# Course notes

Pearson Edexcel · International A-Level · Pure Mathematics

## YPM01: course teaching notes

Version: Issue 3, April 2019; first teaching 2018; unit assessment from 2019

These are original teaching notes, and each lesson is labelled with its tier.

## P1 · Indices, surds and standard form

## How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
- This lesson studies **index** 指数: The power to which a base is raised.

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## The method

- For the same positive base, multiplication adds indices and division subtracts them.
- A negative index means reciprocal; a fractional index represents a root.
- Standard form has $1 \le a < 10$.

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## A worked case

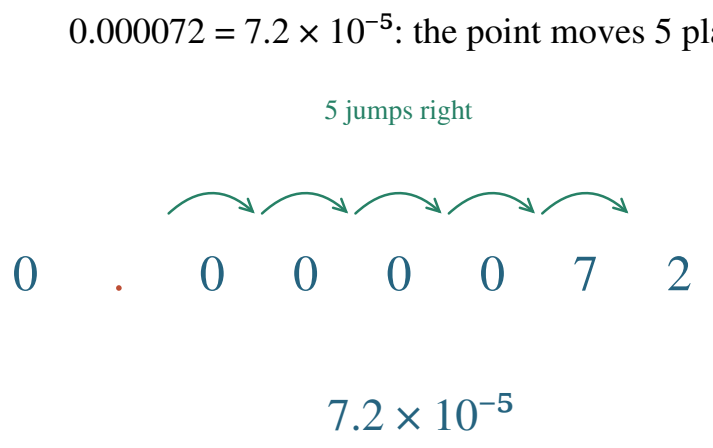
$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = \left(\sqrt[n]{a}\right)^m$$

### ■ Worked example

0.000072=7.2×10^(-5). Also 16^(3/4)=(16^(1/4))^3=2^3=8. Simplify √72=6√2, then rationalise 1/√2=√2/2.



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## Where it goes wrong

- Index laws do not turn a sum into a single power: 2^3+2^4=24, not 2^7.
- Do not round a surd when an exact answer is requested.

> **Watch out**
>
> **Not true:** For every positive a, a^2+a^3 equals a^5. Index laws only add powers when the terms are multiplied: $2^2+2^3=12$, but $2^5=32$.

## Applying it

- Check powers of ten against the original quantity.
- Use surds for exact geometry, and round only the final length when the question asks for a decimal.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ **Key idea**

- **Index**: The power to which a base is raised.
- For the same positive base, multiplication adds indices and division subtracts them.
- Watch for: index laws do not turn a sum into a single power: 2^3+2^4=24, not 2^7.

## P1 · Quadratics and inequalities

## Which widths make enough space?

- A rectangular enclosure has area x(10-x). What widths give at least 21 square metres?
- This lesson studies **discriminant** 判别式: The quantity $b^2$-4ac that determines the real roots of $ax^2$+bx+c=0.

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## The method

- Factor where possible; otherwise complete the square or use the quadratic formula.
- A quadratic inequality needs the sign on intervals, not only the roots.
- The discriminant identifies repeated or missing real roots.

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## A worked case

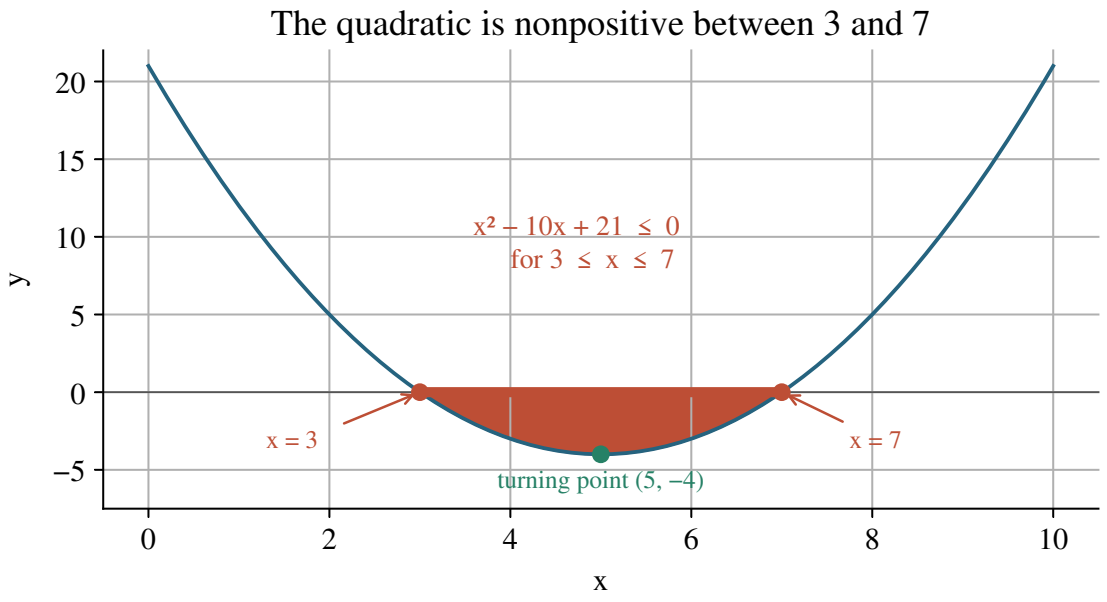
$$ax^2 + bx + c = 0, \qquad \Delta = b^2 - 4ac$$

■ **Worked example**

The condition is x(10-x)≥21, so $x^2$-10x+21≤0. Factor (x-3)(x-7)≤0. The upward parabola is nonpositive between its roots, giving 3≤x≤7. The maximum area is 25 at x=5.



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## Where it goes wrong

- Multiplying an inequality by a negative number reverses its direction.
- A sketch must show which side of each root satisfies the inequality.
- Geometry may restrict x further.

> ■ **Watch out**
>
> **Not true:** A positive discriminant means that a quadratic has no real roots. A positive discriminant means two distinct real roots; a negative discriminant means no real roots.

## Applying it

- Use the vertex to interpret an optimum.
- Check an endpoint and a point between the roots; the algebra and graph should tell the same story.

## In your course

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■ **Key idea**

- **Discriminant**: The quantity $b^2$-4ac that determines the real roots of $ax^2+bx+c=0$.
- Factor where possible; otherwise complete the square or use the quadratic formula.
- Watch for: multiplying an inequality by a negative number reverses its direction.

## P1 · Coordinate geometry and tangents

## How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
- This lesson studies **gradient** 斜率: The change in y divided by the corresponding change in x.

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## The method

- A line through $(x_1,y_1)$ with gradient m has $y-y_1=m(x-x_1)$.
- Parallel lines have equal gradients.
- Finite perpendicular gradients multiply to -1.
- A circle has $(x-a)^2+(y-b)^2=r^2$.

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## A worked case

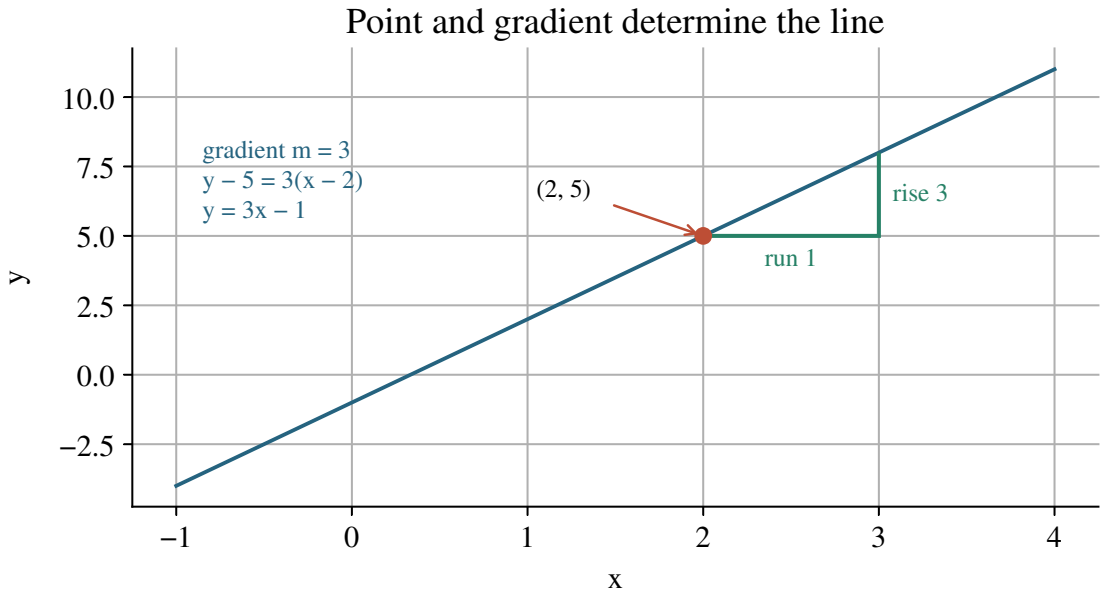
$$y - y_1 = m(x - x_1), \qquad (x - a)^2 + (y - b)^2 = r^2$$

■ **Worked example**

Through (2,5) with gradient 3, y-5=3(x-2), so y=3x-1. A perpendicular through the same point has y-5=-(x-2)/3. The circle $(x-2)^2+(y+1)^2=25$ has centre (2,-1) and radius 5.



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## Where it goes wrong

- A vertical line has no finite gradient; do not force it into y=mx+c.
- Read the signs of a circle's centre carefully.
- The radius to a tangent is perpendicular to the tangent.

> **Watch out**
>
> **Not true:** Perpendicular nonvertical lines have equal gradients. Perpendicular gradients multiply to $-1$: a gradient of 2 needs $-1/2$. Parallel lines have equal gradients.

## Applying it

- Before solving a line-circle intersection, predict whether there are zero, one or two intersections.
- Substitution produces a quadratic whose discriminant checks the prediction.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.



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■ **Key idea**

- **Gradient**: The change in y divided by the corresponding change in x.
- A line through $(x_1,y_1)$ with gradient m has $y-y_1=m(x-x_1)$.
- Watch for: A vertical line has no finite gradient; do not force it into y=mx+c.

## P1 · Right triangles and non-right triangles

## Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse** 斜边: The side opposite the right angle in a right-angled triangle.

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## The method

- In a right triangle $a^2+b^2=c^2$; $\sin\theta$=opposite/hypotenuse, $\cos\theta$=adjacent/hypotenuse and $\tan\theta$=opposite/adjacent.
- For other triangles, use the sine or cosine rule, or area=ab sin C/2.

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## A worked case

$$a^2 + b^2 = c^2, \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

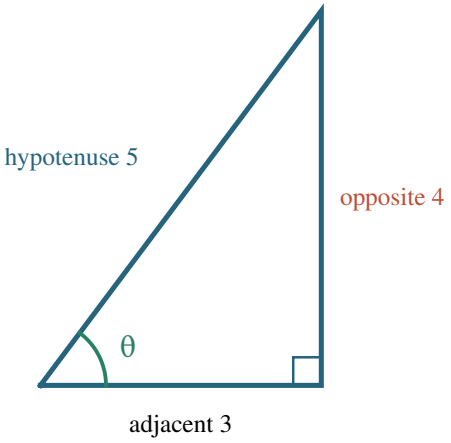
■ **Worked example**

The ladder length is $c=\sqrt{(3^2+4^2)}=5$ m. Its angle to the ground satisfies $\tan\theta=4/3$, so $\theta\approx53.1°$. With two sides 6 and 8 enclosing 60°, $c^2=6^2+8^2-2\times6\times8\cos60°=52$.



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Label the known sides before choosing a rule




opposite: across from θ
adjacent: next to θ
hypotenuse: across from the right angle

$\tan\theta = 4/3$ $\sin\theta = 4/5$
$\cos\theta = 3/5$

## Where it goes wrong

- Label sides relative to the chosen angle.
- Pythagoras needs a right angle.
- A calculator angle mode error can produce a plausible but wrong result.
- Keep unrounded values for later steps.

### ■ Watch out

**Not true:** Pythagoras applies to every triangle, including triangles without a right angle. Pythagoras needs a right angle; for other triangles use the cosine rule.

## Applying it

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule.
- Explain why the chosen triangle contains the required length or angle.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.



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■ **Key idea**

- **Hypotenuse**: The side opposite the right angle in a right-angled triangle.
- In a right triangle $a^2+b^2=c^2$; $\sin\theta$=opposite/hypotenuse, $\cos\theta$=adjacent/hypotenuse and $\tan\theta$=opposite/adjacent.
- Watch for: label sides relative to the chosen angle.

## P1 · Radians, identities and trigonometric equations

## How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
- This lesson studies **radian** 弧度: The angle subtended by an arc whose length equals the radius.

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## The method

- For $\theta$ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$.
- Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve.
- List every solution in the stated interval.

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## A worked case

$$s = r\theta, \qquad A = \frac{1}{2}r^2\theta, \qquad \sin^2\theta + \cos^2\theta = 1$$

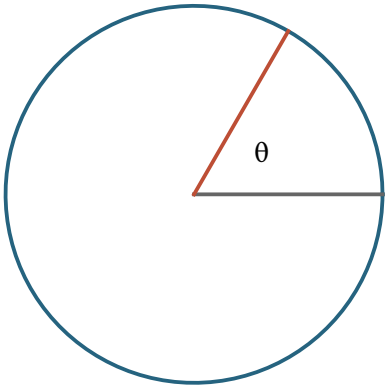
■ **Worked example**

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0\leq x<2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0\leq 2x<4\pi$ before dividing by 2.



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A radius fixes distance from the centre




radius × angle gives arc length

## Where it goes wrong

- A calculator's inverse sine gives one principal value.
- It does not give every solution.
- Dividing by sin x can discard solutions where sin x=0; check that case first.

> ■ **Watch out**
>
> **Not true:** The principal inverse sine value is the only solution in a full turn. Sine takes most values twice in a full turn: sin x=0.5 gives 30° and 150°.

## Applying it

- Choose an identity that reduces the equation to one trigonometric function.
- Keep exact familiar angles where possible, and check each solution against the original equation.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.



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■ **Key idea**

- **Radian**: The angle subtended by an arc whose length equals the radius.
- For $\theta$ in radians, arc length s=r$\theta$ and sector area A=r$^2\theta$/2.
- Watch for: A calculator's inverse sine gives one principal value.

## P1 · Derivatives and stationary points

## What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative** 导数: The instantaneous rate of change, also the gradient of a tangent.

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## The method

- For y=ax^n, dy/dx=anx^(n-1).
- A stationary point satisfies dy/dx=0.
- Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.

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## A worked case

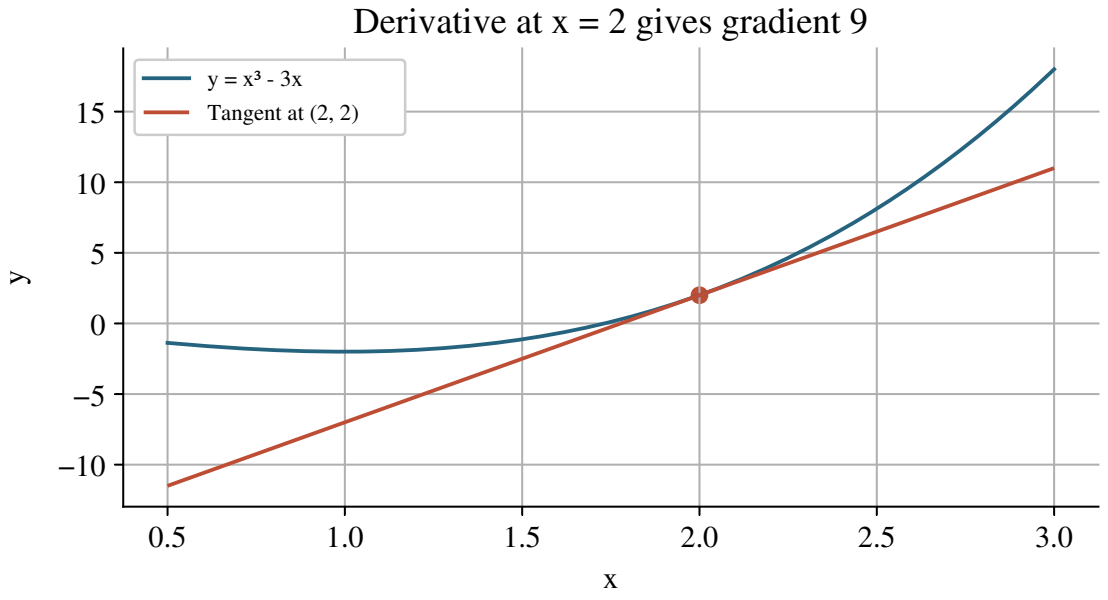
$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad m = f'(x_0)$$

■ **Worked example**

For y=x$^3$-3x, dy/dx=3x$^2$-3. At x=2, the gradient is 9 and y=2. The tangent is y-2=9(x-2). The normal gradient is -1/9.



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## Where it goes wrong

- A derivative gives a gradient, not the ordinate.
- Evaluate y and dy/dx separately at the supplied x-coordinate; the normal gradient is the negative reciprocal of a nonzero tangent gradient.

> ■ **Watch out**
>
> **Not true:** A tangent and normal at the same point have the same gradient. A zero derivative can also be a point of inflection: $y=x^3$ at $x=0$ is neither a maximum nor a minimum.

## Applying it

- P1 covers polynomial derivatives, gradients, tangents and normals.
- Optimization and stationary-point classification are taught in P2; they are not introduced here.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Derivative**: The instantaneous rate of change, also the gradient of a tangent.
- For y=ax^n, dy/dx=anx^(n-1).
- Watch for: A derivative gives a gradient, not the ordinate.

## P1 · Integrals, area and accumulation

## How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative** 原函数: A function whose derivative equals the given integrand.

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## The method

- Integrate polynomial powers by increasing the index by one and dividing by the new index.
- Include an arbitrary constant and use a supplied point to determine it.

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## A worked case

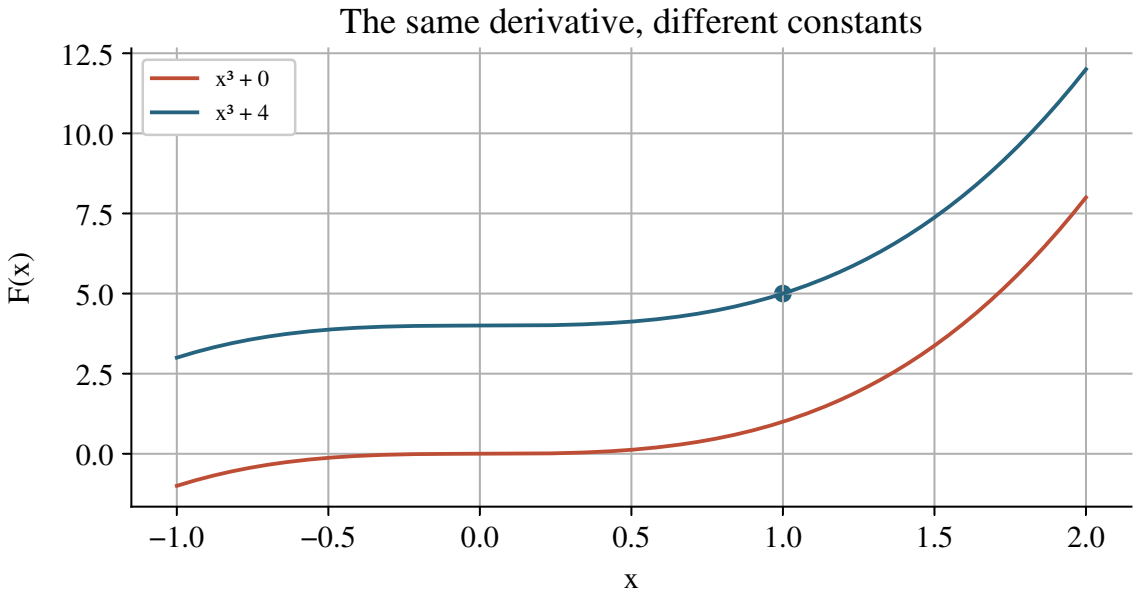
$$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

■ **Worked example**

An antiderivative of $3x^2$ is $x^3+C$ because differentiating $x^3$ gives $3x^2$. If $F(1)=5$, then $1+C=5$, so $C=4$ and $F(x)=x^3+4$.



