times 3 = 12(3x+1)^3$. At $x=0$, the gradient is 12.
For $x^2+y^2=25$, differentiate implicitly: $2x+2y y' = 0$, so $y' = -x/y$ when $y \neq 0$.



Where it goes wrong

- A derivative of a product is not the product of the derivatives.
- In implicit differentiation, every differentiated function of y brings a dy/dx factor.
- A quotient's denominator is squared.

■ Watch out

Not true: The derivative of $u(x)v(x)$ is u' times v prime. The product rule is $(uv)' = u'v + uv'$: for $x \cdot x$ the derivative is $2x$, not $1 \times 1 = 1$.

Applying it

- Choose a useful form before differentiating: expanding a short polynomial may be simpler.
- For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Chain rule:** The rule that multiplies the outer derivative by the inner derivative for a composite function.
- For $y=f(g(x))$, $y' = f'(g(x))g'(x)$.
- Watch for: A derivative of a product is not the product of the derivatives.

FP3 · Substitution, parts and partial fractions

Can we integrate the two factors separately?

- A simple-looking product such as $x e^x$ cannot be integrated by separately integrating each factor.
- This lesson studies **integration by parts** : An integration method based on the derivative of a product.

The method

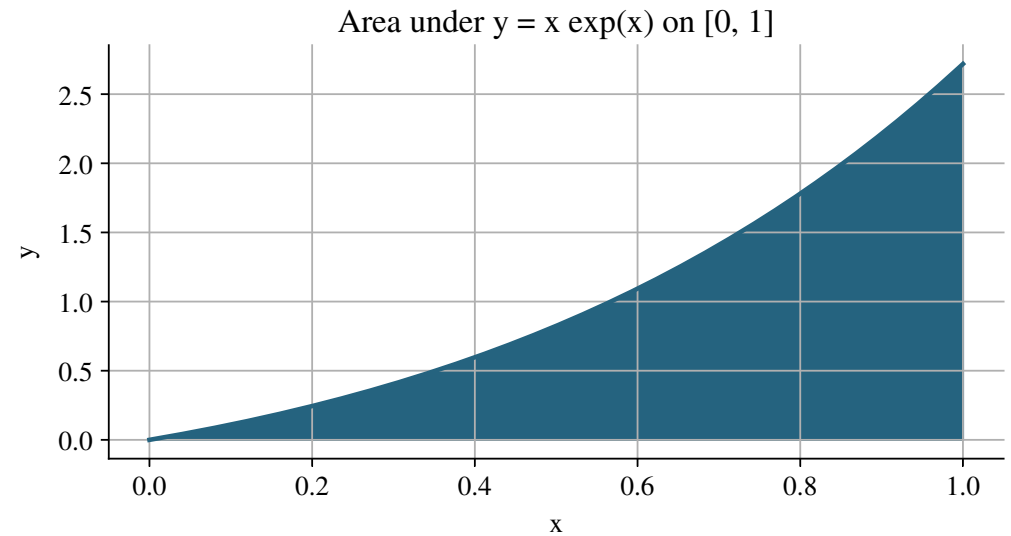
- Use substitution when an inner derivative appears as a factor.
- By parts, $\int u v' = uv - \int u' v$.
- For a rational function, divide first if needed, then decompose into partial fractions.

A worked case

$$\int u v' dx = uv - \int u' v dx$$

■ Worked example

For integral $\int x e^x dx$, take $u=x$ and $v' = e^x$. Then the integral is $x e^x - e^x + C$.
For integral $\int 2x/(x^2+1) dx$, substitute $w=x^2+1$, $dw=2x dx$; the answer is $\ln(x^2+1)+C$.



Where it goes wrong

- Choosing u and v' well matters: the remaining integral should become simpler.
- In a definite substitution, either change the limits or return to x before applying the original limits.

■ Watch out

Not true: The integral of a product equals the product of its separate integrals. A product needs a method such as integration by parts or substitution: $\int x \cdot x dx = x^3/3$, not $(x^2/2)(x^2/2)$.

Applying it

- Check by differentiating.
- For volumes of revolution around the x -axis use $V = \pi \int y^2 dx$; do not confuse the square of a function with the integral of the function.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Integration by parts:** An integration method based on the derivative of a product.
- Use substitution when an inner derivative appears as a factor.
- Watch for: choosing u and v prime well matters: the remaining integral should become simpler.

FP3 · Spatial vectors, lines and angles

Can nonparallel paths still miss each other?

- Two paths in space can be neither parallel nor intersecting. A two-dimensional sketch may conceal their separation.
- This lesson studies **direction vector** : A nonzero vector giving the direction of a line.

The method

- A line has $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ with position vector $\mathbf{a}$ and nonzero direction $\mathbf{b}$.
- To find an intersection, equate all components and solve for both line parameters.
- For an angle use $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ where scalar products are in the named syllabus.

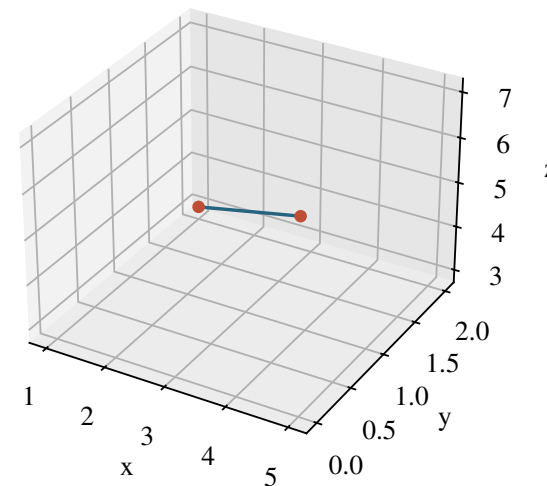
A worked case

$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}, \quad \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

■ Worked example

For $\mathbf{a} = (1, 2, 3)$, $\mathbf{b} = (2, -1, 2)$, $\lambda = 2$ gives $\mathbf{r} = (5, 0, 7)$. The magnitude of $\mathbf{b}$ is $\sqrt{4+1+4}=3$. With $\mathbf{c} = (1, 0, -1)$, $\mathbf{b} \cdot \mathbf{c} = 2 - 2 = 0$, so $\mathbf{b}$ and $\mathbf{c}$ are perpendicular.

$$\mathbf{r} = (1, 2, 3) + \lambda(2, -1, 2)$$



Where it goes wrong

- Satisfying two component equations does not guarantee the third.
- Nonparallel lines may be skew.
- A line's direction vector is not its position vector, and a scalar parameter has no coordinate units.

■ Watch out

Not true: Any two nonparallel lines in three dimensions must intersect. In three dimensions nonparallel lines can be skew: they do not meet.

Applying it

- State whether a requested angle is acute, directed or an angle between lines.
- Use the syllabus-specific plane or distance method only when it is actually required for that unit.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Direction vector:** A nonzero vector giving the direction of a line.
- A line has $r=a+\lambda b$ with position vector a and nonzero direction b .
- Watch for: satisfying two component equations does not guarantee the third.

FP3 · Matrix transformations, inverses and eigenvalues

Where do the basis vectors go?

- A map stretches one direction and shears another. A matrix records the transformation of each basis vector.
- This lesson studies **determinant** : A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.

The method

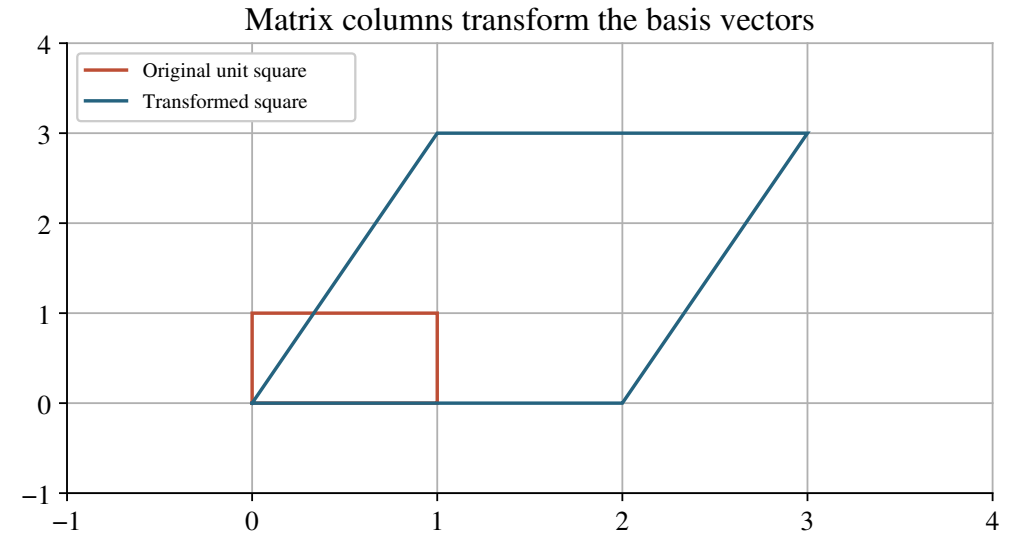
- A 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has determinant $ad-bc$ and inverse $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} / (ad-bc)$ if the determinant is nonzero.
- Transform column vectors by left multiplication.
- The rightmost matrix acts first in a product.

A worked case

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

■ Worked example

For $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $\det A = 6$. $A(1,2) = (4,6)$. Its inverse is $\begin{bmatrix} 1/2 & -1/6 \\ 0 & 1/3 \end{bmatrix}$. The characteristic equation is $(2-\lambda)(3-\lambda)=0$, so the eigenvalues are 2 and 3.



Where it goes wrong

- Matrix multiplication is usually not commutative.
- A determinant of zero means the transformation loses a dimension and has no inverse.
- Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.

■ Watch out

Not true: Every square matrix has an inverse. A square matrix with determinant 0 is singular and has no inverse.

Applying it

- Find an eigenvector by solving $(A-\lambda I)v=0$ with $v \neq 0$.
- Explain the geometrical meaning: this vector keeps its line direction under the transformation.

In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Determinant:** A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
- A 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has determinant $ad - bc$ and inverse $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} / (ad - bc)$ if the determinant is nonzero.
- Watch for: matrix multiplication is usually not commutative.