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## Where it goes wrong

- An indefinite integral needs a constant.
- Integrating each term changes its power; copying the derivative rule gives the wrong result.

> ■ **Watch out**
>
> **Not true:** Every indefinite integral has only one possible antiderivative. Area below the axis counts as negative, so a definite integral can be smaller than the total area, or even zero.

## Applying it

- P1 uses indefinite polynomial integration and a point to determine the constant.
- Definite integrals, areas and the trapezium rule are P2 content.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Antiderivative**: A function whose derivative equals the given integrand.
- Integrate polynomial powers by increasing the index by one and dividing by the new index.
- Watch for: an indefinite integral needs a constant.

## P2 · Deduction, contradiction and induction

## When does a pattern become a proof?

- Checking several integers can suggest a pattern. What turns the pattern into a proof for every integer?
- This lesson studies **counterexample** 反例: A single valid case that disproves a universal claim.

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## The method

- A deductive proof starts from stated definitions or assumptions.
- A counterexample refutes an all-values claim.
- Induction requires a base case and an implication from n=k to n=k+1; checking successive cases alone is not induction.

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## A worked case

$$(2a+1)+(2b+1)=2(a+b+1)$$

■ **Worked example**

For odd integers 2a+1 and 2b+1, their sum is 2(a+b+1), hence even. The claim $n^2$+n+41 is always prime fails at n=41: the value is 41×43=1763. For 1+...+n=n(n+1)/2, the induction step adds k+1 to the assumed sum.



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Odd + odd: (2a + 1) + (2b + 1)

Factor 2: 2(a + b + 1)

The bracket is an integer ⇒ the sum is even

## Where it goes wrong

- Do not assume the conclusion while proving it.
- In contradiction, identify the impossible consequence and reject the initial contrary assumption.
- Induction is not compulsory in AQA 7357; it belongs in its own appropriate further-mathematics scope.

> ■ **Watch out**
>
> **Not true:** Checking the first ten integers proves a claim for every positive integer. A pattern can hold for the first ten cases and then fail; a proof must cover every case.

## Applying it

- AQA proof includes deduction, exhaustion and contradiction.
- IAL P2 introduces exhaustion and counterexample, while P4 introduces contradiction.
- Keep the method matched to the named unit.


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## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

### Key idea

- **Counterexample**: A single valid case that disproves a universal claim.
- A deductive proof starts from stated definitions or assumptions.
- Watch for: do not assume the conclusion while proving it.

# P2 · Quadratics and inequalities

## Which widths make enough space?

- A rectangular enclosure has area x(10-x). What widths give at least 21 square metres?
- This lesson studies **discriminant** 判别式: The quantity $b^2$-4ac that determines the real roots of $ax^2$+bx+c=0.

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## The method

- Factor where possible; otherwise complete the square or use the quadratic formula.
- A quadratic inequality needs the sign on intervals, not only the roots.
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## A worked case

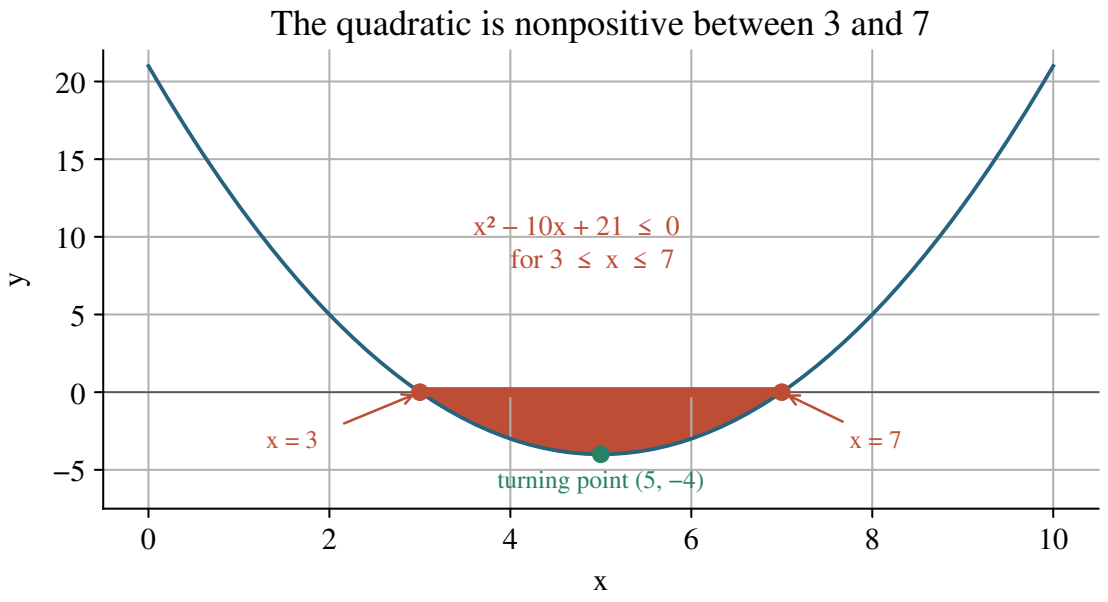
$$ax^2 + bx + c = 0, \qquad \Delta = b^2 - 4ac$$

### Worked example

The condition is x(10-x)≥21, so $x^2$-10x+21≤0. Factor (x-3)(x-7)≤0. The upward parabola is nonpositive between its roots, giving 3≤x≤7. The maximum area is 25 at x=5.



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## Where it goes wrong

- Multiplying an inequality by a negative number reverses its direction.
- A sketch must show which side of each root satisfies the inequality.
- Geometry may restrict x further.

> **Watch out**
>
> **Not true:** A positive discriminant means that a quadratic has no real roots. A positive discriminant means two distinct real roots; a negative discriminant means no real roots.

## Applying it

- Use the vertex to interpret an optimum.
- Check an endpoint and a point between the roots; the algebra and graph should tell the same story.

## In your course

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■ **Key idea**

- **Discriminant**: The quantity $b^2$-4ac that determines the real roots of $ax^2+bx+c=0$.
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- Watch for: multiplying an inequality by a negative number reverses its direction.

## P2 · Coordinate geometry and tangents

## How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
- This lesson studies **gradient** 斜率: The change in y divided by the corresponding change in x.

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## The method

- A line through $(x_1,y_1)$ with gradient m has $y-y_1=m(x-x_1)$.
- Parallel lines have equal gradients.
- Finite perpendicular gradients multiply to -1.
- A circle has $(x-a)^2+(y-b)^2=r^2$.

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## A worked case

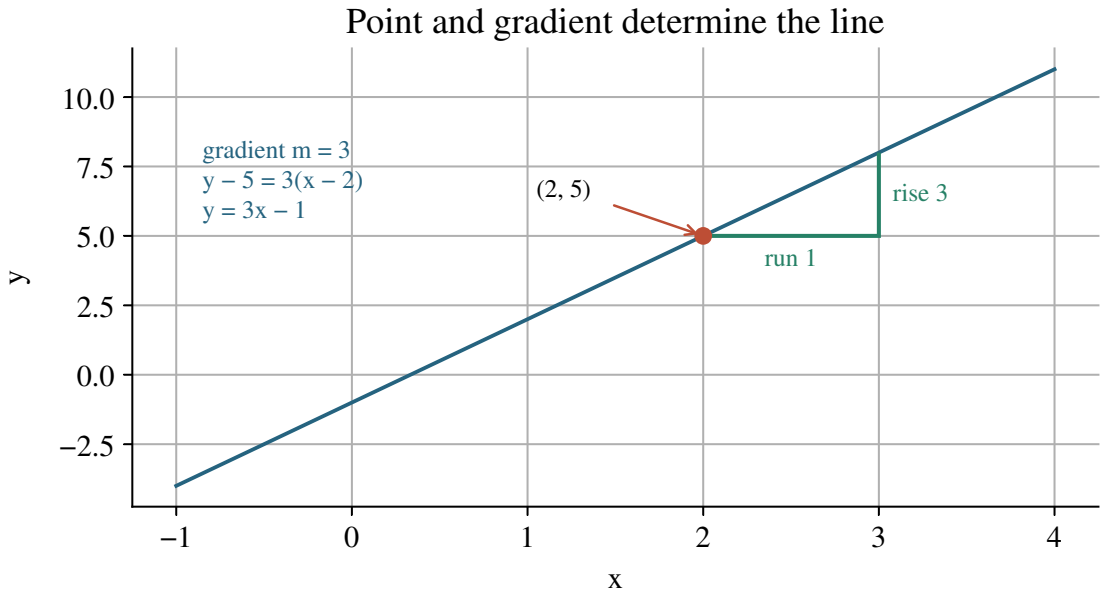
$$y - y_1 = m(x - x_1), \qquad (x - a)^2 + (y - b)^2 = r^2$$

■ **Worked example**

Through (2,5) with gradient 3, y-5=3(x-2), so y=3x-1. A perpendicular through the same point has y-5=-(x-2)/3. The circle $(x-2)^2+(y+1)^2=25$ has centre (2,-1) and radius 5.



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## Where it goes wrong

- A vertical line has no finite gradient; do not force it into y=mx+c.
- Read the signs of a circle's centre carefully.
- The radius to a tangent is perpendicular to the tangent.

### Watch out

**Not true:** Perpendicular nonvertical lines have equal gradients. Perpendicular gradients multiply to $-1$: a gradient of 2 needs $-1/2$. Parallel lines have equal gradients.

## Applying it

- Before solving a line-circle intersection, predict whether there are zero, one or two intersections.
- Substitution produces a quadratic whose discriminant checks the prediction.

## In your course

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### ■ Key idea

- **Gradient**: The change in y divided by the corresponding change in x.
- A line through $(x_1,y_1)$ with gradient m has $y-y_1=m(x-x_1)$.
- Watch for: A vertical line has no finite gradient; do not force it into y=mx+c.

## P2 · Sequences, series and recurrence

## Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common ratio** 公比: The constant multiplier between consecutive terms of a geometric sequence.

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## The method

- For an arithmetic progression, u_n=a+(n-1)d and S_n=n(2a+(n-1)d)/2.
- For a geometric progression, u_n=ar^(n-1) and S_n=a(1-r^n)/(1-r).
- An infinite geometric sum exists only if |r|<1.

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## A worked case

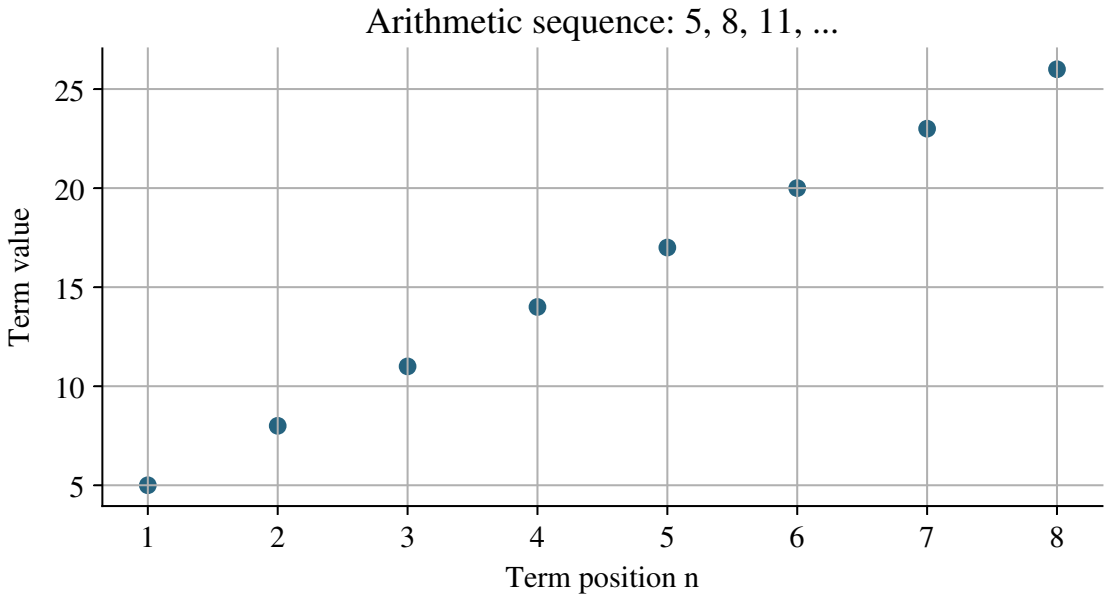
$$u_n = a + (n-1)d, \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

### ■ Worked example

For a=5,d=3,n=8, u_8=5+7×3=26 and S_8=8(10+21)/2=124. For a=12,r=1/2, S infinity=12/(1-1/2)=24. For u_(n+1)=2u_n+1 with u_1=1, the next terms are 3,7,15.



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## Where it goes wrong

- The first term has index 1, so the exponent is n-1.
- A sequence is a list; a series is a sum.
- A geometric sequence can alternate in sign and still converge.

> ■ **Watch out**
>
> **Not true:** Every geometric series has a finite sum to infinity. A sum to infinity exists only when $-1<r<1$; for $r=2$ the terms grow without limit.

## Applying it

- Explain whether the context justifies additive or multiplicative change.
- In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.



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■ **Key idea**

- **Common ratio**: The constant multiplier between consecutive terms of a geometric sequence.
- For an arithmetic progression, u_n=a+(n-1)d and S_n=n(2a+(n-1)d)/2.
- Watch for: the first term has index 1, so the exponent is n-1.

## P2 · Binomial expansion and valid approximations

## How does a small change affect a power?

- A small measurement change affects a power of a quantity. An expansion can show the size of first and second effects.
- This lesson studies **binomial coefficient** 二项式系数: The number of ways to choose a specified number of objects from a set.

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## The method

- For positive integer n, (a+b)^n is a finite binomial expansion.
- For noninteger n, expand (1+x)^n as 1+nx+n(n-1)$x^2$/2+... with |x|<1.
- Factor out constants before using this form.

---

## A worked case

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots$$

■ **Worked example**

(1+2x)^5=1+10x+40$x^2$+80$x^3$+80$x^4$+32$x^5$. For (1+x)^(-1), the first three terms are 1-x+$x^2$; at x=0.1 this gives 0.91 versus the exact 1/1.1≈0.909091.



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$(1 + 2x)^5$: Pascal row × powers of 2

| | | | | | | |
|---|---|---|---|---|---|---|
| Pascal row 5 | 1 | 5 | 10 | 10 | 5 | 1 |
| $\times 2^k$ | 1 | 2 | 4 | 8 | 16 | 32 |
| coefficient | **1** | **10** | **40** | **80** | **80** | **32** |
| | 1 | x | $x^2$ | $x^3$ | $x^4$ | $x^5$ |

## Where it goes wrong

- An expansion in 2x requires $|2x|<1$ for the infinite series, not merely $|x|<1$.
- Positive-integer expansions are finite and do not have that convergence restriction.

> ■ **Watch out**
>
> **Not true:** Every binomial expansion is a finite polynomial, including powers that are not positive integers. For a negative or fractional power the expansion is an infinite series, valid only for $|x|<1$.

## Applying it

- State the range of validity alongside an approximation.
- Retain enough terms to support the requested accuracy, and distinguish a coefficient from the whole term containing x.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.



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■ **Key idea**

- **Binomial coefficient**: The number of ways to choose a specified number of objects from a set.
- For positive integer n, (a+b)^n is a finite binomial expansion.
- Watch for: an expansion in 2x requires |2x|<1 for the infinite series, not merely |x|<1.

## P2 · Exponentials, logarithms and modelling

## Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.
- This lesson studies **half-life** 半衰期: The time for a decaying quantity to fall to half its initial value.

---

## The method

- For y=Ae^(kt), k is a proportional rate.
- Taking logs gives ln y=ln A+kt.
- Logarithms require positive arguments, and log(x+y) is not log x+log y.

---

## A worked case

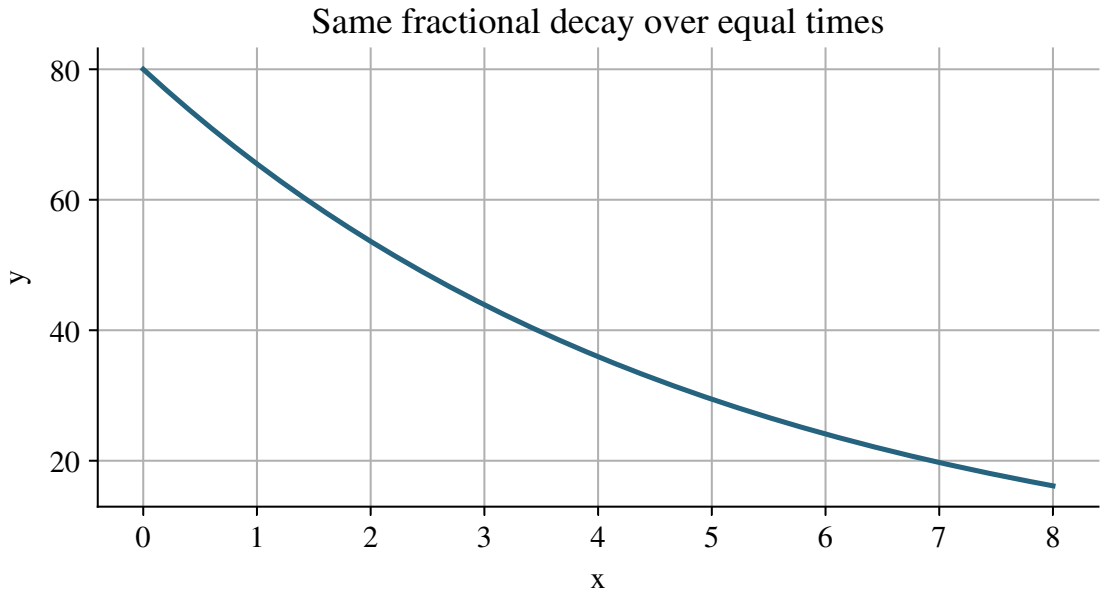
$$y = Ae^{kt}, \qquad \ln y = \ln A + kt$$

■ **Worked example**

If y=80e^(-0.2t), y=40 gives e^(-0.2t)=1/2. Hence t=ln2/0.2≈3.466. For 3^x=20, x=ln20/ln3. Plotting ln y against t linearises this exponential model.



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## Where it goes wrong

- A fitted exponential is a model, not a guarantee.
- Specify the time units and range of use.
- A negative k describes decay; a negative starting amount usually contradicts the context.

**Watch out**

**Not true:** For positive x and y, $\ln(x+y)=\ln x+\ln y$. The log of a sum has no simple law; the product rule is $\ln(xy)=\ln x+\ln y$.

## Applying it

- Compare actual observations with the model.
- Systematic departures may indicate changing conditions.
- In a report, explain what the rate and initial value mean, rather than giving a bare equation.


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## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

### ■ Key idea

- **Half-life**: The time for a decaying quantity to fall to half its initial value.
- For y=Ae^(kt), k is a proportional rate.
- Watch for: A fitted exponential is a model, not a guarantee.

# P2 · Radians, identities and trigonometric equations

## How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
- This lesson studies **radian** 弧度: The angle subtended by an arc whose length equals the radius.

---

## The method

- For $\theta$ in radians, arc length s=r$\theta$ and sector area A=r$^2\theta$/2.
- Use sin$^2\theta$+cos$^2\theta$=1 and angle identities to simplify or solve.
- List every solution in the stated interval.

---

## A worked case

$$s = r\theta, \qquad A = \frac{1}{2}r^2\theta, \qquad \sin^2\theta + \cos^2\theta = 1$$

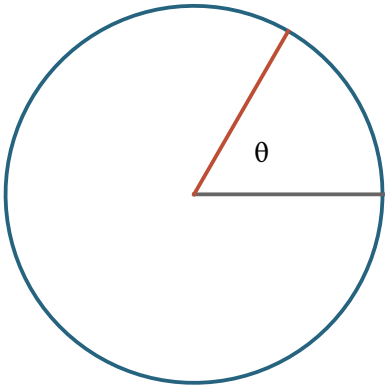
### ■ Worked example

For r=4 and $\theta$=$\pi$/3, s=4$\pi$/3 and A=8$\pi$/3. Solving sin x=1/2 on 0≤x<2$\pi$ gives x=$\pi$/6 and 5$\pi$/6. For sin2x=1/2, solve first on 0≤2x<4$\pi$ before dividing by 2.



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A radius fixes distance from the centre




radius × angle gives arc length

## Where it goes wrong

- A calculator's inverse sine gives one principal value.
- It does not give every solution.
- Dividing by sin x can discard solutions where sin x=0; check that case first.

> ■ **Watch out**
>
> **Not true:** The principal inverse sine value is the only solution in a full turn. Sine takes most values twice in a full turn: sin x=0.5 gives 30° and 150°.

## Applying it

- Choose an identity that reduces the equation to one trigonometric function.
- Keep exact familiar angles where possible, and check each solution against the original equation.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.



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■ **Key idea**

- **Radian**: The angle subtended by an arc whose length equals the radius.
- For $\theta$ in radians, arc length s=r$\theta$ and sector area A=r$^2\theta$/2.
- Watch for: A calculator's inverse sine gives one principal value.

# P2 · Derivatives and stationary points

## What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative** 导数: The instantaneous rate of change, also the gradient of a tangent.

---

## The method

- For y=ax^n, dy/dx=anx^(n-1).
- A stationary point satisfies dy/dx=0.
- Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.

---

## A worked case

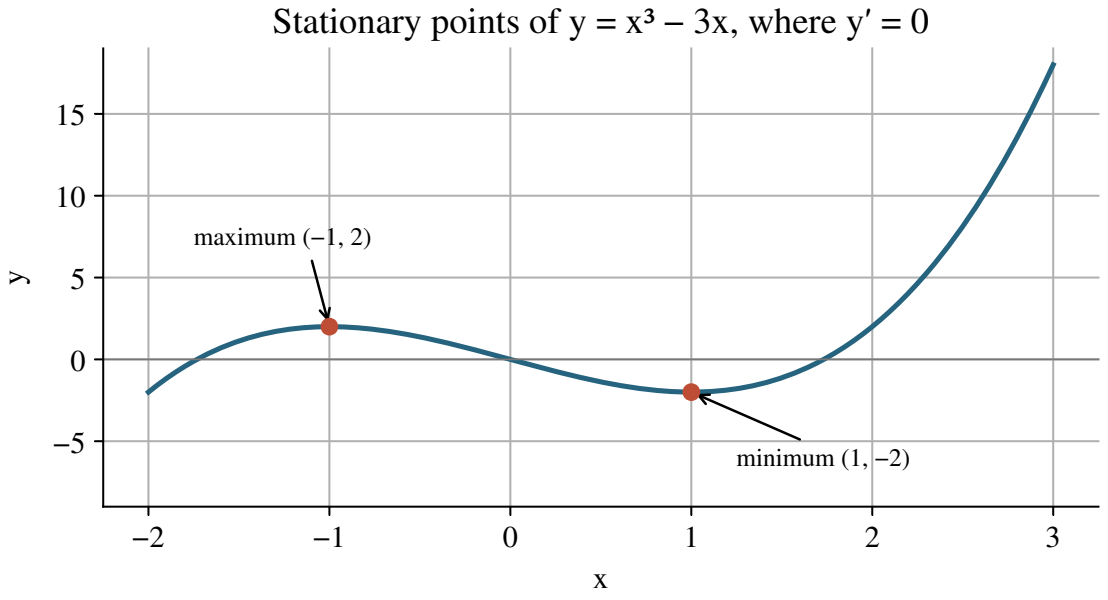
$$\frac{d}{dx}(ax^n) = anx^{n-1}, \qquad f'(x) = 0$$

■ **Worked example**

For y=x$^3$-3x, dy/dx=3x$^2$-3. At x=1, the gradient is 0 and y=-2. The second derivative is 6x, positive at x=1, so this is a local minimum. At x=-1, y=2 and the second derivative is negative, giving a local maximum.



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## Where it goes wrong

- A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.
- An endpoint can also produce an extreme value on a restricted domain.

> ■ **Watch out**
>
> **Not true:** Every point with zero derivative is a local maximum or minimum. A zero derivative can also be a point of inflection: $y=x^3$ at $x=0$ is neither a maximum nor a minimum.

## Applying it

- At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there.
- At advanced level, connect the derivative to rates and optimization with a valid domain.


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## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

### ■ Key idea

- **Derivative**: The instantaneous rate of change, also the gradient of a tangent.
- For y=ax^n, dy/dx=anx^(n-1).
- Watch for: A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.

## P2 · Integrals, area and accumulation

## How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative** 原函数: A function whose derivative equals the given integrand.

---

## The method

- For n≠-1, the integral of ax^n is ax^(n+1)/(n+1)+C.
- A definite integral is signed accumulation.
- Split at sign changes when total area or distance is required.

---

## A worked case

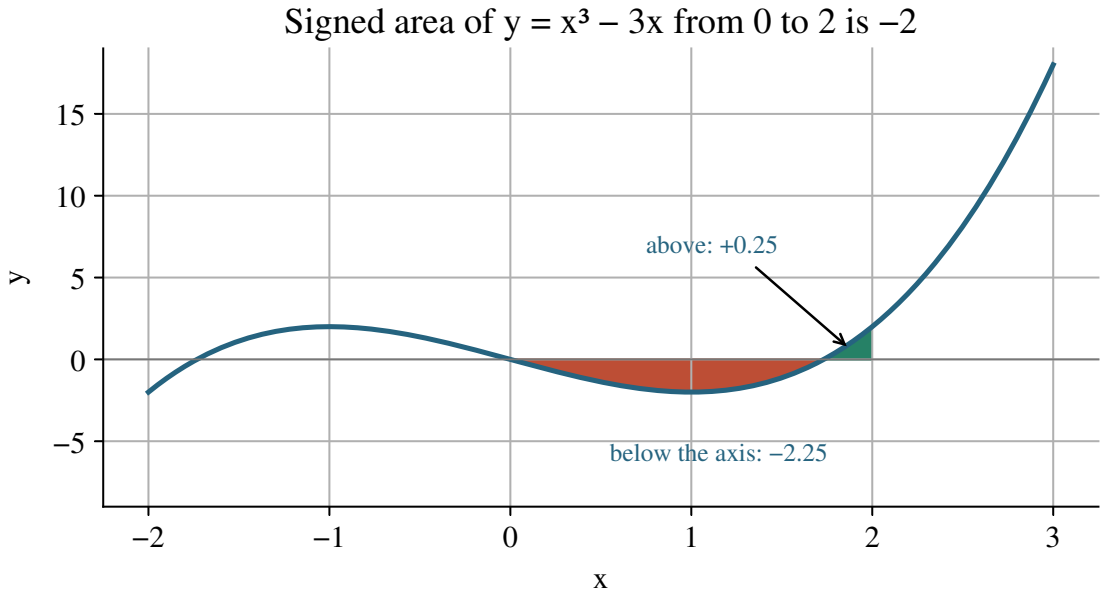
$$\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

### ■ Worked example

For v(t)=3t^2-3 over 0≤t≤2, displacement=[t^3-3t]_0^2=2. Since v changes sign at t=1, distance=-[t^3-3t]_0^1+[t^3-3t]_1^2=2+4=6. The constants cancel only for a definite integral.



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## Where it goes wrong

- The integral of 1/x is ln|x|+C, not the power rule with n=-1.
- Signed area can be zero even when the total enclosed area is positive.

> ■ **Watch out**
>
> **Not true:** A definite integral equals the total positive area, even when the graph crosses the axis. Area below the axis counts as negative, so a definite integral can be smaller than the total area, or even zero.

## Applying it

- Identify what the integral means and include the correct units.
- A rate measured per second integrates to the underlying quantity, not to another rate.
- Differentiate an antiderivative to check it.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.



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■ **Key idea**

- **Antiderivative**: A function whose derivative equals the given integrand.
- For n≠-1, the integral of ax^n is ax^(n+1)/(n+1)+C.
- Watch for: the integral of 1/x is ln|x|+C, not the power rule with n=-1.

# P2 · Root finding and numerical integration

## How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
- This lesson studies **iteration** 迭代: A repeated update in which each new approximation is calculated from the previous one.

---

## The method

- For equal strip width h, the trapezium estimate is h/2 times the sum of the two endpoint heights plus twice the internal heights.

---

## A worked case

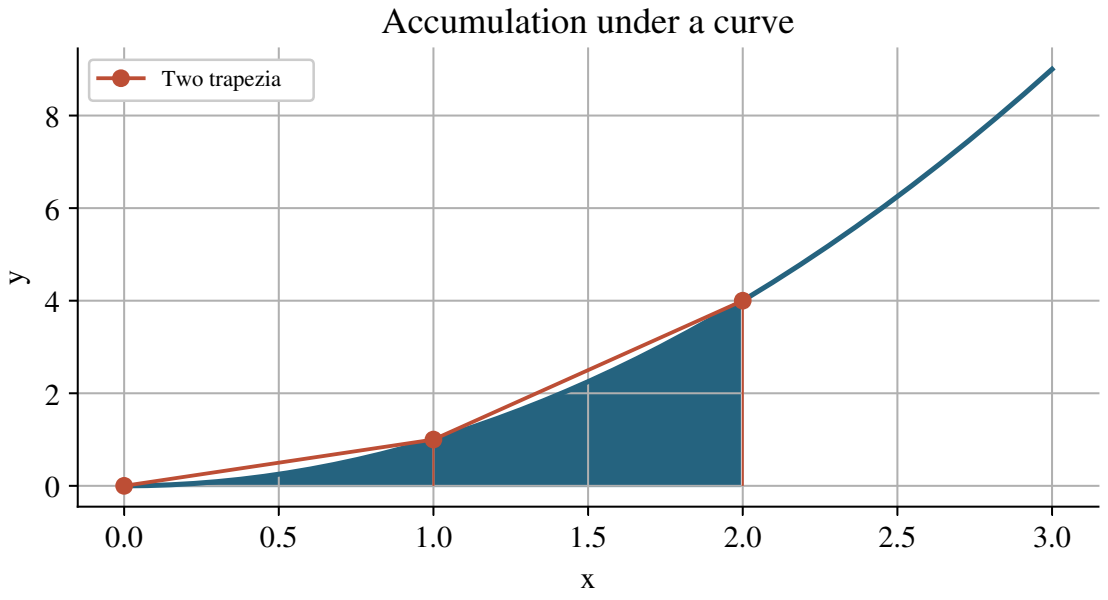
$$A \approx \frac{h}{2}(y_0 + 2y_1 + \cdots + 2y_{n-1} + y_n)$$

■ **Worked example**

For $y=x^2$ on [0,2] with two equal strips, h=1 and the heights are 0,1,4. The estimate is $(1/2)(0+2\times1+4)=3$. The exact area is 8/3, so this convex curve gives an overestimate.



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## Where it goes wrong

- Use equal strip widths and include internal heights twice.
- Curvature decides whether the estimate is above or below the exact area; being increasing alone does not.

### ■ Watch out

**Not true:** An increasing function gives a trapezium overestimate. A sign change shows a root only if the function is continuous there: 1/x changes sign across x=0 but has no root.

## Applying it

- P2 introduces the trapezium rule.
- Newton–Raphson and other root-finding methods belong to other named units; they are not part of this P2 lesson.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Iteration**: A repeated update in which each new approximation is calculated from the previous one.
- For equal strip width h, the trapezium estimate is h/2 times the sum of the two endpoint heights plus twice the internal heights.
- Watch for: use equal strip widths and include internal heights twice.

# P3 · Domains, inverses and composition

## Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
- This lesson studies **domain** 定义域: The set of allowed inputs to a function.

---

## The method

- State the domain and range.
- For an inverse, first ensure the function is one-to-one on its domain.
- Composition fg means apply g first, then f; the intermediate output must be an allowed input to f.

---

## A worked case

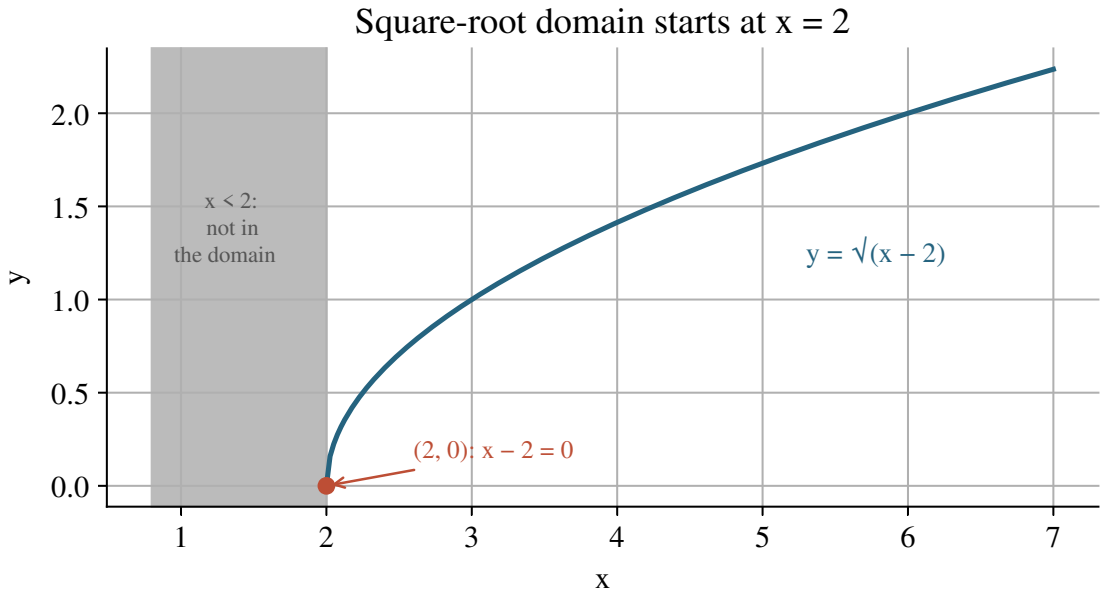
$$f(g(x)) = (f \circ g)(x), \qquad f^{-1}(f(x)) = x$$

■ **Worked example**

For f(x)=√(x-2), x≥2 and the range is y≥0. From y=√(x-2), x=$y^2$+2. Thus f inverse(x)=$x^2$+2 with x≥0. For g(x)=x+3, fg(1)=f(4)=√2.



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## Where it goes wrong

- Squaring can introduce extraneous solutions.
- Restricting a parabola's domain is essential before claiming an inverse.
- A horizontal translation inside f has the opposite sign to the graph's movement.

> **Watch out**
>
> **Not true:** Every quadratic function on all real numbers has an inverse function. A quadratic on all real numbers is many-to-one (f(1)=f(−1) for $x^2$), so its domain must be restricted first.

## Applying it

- Check f(f inverse(x))=x on the inverse domain.
- Use a sketch to test whether a horizontal line meets the original graph more than once.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.



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■ **Key idea**

- **Domain**: The set of allowed inputs to a function.
- State the domain and range.
- Watch for: squaring can introduce extraneous solutions.

## P3 · Radians, identities and trigonometric equations

## How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
- This lesson studies **radian** 弧度: The angle subtended by an arc whose length equals the radius.

---

## The method

- For $\theta$ in radians, arc length s=r$\theta$ and sector area A=r$^2\theta/2$.
- Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve.
- List every solution in the stated interval.

---

## A worked case

$$s = r\theta, \qquad A = \frac{1}{2}r^2\theta, \qquad \sin^2\theta + \cos^2\theta = 1$$

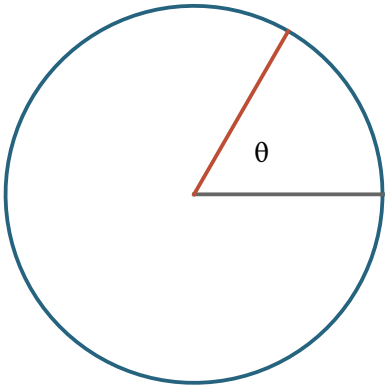
■ **Worked example**

For r=4 and $\theta=\pi/3$, s=$4\pi/3$ and A=$8\pi/3$. Solving sin x=1/2 on $0\leq$x$<2\pi$ gives x=$\pi/6$ and $5\pi/6$. For sin2x=1/2, solve first on $0\leq$2x$<4\pi$ before dividing by 2.



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A radius fixes distance from the centre




radius × angle gives arc length

## Where it goes wrong

- A calculator's inverse sine gives one principal value.
- It does not give every solution.
- Dividing by sin x can discard solutions where sin x=0; check that case first.

> ■ **Watch out**
>
> **Not true:** The principal inverse sine value is the only solution in a full turn. Sine takes most values twice in a full turn: sin x=0.5 gives 30° and 150°.

## Applying it

- Choose an identity that reduces the equation to one trigonometric function.
- Keep exact familiar angles where possible, and check each solution against the original equation.

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- Pearson Edexcel International A Level Pure Mathematics, unit P3.
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■ **Key idea**

- **Radian**: The angle subtended by an arc whose length equals the radius.
- For $\theta$ in radians, arc length s=r$\theta$ and sector area A=r$^2\theta$/2.
- Watch for: A calculator's inverse sine gives one principal value.

## P3 · Exponentials, logarithms and modelling

## Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.
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## The method

- For y=Ae^(kt), k is a proportional rate.
- Taking logs gives ln y=ln A+kt.
- Logarithms require positive arguments, and log(x+y) is not log x+log y.

---

## A worked case

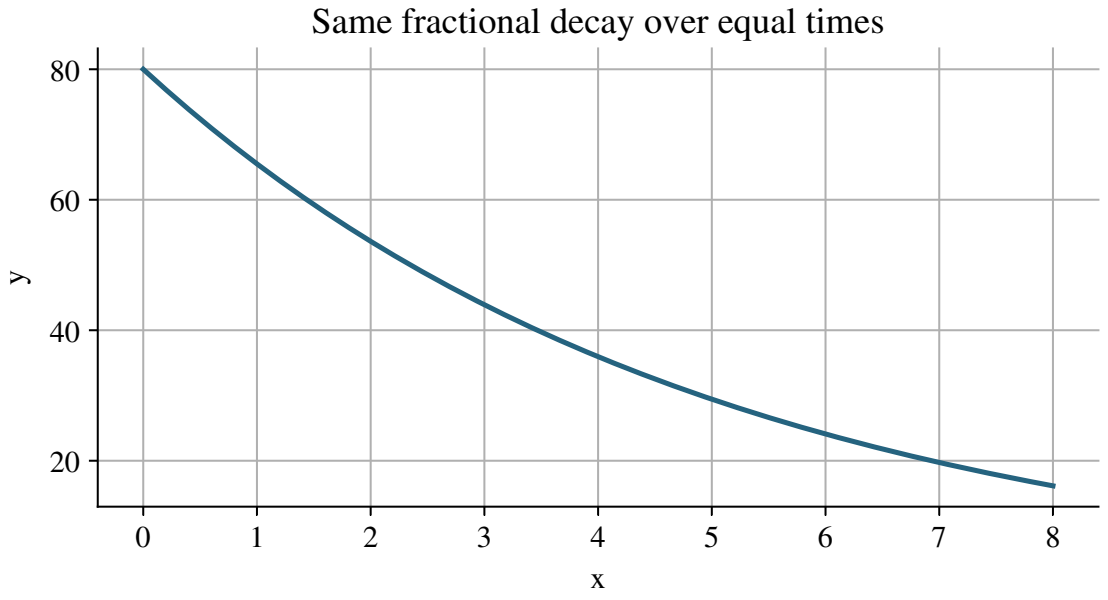
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## Where it goes wrong

- A fitted exponential is a model, not a guarantee.
- Specify the time units and range of use.
- A negative k describes decay; a negative starting amount usually contradicts the context.

**Watch out**

**Not true:** For positive x and y, $\ln(x+y)=\ln x+\ln y$. The log of a sum has no simple law; the product rule is $\ln(xy)=\ln x+\ln y$.

## Applying it

- Compare actual observations with the model.
- Systematic departures may indicate changing conditions.
- In a report, explain what the rate and initial value mean, rather than giving a bare equation.


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## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

### Key idea

- **Half-life**: The time for a decaying quantity to fall to half its initial value.
- For y=Ae^(kt), k is a proportional rate.
- Watch for: A fitted exponential is a model, not a guarantee.

## P3 · Chain, product, quotient and implicit differentiation

## Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
- This lesson studies **chain rule** 链式法则: The rule that multiplies the outer derivative by the inner derivative for a composite function.

---

## The method

- For y=f(g(x)), y prime=f prime(g(x))g prime(x).
- For uv, differentiate to u prime v+uv prime.
- For u/v, use (u prime v-uv prime)/$v^2$, where v≠0.

---

## A worked case

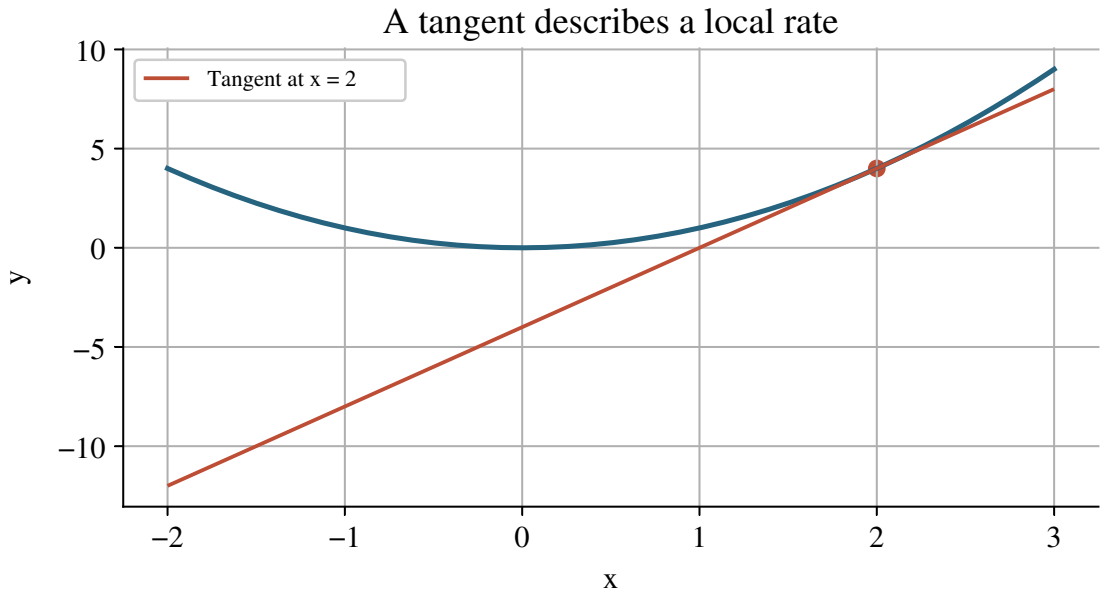
$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

### Worked example

For y=(3x+1)^4, y prime=4(3x+1)^3×3=12(3x+1)^3. At x=0, the gradient is 12. For $x^2+y^2=25$, differentiate implicitly: 2x+2y y prime=0, so y prime=-x/y when y≠0.



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## Where it goes wrong

- A derivative of a product is not the product of the derivatives.
- In implicit differentiation, every differentiated function of y brings a dy/dx factor.
- A quotient's denominator is squared.

> ■ **Watch out**
>
> **Not true:** The derivative of u(x)v(x) is u prime times v prime. The product rule is $(uv)'=u'v+uv'$: for $x \cdot x$ the derivative is 2x, not $1\times1=1$.

## Applying it

- Choose a useful form before differentiating: expanding a short polynomial may be simpler.
- For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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### Key idea

- **Chain rule**: The rule that multiplies the outer derivative by the inner derivative for a composite function.
- For y=f(g(x)), y prime=f prime(g(x))g prime(x).
- Watch for: A derivative of a product is not the product of the derivatives.

## P3 · Integrals, area and accumulation

## How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative** 原函数: A function whose derivative equals the given integrand.

---

## The method

- For n≠-1, the integral of ax^n is ax^(n+1)/(n+1)+C.
- A definite integral is signed accumulation.
- Split at sign changes when total area or distance is required.

---

## A worked case

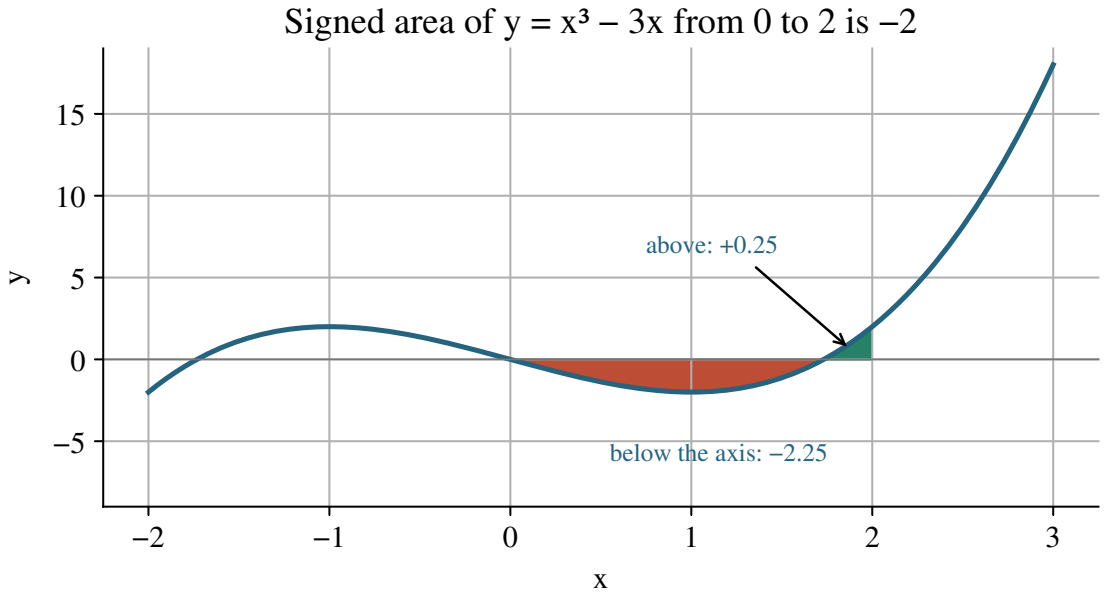
$$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

### Worked example

For v(t)=3t$^2$-3 over 0≤t≤2, displacement=[t$^3$-3t]_0^2=2. Since v changes sign at t=1, distance=-[t$^3$-3t]_0^1+[t$^3$-3t]_1^2=2+4=6. The constants cancel only for a definite integral.



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## Where it goes wrong

- The integral of 1/x is ln|x|+C, not the power rule with n=-1.
- Signed area can be zero even when the total enclosed area is positive.

> ■ **Watch out**
>
> **Not true:** A definite integral equals the total positive area, even when the graph crosses the axis. Area below the axis counts as negative, so a definite integral can be smaller than the total area, or even zero.

## Applying it

- Identify what the integral means and include the correct units.
- A rate measured per second integrates to the underlying quantity, not to another rate.
- Differentiate an antiderivative to check it.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Antiderivative**: A function whose derivative equals the given integrand.
- For n≠-1, the integral of ax^n is ax^(n+1)/(n+1)+C.
- Watch for: the integral of 1/x is ln|x|+C, not the power rule with n=-1.

## P3 · Fixed-point iteration

## How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
- This lesson studies **iteration** 迭代: A repeated update in which each new approximation is calculated from the previous one.

---

## The method

- Rearrange the equation as x=g(x), choose a starting value and iterate.
- A limit L must satisfy L=g(L).
- Near a fixed point, |g prime(L)|<1 provides a local convergence check; a sign bracket can validate the reported rounded root.

---

## A worked case

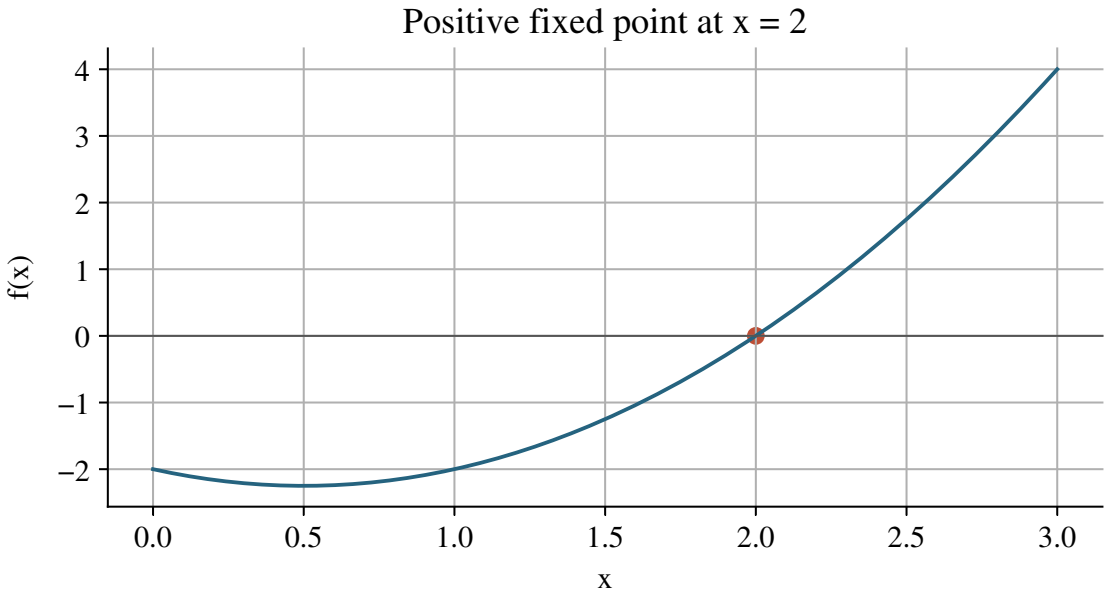
$$x_{n+1} = \sqrt{2 + x_n}$$

■ **Worked example**

For $x^2$-x-2=0 choose x next=√(2+x), starting at $x_0$=1. Then $x_1$=√3≈1.73205, $x_2$≈1.93185 and the positive fixed point is 2. At 2, g prime=1/(2√4)=1/4, so small local errors shrink. The square-root rearrangement seeks the positive root; it does not give the negative root -1.



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## Where it goes wrong

- Different rearrangements can have different convergence behaviour.
- Check the iterates, domain and original equation; a square-root update cannot reach a negative fixed point.

> **Watch out**
>
> **Not true:** Every rearrangement of an equation converges to the same root from every starting value. A sign change shows a root only if the function is continuous there: $1/x$ changes sign across $x=0$ but has no root.

## Applying it

- P3 uses equation rearrangement and numerical iteration.
- This lesson excludes Newton and numerical integration, which belong to other units.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Iteration**: A repeated update in which each new approximation is calculated from the previous one.
- Rearrange the equation as x=g(x), choose a starting value and iterate.
- Watch for: different rearrangements can have different convergence behaviour.

## P4 · Deduction, contradiction and induction

## When does a pattern become a proof?

- Checking several integers can suggest a pattern. What turns the pattern into a proof for every integer?
- This lesson studies **counterexample** 反例: A single valid case that disproves a universal claim.

---

## The method

- A deductive proof starts from stated definitions or assumptions.
- A counterexample refutes an all-values claim.
- Induction requires a base case and an implication from n=k to n=k+1; checking successive cases alone is not induction.

---

## A worked case

$$(2a+1)+(2b+1)=2(a+b+1)$$

■ **Worked example**

For odd integers 2a+1 and 2b+1, their sum is 2(a+b+1), hence even. The claim $n^2$+n+41 is always prime fails at n=41: the value is 41×43=1763. For 1+...+n=n(n+1)/2, the induction step adds k+1 to the assumed sum.



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Odd + odd: (2a + 1) + (2b + 1)

Factor 2: 2(a + b + 1)

The bracket is an integer ⇒ the sum is even

## Where it goes wrong

- Do not assume the conclusion while proving it.
- In contradiction, identify the impossible consequence and reject the initial contrary assumption.
- Induction is not compulsory in AQA 7357; it belongs in its own appropriate further-mathematics scope.

### ■ Watch out

**Not true:** Checking the first ten integers proves a claim for every positive integer. A pattern can hold for the first ten cases and then fail; a proof must cover every case.

## Applying it

- AQA proof includes deduction, exhaustion and contradiction.
- IAL P2 introduces exhaustion and counterexample, while P4 introduces contradiction.
- Keep the method matched to the named unit.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

### ■ Key idea

- **Counterexample**: A single valid case that disproves a universal claim.
- A deductive proof starts from stated definitions or assumptions.
- Watch for: do not assume the conclusion while proving it.

# P4 · Equations, identities and rearrangement

## When do two plans cost the same?

- Two mobile plans cost 20+3x and 44+x yuan for x GB. When do they cost the same?
- This lesson studies **identity** 恒等式: An equality that holds for every allowed value of its variable.

---

## The method

- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs.
- Preserve equality by applying the same operation to both sides.
- State restrictions before dividing by a variable.

---

## A worked case

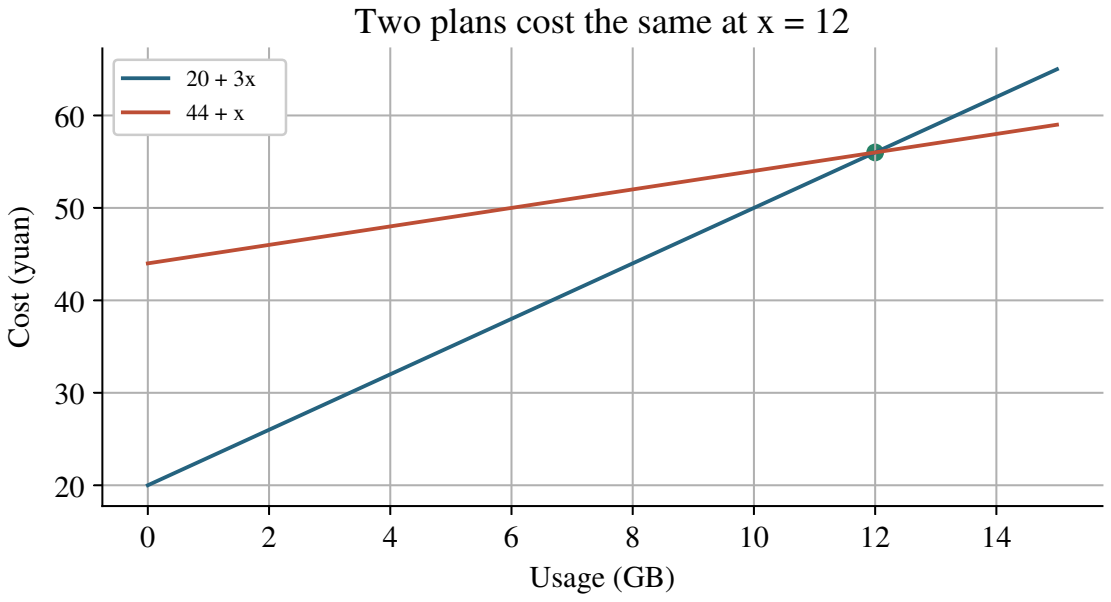
$$C_1 = 20 + 3x, \qquad C_2 = 44 + x, \qquad C_1 = C_2$$

### ■ Worked example

20+3x=44+x gives 2x=24 and x=12. Both plans then cost 56. In $A=\pi r^2$, divide by $\pi$ and take the positive square root to obtain $r=\sqrt{(A/\pi)}$, because r is a length.



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## Where it goes wrong

- Cancelling a term is not the same as cancelling a factor.
- In $(x^2+2x)/x$, factor the numerator and retain $x \neq 0$.
- Check a rearrangement by substitution.

### ■ Watch out

**Not true:** Cancelling x from $(x+3)/x$ leaves 3 for every nonzero x. x is a term of the numerator, not a factor, so it cannot be cancelled: at $x=1$, $(1+3)/1=4$, not 3.

## Applying it

- Set up the equation from units and the meaning of the unknown.
- A negative or fractional solution may be algebraically correct but impossible for a count.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Identity**: An equality that holds for every allowed value of its variable.
- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs.
- Watch for: cancelling a term is not the same as cancelling a factor.

## P4 · Binomial expansion and valid approximations

## How does a small change affect a power?

- A small measurement change affects a power of a quantity. An expansion can show the size of first and second effects.
- This lesson studies **binomial coefficient** 二项式系数: The number of ways to choose a specified number of objects from a set.

---

## The method

- For positive integer n, (a+b)^n is a finite binomial expansion.
- For noninteger n, expand (1+x)^n as 1+nx+n(n-1)$x^2$/2+... with |x|<1.
- Factor out constants before using this form.

---

## A worked case

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots$$

■ **Worked example**

(1+2x)^5=1+10x+40$x^2$+80$x^3$+80$x^4$+32$x^5$. For (1+x)^(-1), the first three terms are 1-x+$x^2$; at x=0.1 this gives 0.91 versus the exact 1/1.1≈0.909091.



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$(1 + 2x)^5$: Pascal row × powers of 2

| | | | | | | |
|---|---|---|---|---|---|---|
| Pascal row 5 | 1 | 5 | 10 | 10 | 5 | 1 |
| $\times 2^k$ | 1 | 2 | 4 | 8 | 16 | 32 |
| coefficient | **1** | **10** | **40** | **80** | **80** | **32** |
| | 1 | x | $x^2$ | $x^3$ | $x^4$ | $x^5$ |

## Where it goes wrong

- An expansion in 2x requires $|2x|<1$ for the infinite series, not merely $|x|<1$.
- Positive-integer expansions are finite and do not have that convergence restriction.

> ■ **Watch out**
>
> **Not true:** Every binomial expansion is a finite polynomial, including powers that are not positive integers. For a negative or fractional power the expansion is an infinite series, valid only for $|x|<1$.

## Applying it

- State the range of validity alongside an approximation.
- Retain enough terms to support the requested accuracy, and distinguish a coefficient from the whole term containing x.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Binomial coefficient**: The number of ways to choose a specified number of objects from a set.
- For positive integer n, (a+b)^n is a finite binomial expansion.
- Watch for: an expansion in 2x requires |2x|<1 for the infinite series, not merely |x|<1.

## P4 · Conics, parametric curves and tangent reasoning

## How can a rotating parameter trace an ellipse?

- An ellipse can be traced by a rotating parameter even when it is awkward to describe as a single y=f(x).
- This lesson studies **parametric equation** 参数方程: A representation in which coordinates are expressed using a common parameter.

---

## The method

- For x=a cos t,y=b sin t, eliminating t gives $x^2/a^2+y^2/b^2=1$.
- For parametric curves, dy/dx=(dy/dt)/(dx/dt), where dx/dt≠0.
- A zero denominator may signal a vertical tangent.

---

## A worked case

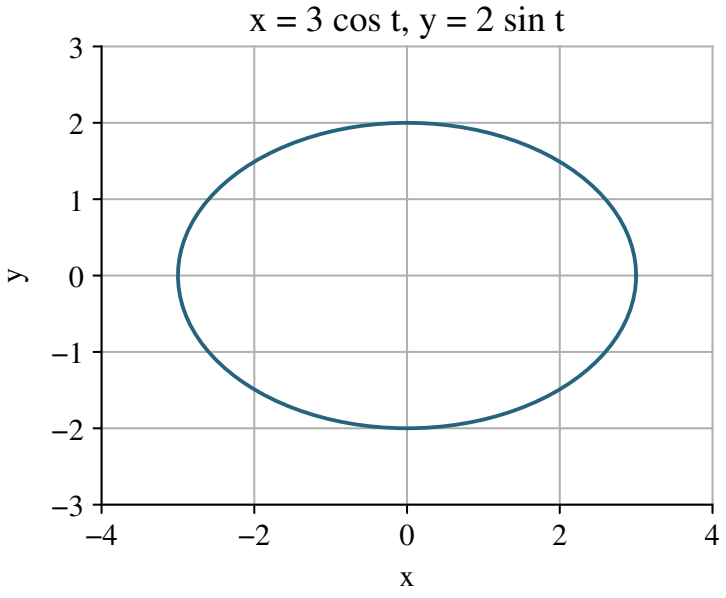
$$x = a\cos t, \qquad y = b\sin t, \qquad \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

■ **Worked example**

For x=3cos t,y=2sin t at t=π/4, the gradient is (2cos t)/(-3sin t)=-2/3. The point is (3/√2,√2). The area inside the ellipse is πab=6π; its semiaxes are 3 and 2.



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## Where it goes wrong

- Eliminating a parameter may lose a domain restriction or direction of travel.
- A vertical tangent cannot be assigned a finite dy/dx.
- Distinguish semiaxes from full widths.

> ■ **Watch out**
>
> **Not true:** Eliminating a parameter preserves every restriction and direction automatically. Elimination can lose restrictions: $x=t^2$, $y=t$ gives $x=y^2$, but $x\geq 0$ and the direction of motion must be stated separately.

## Applying it

- For advanced coordinate geometry, use the defining equation and a consistent parameter.
- For IAL P4 parametric integration, use the specification's restricted requirements rather than importing all Further Pure conics.


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## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

### ■ Key idea

- **Parametric equation**: A representation in which coordinates are expressed using a common parameter.
- For x=a cos t,y=b sin t, eliminating t gives $x^2/a^2+y^2/b^2=1$.
- Watch for: eliminating a parameter may lose a domain restriction or direction of travel.

# P4 · Chain, product, quotient and implicit differentiation

## Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
- This lesson studies **chain rule** 链式法则: The rule that multiplies the outer derivative by the inner derivative for a composite function.

---

## The method

- For y=f(g(x)), y prime=f prime(g(x))g prime(x).
- For uv, differentiate to u prime v+uv prime.
- For u/v, use (u prime v-uv prime)/$v^2$, where $v\neq0$.

---

## A worked case

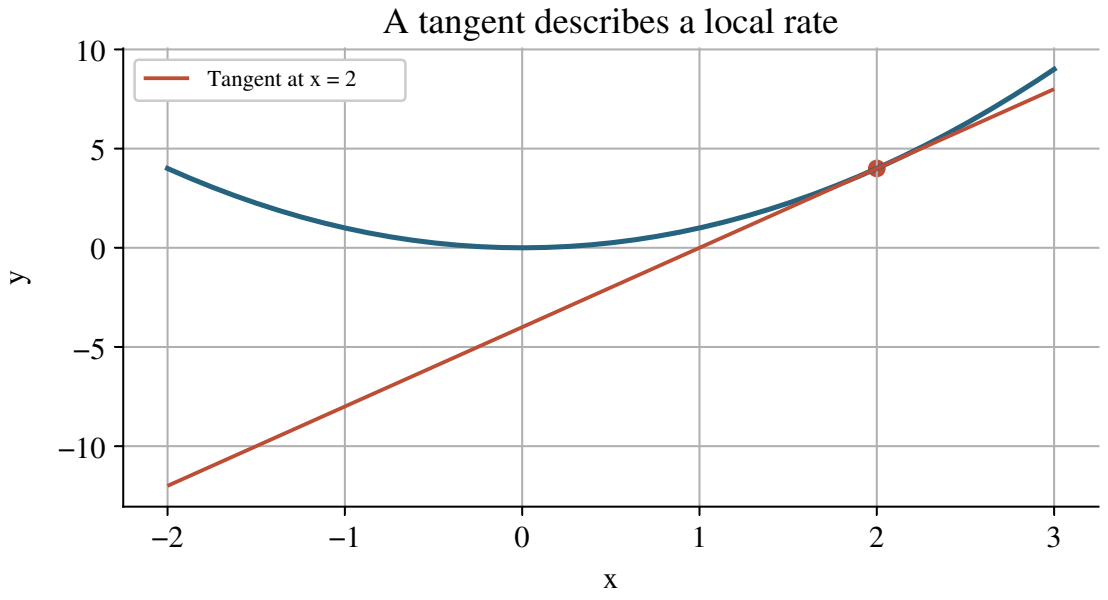
$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

### ■ Worked example

For y=(3x+1)^4, y prime=4(3x+1)^3×3=12(3x+1)^3. At x=0, the gradient is 12. For $x^2+y^2=25$, differentiate implicitly: 2x+2y y prime=0, so y prime=-x/y when $y\neq0$.



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## Where it goes wrong

- A derivative of a product is not the product of the derivatives.
- In implicit differentiation, every differentiated function of y brings a dy/dx factor.
- A quotient's denominator is squared.

> **Watch out**
>
> **Not true:** The derivative of u(x)v(x) is u prime times v prime. The product rule is $(uv)'=u'v+uv'$: for $x \cdot x$ the derivative is 2x, not $1\times1=1$.

## Applying it

- Choose a useful form before differentiating: expanding a short polynomial may be simpler.
- For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Chain rule**: The rule that multiplies the outer derivative by the inner derivative for a composite function.
- For y=f(g(x)), y prime=f prime(g(x))g prime(x).
- Watch for: A derivative of a product is not the product of the derivatives.

## P4 · Substitution, parts and partial fractions

## Can we integrate the two factors separately?

- A simple-looking product such as xe^x cannot be integrated by separately integrating each factor.
- This lesson studies **integration by parts** 分部积分法: An integration method based on the derivative of a product.

---

## The method

- Use substitution when an inner derivative appears as a factor.
- By parts, integral u v prime =uv-integral u prime v.
- For a rational function, divide first if needed, then decompose into partial fractions.

---

## A worked case

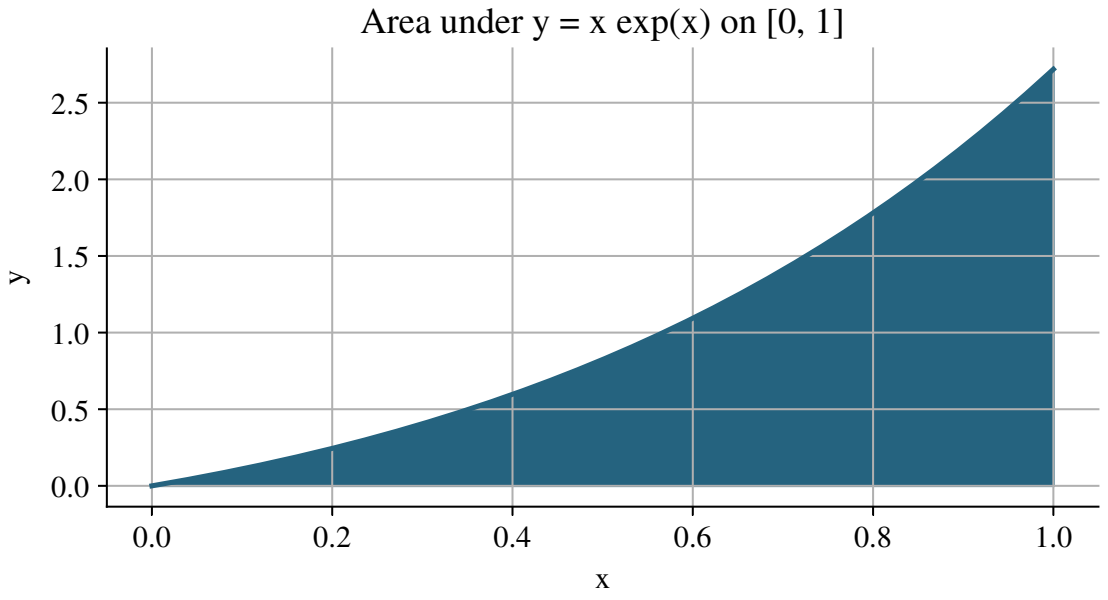
$$\int uv'\,dx = uv - \int u'v\,dx$$

■ **Worked example**

For integral xe^x dx, take u=x and v prime=e^x. Then the integral is xe^x-e^x+C. For integral $2x/(x^2+1)$ dx, substitute $w=x^2+1$, dw=2x dx; the answer is $\ln(x^2+1)+C$.



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## Where it goes wrong

- Choosing u and v prime well matters: the remaining integral should become simpler.
- In a definite substitution, either change the limits or return to x before applying the original limits.

> ■ **Watch out**
>
> **Not true:** The integral of a product equals the product of its separate integrals. A product needs a method such as integration by parts or substitution: $\int x \cdot x\,dx = x^3/3$, not $(x^2/2)(x^2/2)$.

## Applying it

- Check by differentiating.
- For volumes of revolution around the x-axis use $V = \pi \int y^2\,dx$; do not confuse the square of a function with the integral of the function.


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## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

### Key idea

- **Integration by parts**: An integration method based on the derivative of a product.
- Use substitution when an inner derivative appears as a factor.
- Watch for: choosing u and v prime well matters: the remaining integral should become simpler.

# P4 · Differential equations and numerical solutions

## What does the starting value decide?

- A growing population's rate is proportional to its current size. The rate equation describes a whole family until an initial population is supplied.
- This lesson studies **initial condition** 初始条件: A specified value that selects a particular solution from a family.

---

## The method

- Separate the variables in dy/dx=ky, integrate both sides and apply an initial condition.
- Include any equilibrium solution excluded when dividing by y.

---

## A worked case

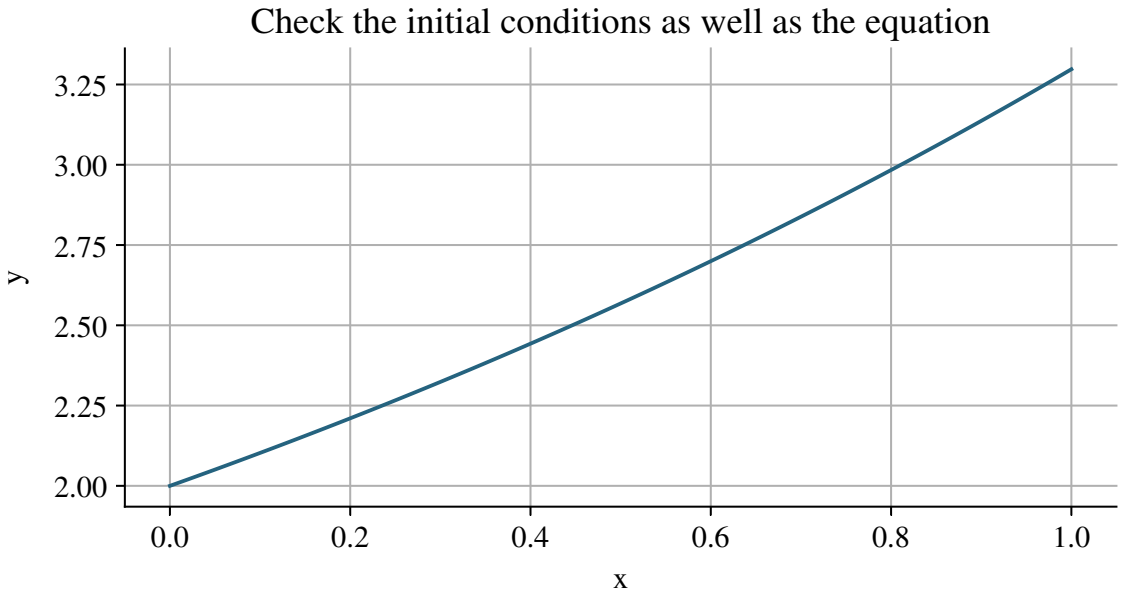
$$\frac{dy}{dx} = ky, \qquad y = Ae^{kx}$$

### Worked example

For dy/dx=0.5y and y(0)=2, separation gives ln|y|=0.5x+C, hence y=Ae^(0.5x). The initial value gives A=2. Differentiate the result to check the equation and substitute x=0 to check the initial condition.



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## Where it goes wrong

- An initial condition determines the integration constant; it is not a replacement for integrating.
- Dividing by y can omit an equilibrium solution y=0.
- Euler accuracy depends on step size and the equation.

> ■ **Watch out**
>
> **Not true:** An initial condition never changes the constant in a differential-equation solution. Euler's method depends on the step size; smaller steps usually give a more accurate value.

## Applying it

- This unit uses first-order separation and initial conditions.
- Euler numerical integration and second-order complementary functions are excluded from this lesson.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

### ■ Key idea

- **Initial condition**: A specified value that selects a particular solution from a family.
- Separate the variables in dy/dx=ky, integrate both sides and apply an initial condition.
- Watch for: an initial condition determines the integration constant; it is not a replacement for integrating.

# P4 · Spatial vectors, lines and angles

## Can nonparallel paths still miss each other?

- Two paths in space can be neither parallel nor intersecting. A two-dimensional sketch may conceal their separation.
- This lesson studies **direction vector** 方向向量: A nonzero vector giving the direction of a line.

---

## The method

- A line has r=a+$\lambda$b with position vector a and nonzero direction b.
- To find an intersection, equate all components and solve for both line parameters.
- For an angle use a · b=|a||b|cos$\theta$ where scalar products are in the named syllabus.

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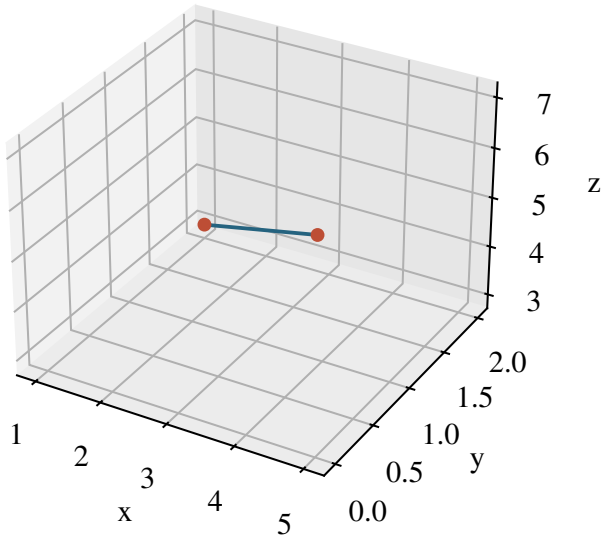
## A worked case

$$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}, \qquad \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$$

### ■ Worked example

For a=(1,2,3),b=(2,-1,2), $\lambda$=2 gives r=(5,0,7). The magnitude of b is $\sqrt{}$(4+1+4)=3. With c=(1,0,-1), b · c=2-2=0, so b and c are perpendicular.

$$\mathbf{r} = (1, 2, 3) + \lambda(2, -1, 2)$$




## Where it goes wrong

- Satisfying two component equations does not guarantee the third.
- Nonparallel lines may be skew.
- A line's direction vector is not its position vector, and a scalar parameter has no coordinate units.

> **Watch out**
>
> **Not true:** Any two nonparallel lines in three dimensions must intersect. In three dimensions nonparallel lines can be skew: they do not meet.

## Applying it

- State whether a requested angle is acute, directed or an angle between lines.
- Use the syllabus-specific plane or distance method only when it is actually required for that unit.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit P4.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Direction vector**: A nonzero vector giving the direction of a line.
- A line has r=a+λb with position vector a and nonzero direction b.
- Watch for: satisfying two component equations does not guarantee the third.

# FP1 · Complex arithmetic and quadratic roots

## What if the root is not real?

- The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots.
- This lesson studies **complex conjugate** 共轭复数: The number obtained by reversing the sign of the imaginary part.

---

## The method

- Write z=a+bi with $i^2=-1$.
- Add components, multiply brackets and use a conjugate to make a division denominator real.
- Non-real roots of a real-coefficient quadratic occur in conjugate pairs.

---

## A worked case

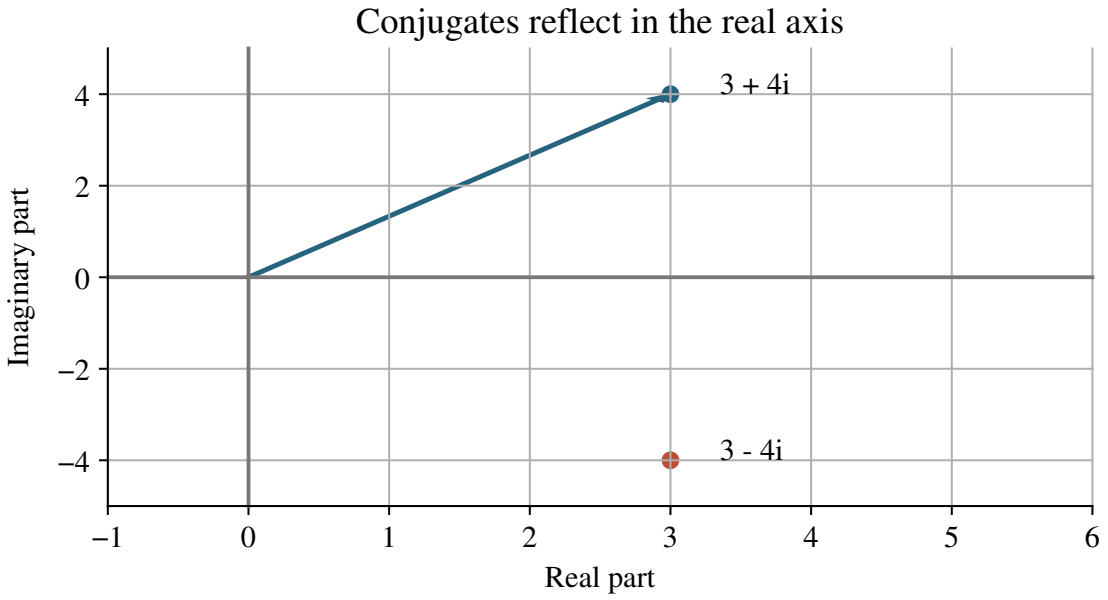
$$z = a + bi, \qquad |z| = \sqrt{a^2 + b^2}, \qquad z\overline{z} = a^2 + b^2$$

■ **Worked example**

For z=3+4i, |z|=5 and z conjugate=3-4i. The reciprocal is (3-4i)/25. For $x^2-6x+13=0$, the roots are 3±2i. Multiplying by the conjugate makes a complex denominator real.



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## Where it goes wrong

- The square root of a sum is not the sum of the square roots.
- An inverse tangent alone may return the wrong quadrant.
- Do not confuse modulus with squared modulus.

> ■ **Watch out**
>
> **Not true:** For a+bi, its modulus is a+b. The modulus is $\sqrt{(a^2+b^2)}$: for 3+4i it is 5, not 7.

## Applying it

- FP1 uses complex arithmetic, modulus, Argand representation and quadratic roots.
- De Moivre powers and roots of unity are reserved for FP2.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Complex conjugate**: The number obtained by reversing the sign of the imaginary part.
- Write z=a+bi with $i^2$=-1.
- Watch for: the square root of a sum is not the sum of the square roots.

## FP1 · Parabolas and rectangular hyperbolas

## How can a rotating parameter trace an ellipse?

- An ellipse can be traced by a rotating parameter even when it is awkward to describe as a single y=f(x).
- This lesson studies **parabola** 抛物线: The locus of points equally distant from a fixed focus and a fixed directrix.

---

## The method

- For $y^2$=4ax, the focus is (a,0), directrix x=-a and vertex (0,0).
- For xy=$c^2$, the coordinate axes are asymptotes.
- Read the parameter from the coefficient rather than assuming it equals 4a.

---

## A worked case

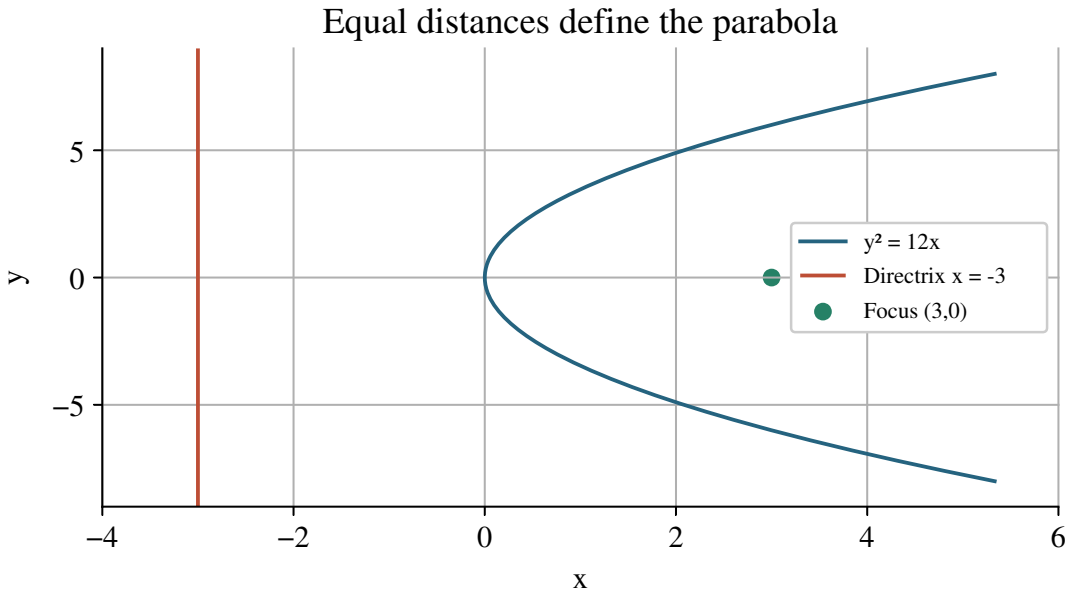
$$y^2 = 12x, \quad xy = 9$$

■ **Worked example**

For $y^2$=12x, a=3: the focus is (3,0) and directrix x=-3. At x=3, y=±6; at (3,6), distance to the focus and directrix is 6. For xy=9, the point (3,3) lies on the hyperbola and x=0,y=0 are its asymptotes.



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## Where it goes wrong

- The coefficient of x is 4a, not a.
- Include both signs when solving $y^2$.
- An asymptote is approached; it is not a finite intercept of $xy=c^2$.

> **Watch out**
>
> **Not true:** For $y^2=4ax$, the focus x-coordinate is 4a. Elimination can lose restrictions: $x=t^2$, $y=t$ gives $x=y^2$, but $x\geq 0$ and the direction of motion must be stated separately.

## Applying it

- FP1 coordinate-geometry support for parabolas and rectangular hyperbolas.
- Ellipse area and ellipse parametrisation are excluded.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Parabola**: The locus of points equally distant from a fixed focus and a fixed directrix.
- For $y^2=4ax$, the focus is (a,0), directrix x=-a and vertex (0,0).
- Watch for: the coefficient of x is 4a, not a.

# FP1 · Matrix operations, inverses and plane transformations

## Where do the basis vectors go?

- A map stretches one direction and shears another. A matrix records the transformation of each basis vector.
- This lesson studies **determinant** 行列式: A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.

---

## The method

- A 2×2 matrix [[a,b],[c,d]] has determinant ad-bc and inverse [[d,-b],[-c,a]]/(ad-bc) if the determinant is nonzero.
- Transform column vectors by left multiplication.
- The rightmost matrix acts first in a product.

---

## A worked case

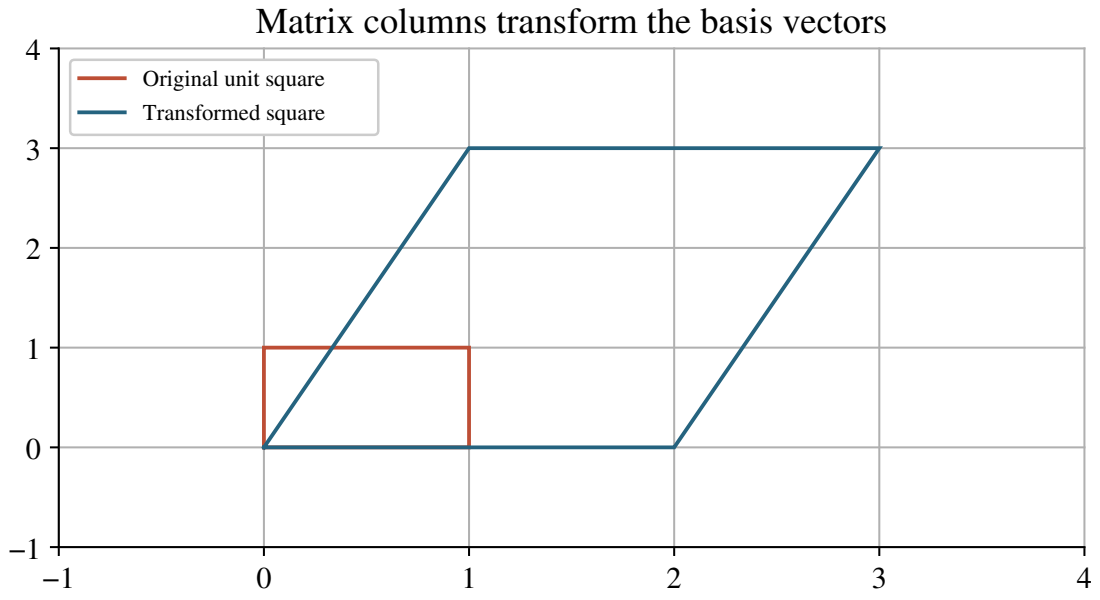
$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

■ **Worked example**

For A=[[2,1],[0,3]], det A=6. A(1,2)=(4,6). Its inverse is [[1/2,-1/6],[0,1/3]]. Check by multiplying A by its inverse to obtain the identity matrix.



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## Where it goes wrong

- Matrix multiplication is usually not commutative.
- A determinant of zero means the transformation loses a dimension and has no inverse.
- Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.

### ■ Watch out

**Not true:** Every square matrix has an inverse. A square matrix with determinant 0 is singular and has no inverse.

## Applying it

- FP1 uses 2×2 matrix operations, inverses and plane transformations.
- Eigenvalues and 3×3 diagonalisation belong to FP3, not this unit.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Determinant**: A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
- A 2×2 matrix [[a,b],[c,d]] has determinant ad-bc and inverse [[d,-b],[-c,a]]/(ad-bc) if the determinant is nonzero.
- Watch for: matrix multiplication is usually not commutative.

## FP1 · Bisection and Newton root finding

## How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
- This lesson studies **iteration** 迭代: A repeated update in which each new approximation is calculated from the previous one.

---

## The method

- For a continuous function with a sign change, bisect a root bracket and keep the half with opposite endpoint signs.
- Newton updates x to x-f(x)/f prime(x), provided the derivative is nonzero.

---

## A worked case

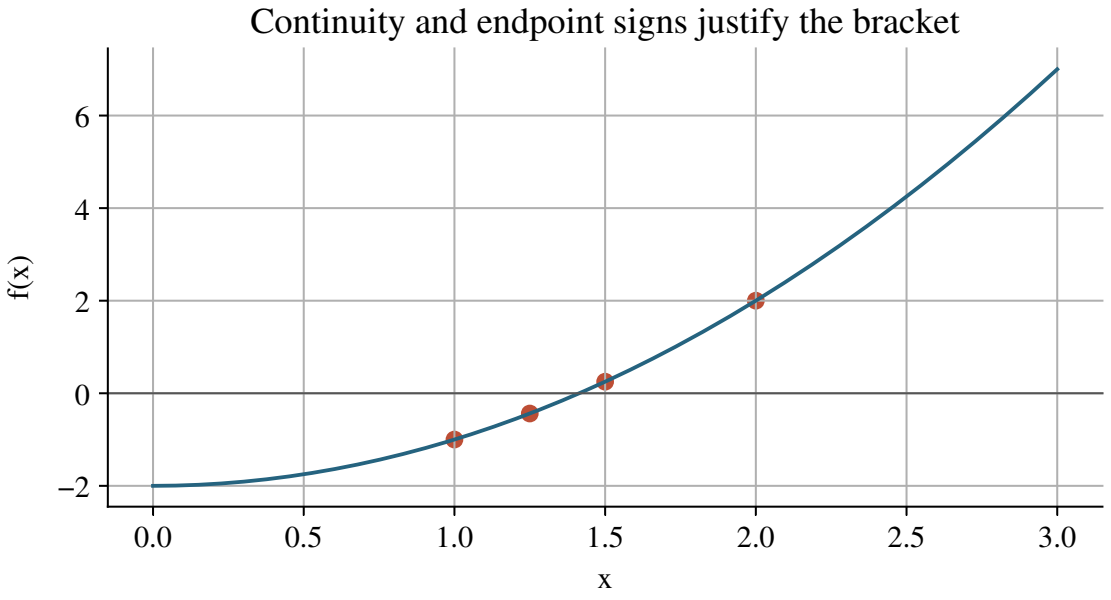
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

■ **Worked example**

For f(x)=x$^2$-2, f(1)=-1 and f(2)=2. At midpoint 1.5, f=0.25, so the new bracket is [1,1.5]. At midpoint 1.25, f=-0.4375, so the next bracket is [1.25,1.5]. Newton from 1.5 gives 1.4166667.



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## Where it goes wrong

- A sign change must occur on an interval where the function is continuous.
- Newton can fail if its derivative is zero, or if its iterates leave the useful domain.

> ■ **Watch out**
>
> **Not true:** Every sign change proves a root, including one across a discontinuity. A sign change shows a root only if the function is continuous there: 1/x changes sign across x=0 but has no root.

## Applying it

- FP1 numerical methods concern roots: bisection, interpolation and Newton methods.
- Numerical integration is not included in this lesson.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Iteration**: A repeated update in which each new approximation is calculated from the previous one.
- For a continuous function with a sign change, bisect a root bracket and keep the half with opposite endpoint signs.
- Watch for: A sign change must occur on an interval where the function is continuous.

## FP1 · Finite sums and induction

## Can a polynomial replace an exponential nearby?

- Near a known point, a complicated function can be replaced by a polynomial approximation. The approximation needs a centre and a valid scale.
- This lesson studies **sigma notation** 求和符号: Notation that adds indexed terms over a stated finite range.

---

## The method

- For k=1 to n, sum k=n(n+1)/2, sum $k^2$=n(n+1)(2n+1)/6 and sum $k^3$=[n(n+1)/2]$^2$.
- Differences between consecutive terms can telescope.
- In induction, verify the starting case and show the next case follows from the assumed case.

---

## A worked case

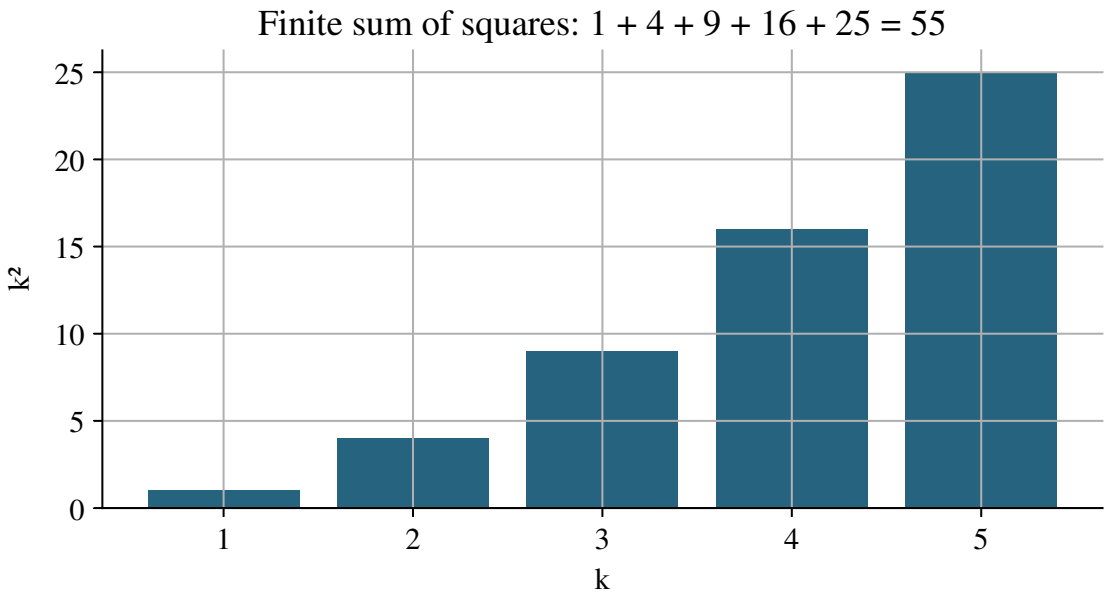
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

■ **Worked example**

For n=5, sum k=15, sum $k^2$=1+4+9+16+25=55 and sum $k^3$=1+8+27+64+125=225. In proving a sum formula, adding the (n+1)th term to the assumed expression must produce the formula with n replaced by n+1.



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## Where it goes wrong

- The sum of squares is not the square of the sum.
- State the index range and check the first case; an inductive step by itself is not a complete induction proof.

> ■ **Watch out**
>
> **Not true:** The sum of squared terms equals the square of their sum. A Taylor polynomial is an approximation; it is usually accurate only near its centre.

## Applying it

- FP1 finite series and induction support only.
- Maclaurin and Taylor approximation belong to later further-pure units and are excluded.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Sigma notation**: Notation that adds indexed terms over a stated finite range.
- For k=1 to n, sum k=n(n+1)/2, sum $k^2$=n(n+1)(2n+1)/6 and sum $k^3$=[n(n+1)/2]$^2$.
- Watch for: the sum of squares is not the square of the sum.

## FP1 · Deduction, contradiction and induction

## When does a pattern become a proof?

- Checking several integers can suggest a pattern. What turns the pattern into a proof for every integer?
- This lesson studies **counterexample** 反例: A single valid case that disproves a universal claim.

---

## The method

- A deductive proof starts from stated definitions or assumptions.
- A counterexample refutes an all-values claim.
- Induction requires a base case and an implication from n=k to n=k+1; checking successive cases alone is not induction.

---

## A worked case

$$(2a+1)+(2b+1)=2(a+b+1)$$

■ **Worked example**

For odd integers 2a+1 and 2b+1, their sum is 2(a+b+1), hence even. The claim $n^2$+n+41 is always prime fails at n=41: the value is 41×43=1763. For 1+...+n=n(n+1)/2, the induction step adds k+1 to the assumed sum.



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Odd + odd: (2a + 1) + (2b + 1)

Factor 2: 2(a + b + 1)

The bracket is an integer ⇒ the sum is even

## Where it goes wrong

- Do not assume the conclusion while proving it.
- In contradiction, identify the impossible consequence and reject the initial contrary assumption.
- Induction is not compulsory in AQA 7357; it belongs in its own appropriate further-mathematics scope.

> ■ **Watch out**
>
> **Not true:** Checking the first ten integers proves a claim for every positive integer. A pattern can hold for the first ten cases and then fail; a proof must cover every case.

## Applying it

- AQA proof includes deduction, exhaustion and contradiction.
- IAL P2 introduces exhaustion and counterexample, while P4 introduces contradiction.
- Keep the method matched to the named unit.



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## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

### Key idea

- **Counterexample**: A single valid case that disproves a universal claim.
- A deductive proof starts from stated definitions or assumptions.
- Watch for: do not assume the conclusion while proving it.

# FP2 · Quadratics and inequalities

## Which widths make enough space?

- A rectangular enclosure has area x(10-x). What widths give at least 21 square metres?
- This lesson studies **discriminant** 判别式: The quantity $b^2$-4ac that determines the real roots of $ax^2$+bx+c=0.

---

## The method

- Factor where possible; otherwise complete the square or use the quadratic formula.
- A quadratic inequality needs the sign on intervals, not only the roots.
- The discriminant identifies repeated or missing real roots.

---

## A worked case

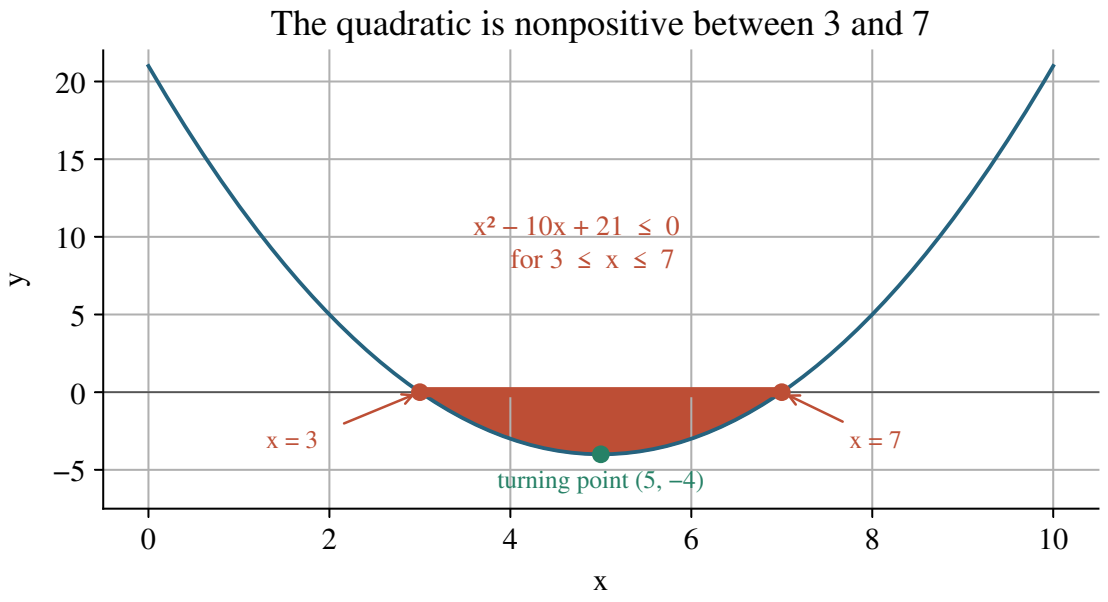
$$ax^2 + bx + c = 0, \qquad \Delta = b^2 - 4ac$$

### Worked example

The condition is x(10-x)≥21, so $x^2$-10x+21≤0. Factor (x-3)(x-7)≤0. The upward parabola is nonpositive between its roots, giving 3≤x≤7. The maximum area is 25 at x=5.



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## Where it goes wrong

- Multiplying an inequality by a negative number reverses its direction.
- A sketch must show which side of each root satisfies the inequality.
- Geometry may restrict x further.

> **Watch out**
>
> **Not true:** A positive discriminant means that a quadratic has no real roots. A positive discriminant means two distinct real roots; a negative discriminant means no real roots.

## Applying it

- Use the vertex to interpret an optimum.
- Check an endpoint and a point between the roots; the algebra and graph should tell the same story.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Discriminant**: The quantity $b^2$-4ac that determines the real roots of $ax^2$+bx+c=0.
- Factor where possible; otherwise complete the square or use the quadratic formula.
- Watch for: multiplying an inequality by a negative number reverses its direction.

## FP2 · Series, finite differences and Taylor expansions

## Can a polynomial replace an exponential nearby?

- Near a known point, a complicated function can be replaced by a polynomial approximation. The approximation needs a centre and a valid scale.
- This lesson studies **Taylor polynomial** 泰勒多项式: A polynomial formed from a function's derivatives at a chosen centre.

---

## The method

- About x=a, f(x)=f(a)+f prime(a)(x-a)+f double prime(a)$(x-a)^2/2!$+... when the expansion is valid.
- At a=0 it is a Maclaurin expansion.
- Finite sums may also simplify by cancellation or standard sum formulae.

---

## A worked case

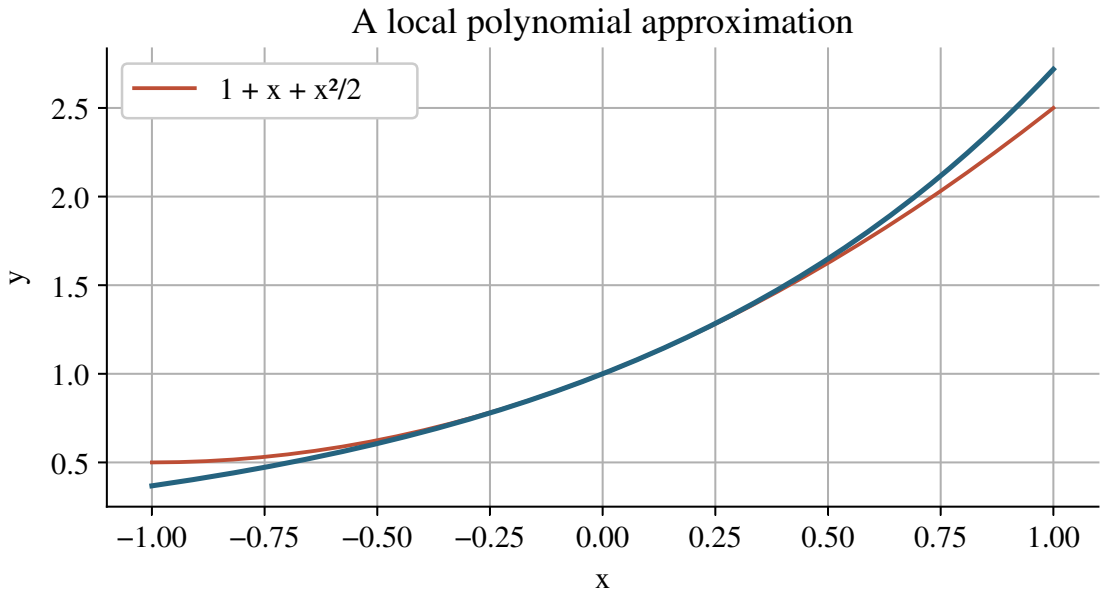
$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots$$

■ **Worked example**

The first three terms of e^x are $1+x+x^2/2$. At x=0.1 this gives 1.105, close to e^0.1≈1.105170. For the sum of $k^2$ from k=1 to 5, n(n+1)(2n+1)/6 gives 5×6×11/6=55.



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## Where it goes wrong

- The factorial belongs in the denominator of every Taylor coefficient.
- The expansion centre is not always zero.
- An approximation close to its centre can be poor far away.

> ■ **Watch out**
>
> **Not true:** A Taylor polynomial is exact for the original function at every input. A Taylor polynomial is an approximation; it is usually accurate only near its centre.

## Applying it

- For a telescoping series, write several terms and identify both surviving ends.
- For a Taylor approximation, state its order and compare with a known value or a remainder estimate where required.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Taylor polynomial**: A polynomial formed from a function's derivatives at a chosen centre.
- About x=a, f(x)=f(a)+f prime(a)(x-a)+f double prime(a)(x-a)$^2$/2!+... when the expansion is valid.
- Watch for: the factorial belongs in the denominator of every Taylor coefficient.

## FP2 · Complex numbers and roots

## What if the root is not real?

- The equation x$^2$+1=0 has no real roots. Extending the number system lets us represent and calculate with both roots.
- This lesson studies **complex conjugate** 共轭复数: The number obtained by reversing the sign of the imaginary part.

---

## The method

- Write z=a+bi with i$^2$=-1.
- The conjugate is a-bi and z times its conjugate=a$^2$+b$^2$.
- The modulus is √(a$^2$+b$^2$); the argument needs the correct quadrant.
- Conjugate roots occur for polynomials with real coefficients.

---

## A worked case

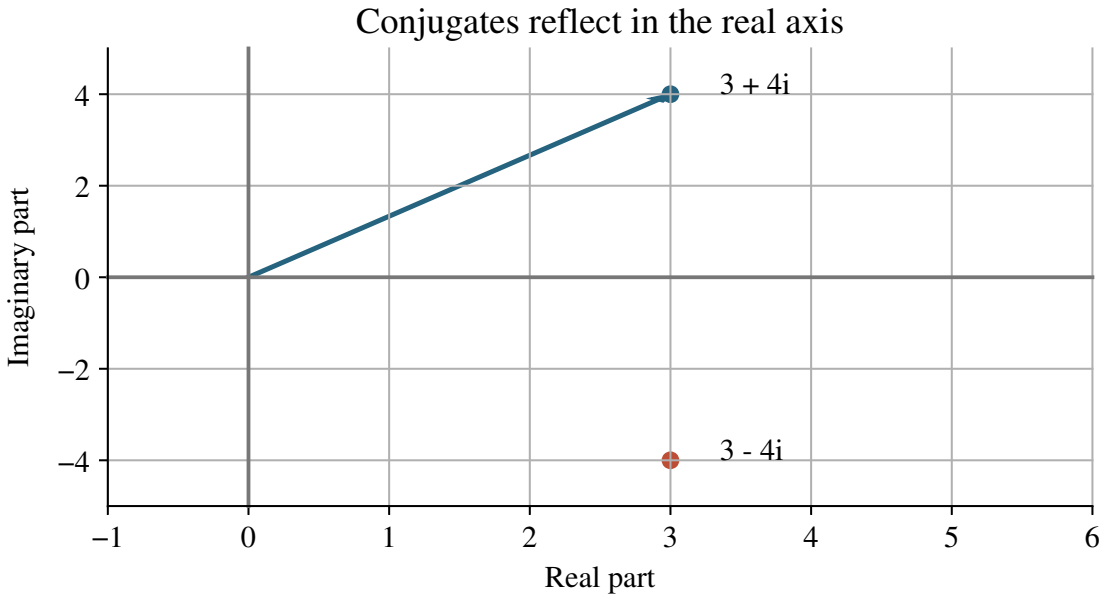
$$z = a + bi, \qquad |z| = \sqrt{a^2 + b^2}, \qquad z\overline{z} = a^2 + b^2$$

■ **Worked example**

For z=3+4i, |z|=5 and z conjugate=3-4i. The reciprocal is (3-4i)/25. For x$^2$-6x+13=0, the roots are 3±2i. Multiplying by the conjugate makes a complex denominator real.



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## Where it goes wrong

- The square root of a sum is not the sum of the square roots.
- An inverse tangent alone may return the wrong quadrant.
- Do not confuse modulus with squared modulus.

> ■ **Watch out**
>
> **Not true:** For a+bi, its modulus is a+b. The modulus is $\sqrt{(a^2+b^2)}$: for 3+4i it is 5, not 7.

## Applying it

- Draw the real axis horizontally and imaginary axis vertically.
- For products, moduli multiply and arguments add; powers extend this geometric pattern through de Moivre's theorem.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.



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■ **Key idea**

- **Complex conjugate**: The number obtained by reversing the sign of the imaginary part.
- Write z=a+bi with $i^2$=-1.
- Watch for: the square root of a sum is not the sum of the square roots.

## FP2 · Differential equations and numerical solutions

## What does the starting value decide?

- A growing population's rate is proportional to its current size. The rate equation describes a whole family until an initial population is supplied.
- This lesson studies **initial condition** 初始条件: A specified value that selects a particular solution from a family.

---

## The method

- For dy/dx=ky, separate variables: dy/y=k dx, giving y=Ae^(kx).
- Use an initial condition to find A.
- Euler's method takes y next=y+h f(x,y), with a chosen step h.

---

## A worked case

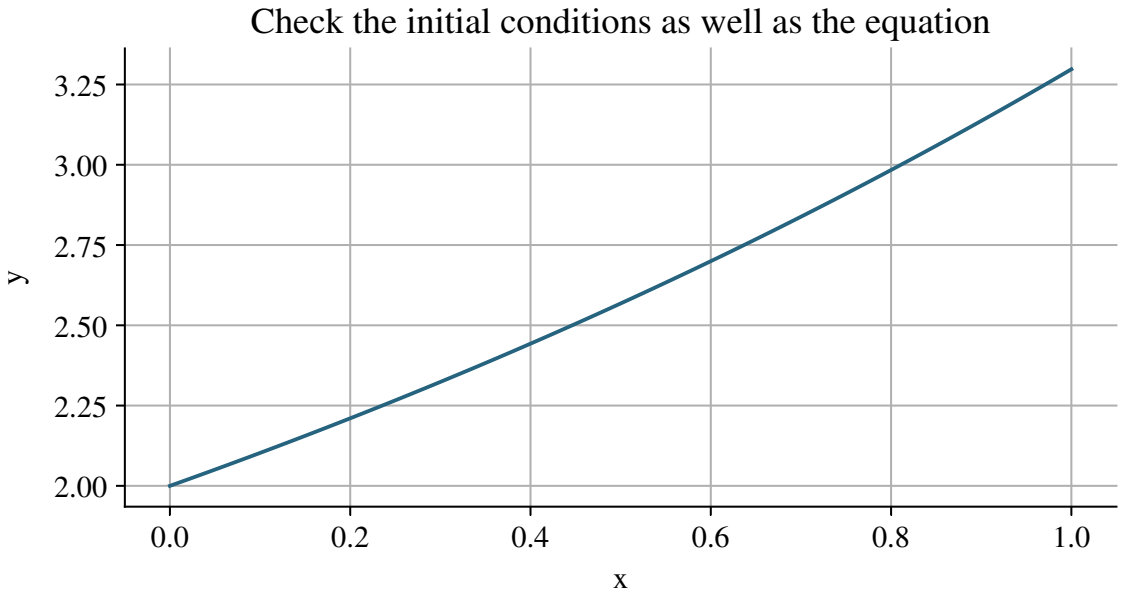
$$\frac{dy}{dx} = ky, \qquad y = Ae^{kx}$$

■ **Worked example**

With dy/dx=0.5y and y(0)=2, y=2e^(0.5x). Euler with h=0.2 gives y(0.2)≈2+0.2×1=2.2 and y(0.4)≈2.2+0.2×1.1=2.42. The exact second value is about 2.4428.



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## Where it goes wrong

- An initial condition determines the integration constant; it is not a replacement for integrating.
- Dividing by y can omit an equilibrium solution y=0.
- Euler accuracy depends on step size and the equation.

> ■ **Watch out**
>
> **Not true:** Every step size gives exactly the same Euler approximation. Euler's method depends on the step size; smaller steps usually give a more accurate value.

## Applying it

- For second-order linear equations, combine the complementary function with an appropriate particular integral, then apply the required initial conditions.
- This is Further Pure scope; check the named unit.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Initial condition**: A specified value that selects a particular solution from a family.
- For dy/dx=ky, separate variables: dy/y=k dx, giving y=Ae^(kx).
- Watch for: an initial condition determines the integration constant; it is not a replacement for integrating.

## FP2 · Second-order linear differential equations

## Why do we need two initial conditions?

- A displacement model involves both velocity and acceleration. Solving only a first-order rate equation cannot capture both initial conditions.
- This lesson studies **complementary function** 互补函数: The general solution of the associated homogeneous linear differential equation.

---

## The method

- For y double prime+ay prime+by=f(x), solve the auxiliary quadratic for the homogeneous part.
- Add a suitable particular integral for the forcing term.
- Repeated and complex roots require their correct forms.

---

## A worked case

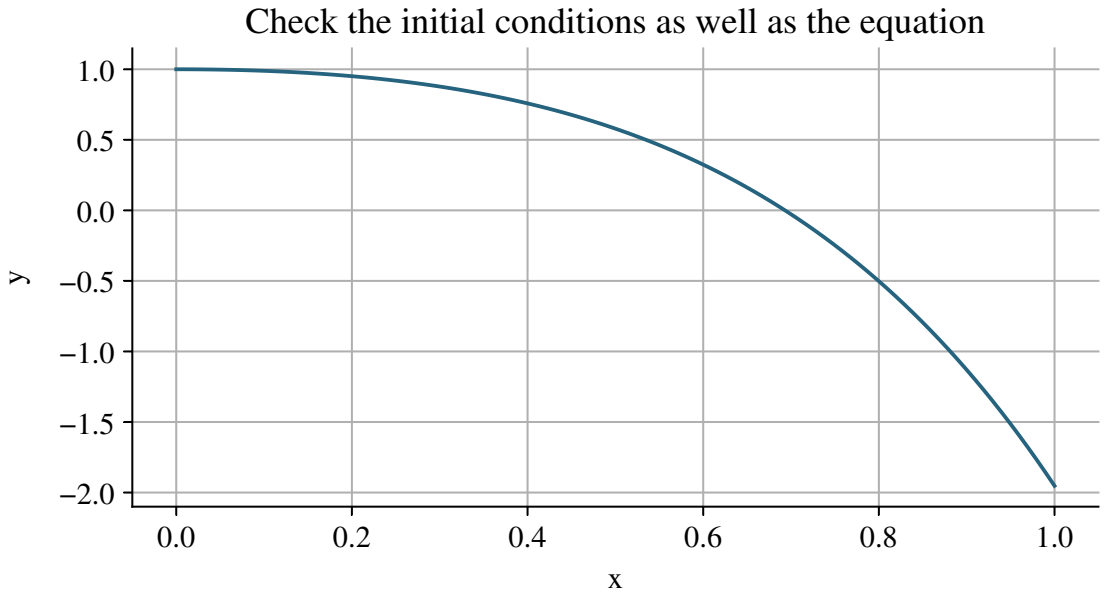
$$y'' + ay' + by = f(x), \qquad m^2 + am + b = 0$$

■ **Worked example**

For y double prime-3y prime+2y=0, the auxiliary equation is $m^2$-3m+2=0 with roots 1,2. Hence y=Ae^x+Be^(2x). With y(0)=1 and y prime(0)=0, A+B=1 and A+2B=0, so A=2,B=-1.



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## Where it goes wrong

- Two arbitrary constants need two independent conditions.
- If the trial particular integral duplicates a complementary-function term, multiply the trial by x as required.
- Do not discard a valid oscillatory solution because the auxiliary roots are complex.

> ■ **Watch out**
>
> **Not true:** One initial value is sufficient to determine both constants in a second-order general solution. A second-order solution has two constants, so it needs two conditions.

## Applying it

- Differentiate the final expression and substitute into the original differential equation.
- Check both initial conditions separately, and interpret the permitted solution interval.


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## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

### ■ Key idea

- **Complementary function**: The general solution of the associated homogeneous linear differential equation.
- For y double prime+ay prime+by=f(x), solve the auxiliary quadratic for the homogeneous part.
- Watch for: two arbitrary constants need two independent conditions.

## FP2 · Polar curves and area

## Why describe a curve by direction?

- A petal-shaped curve is simpler when its radius depends on direction. Cartesian coordinates can hide this structure.
- This lesson studies **polar coordinate** 极坐标: A position described by distance r and angle $\theta$ from a chosen origin and reference ray.

---

## The method

- Use x=r cos$\theta$ and y=r sin$\theta$.
- A polar area is one half the integral of $r^2$ with respect to $\theta$ over a correctly chosen interval.
- Identify symmetry and repeated tracing before selecting limits.

---

## A worked case

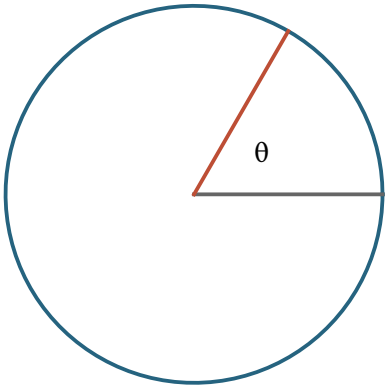
$$x = r\cos\theta, \quad y = r\sin\theta, \quad A = \frac{1}{2}\int r^2\, d\theta$$

### ■ Worked example

For r=2 and $\theta=\pi/3$, x=1 and y=$\sqrt{3}$. For the circle r=2 over a complete turn, A=(1/2) integral_0^(2$\pi$) 4 d$\theta$=4$\pi$. A half-turn gives 2$\pi$, exactly half the disk.


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A radius fixes distance from the centre




radius × angle gives arc length

## Where it goes wrong

- Negative r places a point in the opposite direction; it is not an ordinary negative distance along the same ray.
- A parametrisation may trace the same region more than once, so a full parameter interval can overcount area.

### ■ Watch out

**Not true:** Using any full parameter interval counts each polar region exactly once. The interval must match the curve's period; too long an interval counts a region twice.

## Applying it

- Sketch enough points to establish orientation and bounds.
- For an enclosed region between two curves, determine intersections and which radial square contributes the outer boundary on each interval.



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## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

### Key idea

- **Polar coordinate**: A position described by distance r and angle $\theta$ from a chosen origin and reference ray.
- Use x=r cos$\theta$ and y=r sin$\theta$.
- Watch for: negative r places a point in the opposite direction; it is not an ordinary negative distance along the same ray.

# FP3 · Hyperbolic functions and inverse relations

## How can growth and decay make a symmetric curve?

- A hanging cable has a curved profile related to exponentials. Hyperbolic functions combine growth and decay symmetrically.
- This lesson studies **hyperbolic cosine** 双曲余弦: The function cosh x=(e^x+e^(-x))/2.

---

## The method

- Define sinh x=(e^x-e^(-x))/2 and cosh x=(e^x+e^(-x))/2.
- Their identity is cosh$^2$x-sinh$^2$x=1.
- Derivatives are sinh prime=cosh and cosh prime=sinh.

---

## A worked case

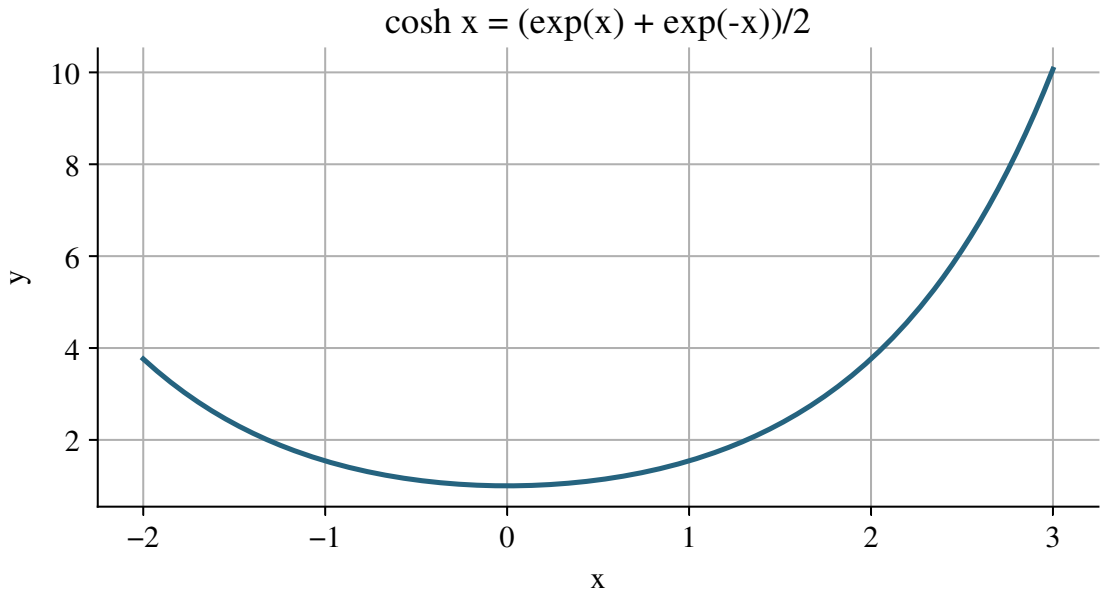
$$\cosh x = \frac{e^{x} + e^{-x}}{2}, \qquad \cosh^{2} x - \sinh^{2} x = 1$$

### Worked example

At x=ln2, e^x=2 and e^(-x)=1/2. Thus cosh x=1.25 and sinh x=0.75. Their squared difference is 1.5625-0.5625=1. To invert y=sinh x, solve a quadratic in e^x and choose the positive root.



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## Where it goes wrong

- The hyperbolic identity has a minus sign. cosh is not one-to-one on all real inputs; its usual inverse uses $x \geq 0$.
- Ordinary circular-trigonometric identities cannot be substituted unchanged.

> ■ **Watch out**
>
> **Not true:** The identity for hyperbolic functions is $\cosh^2 x + \sinh^2 x = 1$. The hyperbolic identity is $\cosh^2 x - \sinh^2 x = 1$.

## Applying it

- Use exponential definitions to prove identities and solve equations.
- State domain restrictions for inverse functions before differentiating or integrating them.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Hyperbolic cosine**: The function cosh x=(e^x+e^(-x))/2.
- Define sinh x=(e^x-e^(-x))/2 and cosh x=(e^x+e^(-x))/2.
- Watch for: the hyperbolic identity has a minus sign. cosh is not one-to-one on all real inputs; its usual inverse uses x≥0.

## FP3 · Conics, parametric curves and tangent reasoning

## How can a rotating parameter trace an ellipse?

- An ellipse can be traced by a rotating parameter even when it is awkward to describe as a single y=f(x).
- This lesson studies **parametric equation** 参数方程: A representation in which coordinates are expressed using a common parameter.

---

## The method

- For x=a cos t,y=b sin t, eliminating t gives $x^2/a^2+y^2/b^2=1$.
- For parametric curves, dy/dx=(dy/dt)/(dx/dt), where dx/dt≠0.
- A zero denominator may signal a vertical tangent.

---

## A worked case

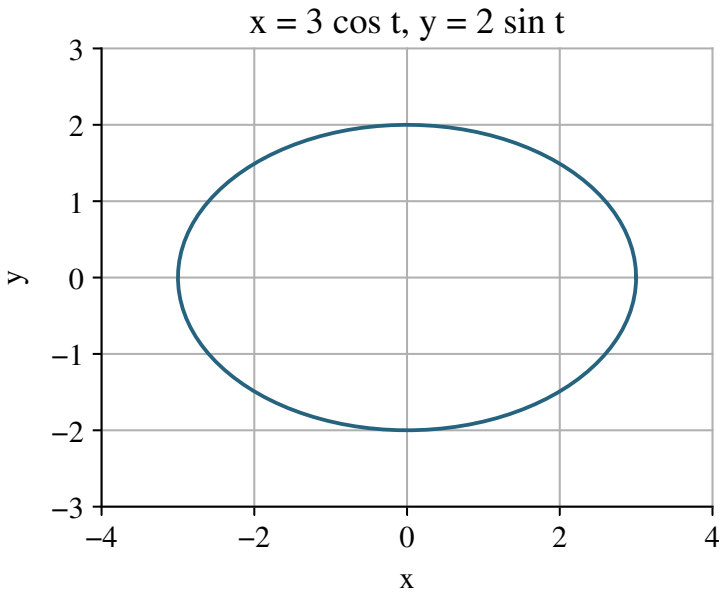
$$x = a\cos t, \qquad y = b\sin t, \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

■ **Worked example**

For x=3cos t,y=2sin t at t=π/4, the gradient is (2cos t)/(-3sin t)=-2/3. The point is (3/√2,√2). The area inside the ellipse is πab=6π; its semiaxes are 3 and 2.



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## Where it goes wrong

- Eliminating a parameter may lose a domain restriction or direction of travel.
- A vertical tangent cannot be assigned a finite dy/dx.
- Distinguish semiaxes from full widths.

> ■ **Watch out**
>
> **Not true:** Eliminating a parameter preserves every restriction and direction automatically. Elimination can lose restrictions: $x=t^2$, $y=t$ gives $x=y^2$, but $x\geq 0$ and the direction of motion must be stated separately.

## Applying it

- For advanced coordinate geometry, use the defining equation and a consistent parameter.
- For IAL P4 parametric integration, use the specification's restricted requirements rather than importing all Further Pure conics.


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## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

### Key idea

- **Parametric equation**: A representation in which coordinates are expressed using a common parameter.
- For x=a cos t,y=b sin t, eliminating t gives $x^2/a^2+y^2/b^2=1$.
- Watch for: eliminating a parameter may lose a domain restriction or direction of travel.

# FP3 · Chain, product, quotient and implicit differentiation

## Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
- This lesson studies **chain rule** 链式法则: The rule that multiplies the outer derivative by the inner derivative for a composite function.

---

## The method

- For y=f(g(x)), y prime=f prime(g(x))g prime(x).
- For uv, differentiate to u prime v+uv prime.
- For u/v, use (u prime v-uv prime)/$v^2$, where $v\neq0$.

---

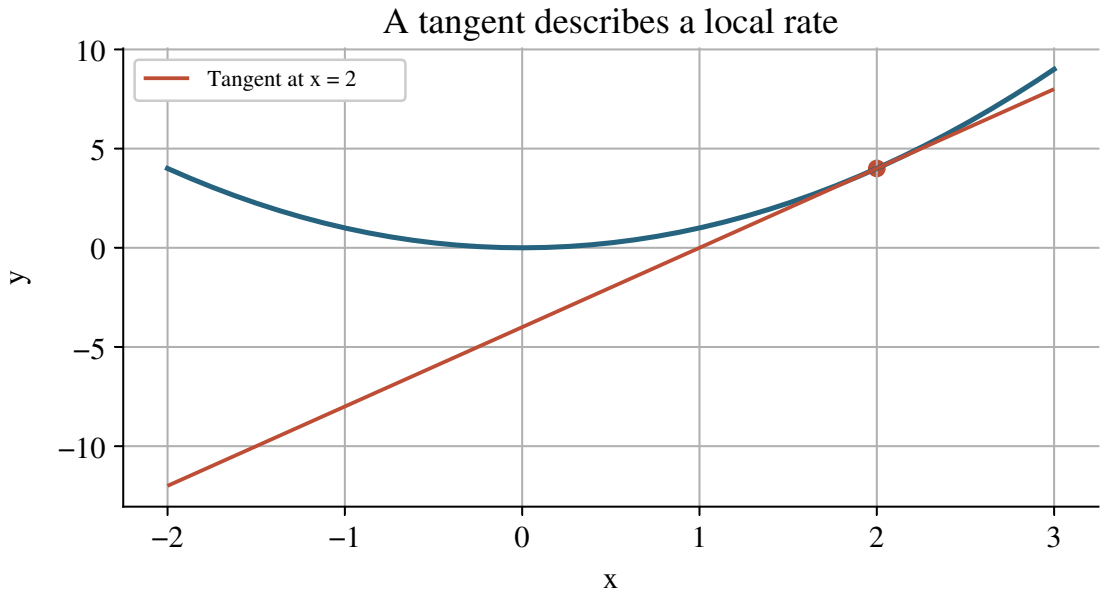
## A worked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

### Worked example

For y=(3x+1)^4, y prime=4(3x+1)^3×3=12(3x+1)^3. At x=0, the gradient is 12. For $x^2+y^2=25$, differentiate implicitly: 2x+2y y prime=0, so y prime=-x/y when $y\neq0$.


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## Where it goes wrong

- A derivative of a product is not the product of the derivatives.
- In implicit differentiation, every differentiated function of y brings a dy/dx factor.
- A quotient's denominator is squared.

> ■ **Watch out**
>
> **Not true:** The derivative of u(x)v(x) is u prime times v prime. The product rule is $(uv)'=u'v+uv'$: for $x \cdot x$ the derivative is 2x, not $1\times1=1$.

## Applying it

- Choose a useful form before differentiating: expanding a short polynomial may be simpler.
- For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.


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■ **Key idea**

- **Chain rule**: The rule that multiplies the outer derivative by the inner derivative for a composite function.
- For y=f(g(x)), y prime=f prime(g(x))g prime(x).
- Watch for: A derivative of a product is not the product of the derivatives.

## FP3 · Substitution, parts and partial fractions

## Can we integrate the two factors separately?

- A simple-looking product such as xe^x cannot be integrated by separately integrating each factor.
- This lesson studies **integration by parts** 分部积分法: An integration method based on the derivative of a product.

---

## The method

- Use substitution when an inner derivative appears as a factor.
- By parts, integral u v prime =uv-integral u prime v.
- For a rational function, divide first if needed, then decompose into partial fractions.

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## A worked case

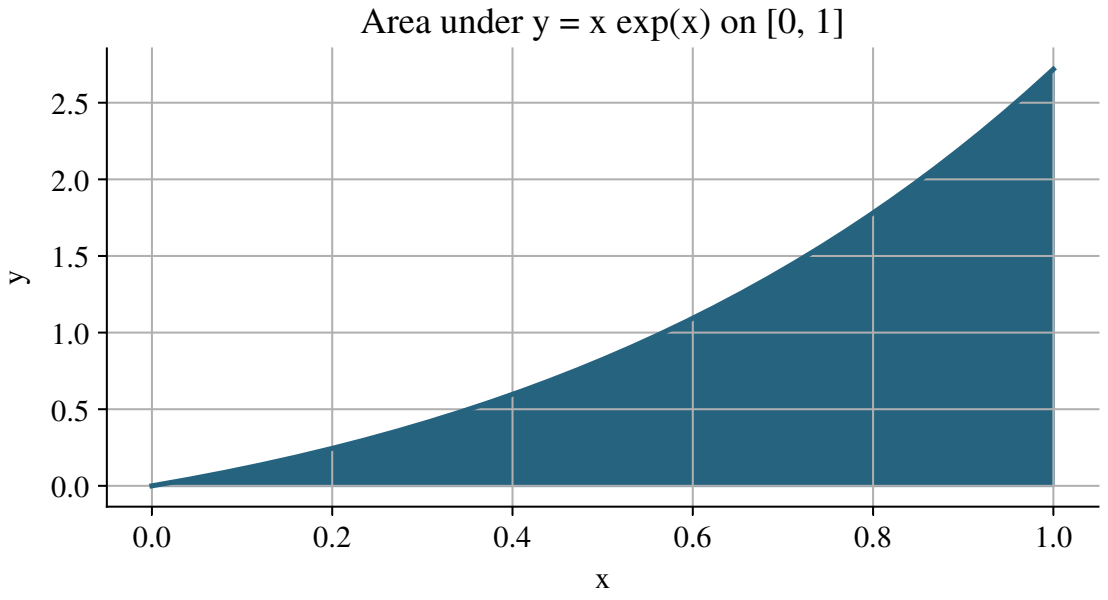
$$\int uv'\,dx = uv - \int u'v\,dx$$

■ **Worked example**

For integral xe^x dx, take u=x and v prime=e^x. Then the integral is xe^x-e^x+C. For integral $2x/(x^2+1)$ dx, substitute $w=x^2+1$, dw=2x dx; the answer is $\ln(x^2+1)+C$.



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## Where it goes wrong

- Choosing u and v prime well matters: the remaining integral should become simpler.
- In a definite substitution, either change the limits or return to x before applying the original limits.

> ■ **Watch out**
>
> **Not true:** The integral of a product equals the product of its separate integrals. A product needs a method such as integration by parts or substitution: $\int x \cdot x\,dx = x^3/3$, not $(x^2/2)(x^2/2)$.

## Applying it

- Check by differentiating.
- For volumes of revolution around the x-axis use $V = \pi \int y^2\,dx$; do not confuse the square of a function with the integral of the function.



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## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

### ■ Key idea

- **Integration by parts**: An integration method based on the derivative of a product.
- Use substitution when an inner derivative appears as a factor.
- Watch for: choosing u and v prime well matters: the remaining integral should become simpler.

# FP3 · Spatial vectors, lines and angles

## Can nonparallel paths still miss each other?

- Two paths in space can be neither parallel nor intersecting. A two-dimensional sketch may conceal their separation.
- This lesson studies **direction vector** 方向向量: A nonzero vector giving the direction of a line.

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## The method

- A line has r=a+$\lambda$b with position vector a and nonzero direction b.
- To find an intersection, equate all components and solve for both line parameters.
- For an angle use a · b=|a||b|cos$\theta$ where scalar products are in the named syllabus.

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## A worked case

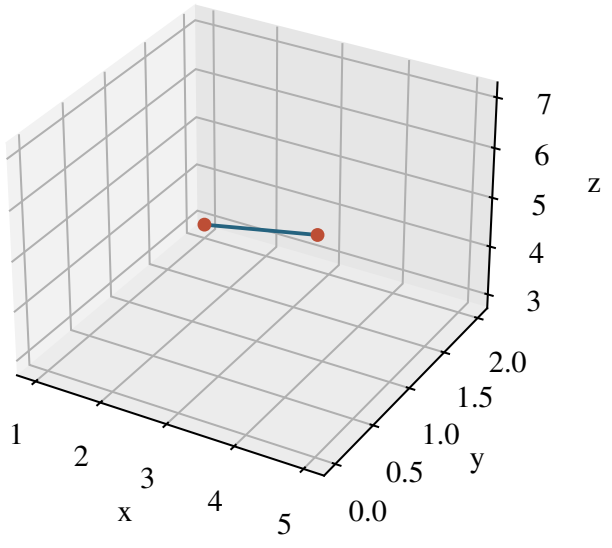
$$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}, \qquad \mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

### ■ Worked example

For a=(1,2,3),b=(2,-1,2), $\lambda$=2 gives r=(5,0,7). The magnitude of b is $\sqrt{}$(4+1+4)=3. With c=(1,0,-1), b · c=2-2=0, so b and c are perpendicular.


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$$\mathbf{r} = (1, 2, 3) + \lambda(2, -1, 2)$$




## Where it goes wrong

- Satisfying two component equations does not guarantee the third.
- Nonparallel lines may be skew.
- A line's direction vector is not its position vector, and a scalar parameter has no coordinate units.

> **Watch out**
>
> **Not true:** Any two nonparallel lines in three dimensions must intersect. In three dimensions nonparallel lines can be skew: they do not meet.

## Applying it

- State whether a requested angle is acute, directed or an angle between lines.
- Use the syllabus-specific plane or distance method only when it is actually required for that unit.

## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.



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■ **Key idea**

- **Direction vector**: A nonzero vector giving the direction of a line.
- A line has r=a+$\lambda$b with position vector a and nonzero direction b.
- Watch for: satisfying two component equations does not guarantee the third.

## FP3 · Matrix transformations, inverses and eigenvalues

## Where do the basis vectors go?

- A map stretches one direction and shears another. A matrix records the transformation of each basis vector.
- This lesson studies **determinant** 行列式: A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.

---

## The method

- A 2×2 matrix [[a,b],[c,d]] has determinant ad-bc and inverse [[d,-b],[-c,a]]/(ad-bc) if the determinant is nonzero.
- Transform column vectors by left multiplication.
- The rightmost matrix acts first in a product.

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## A worked case

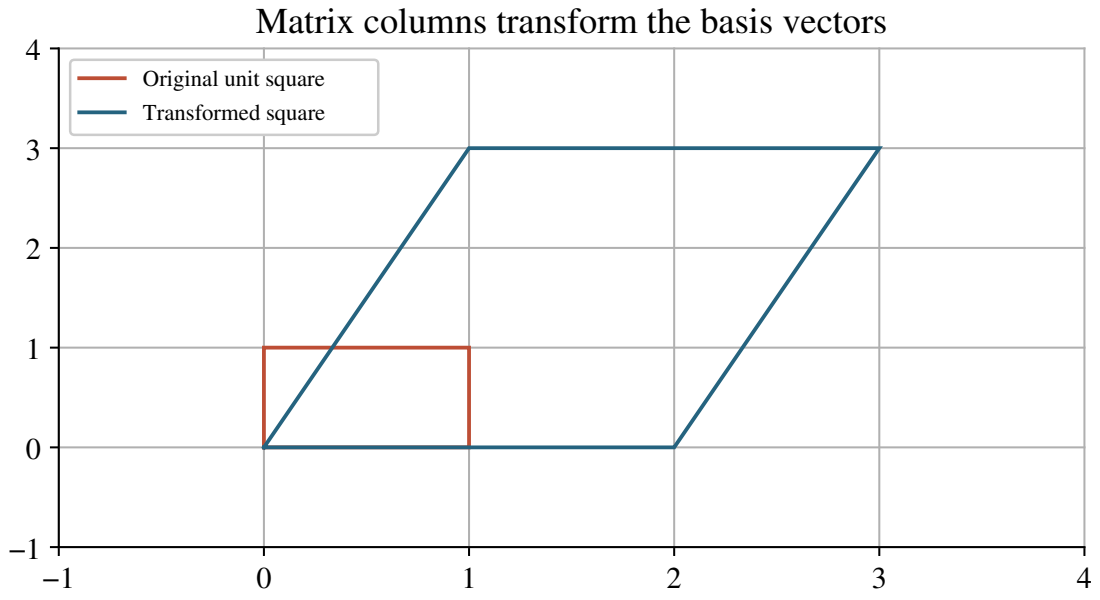
$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

■ **Worked example**

For A=[[2,1],[0,3]], det A=6. A(1,2)=(4,6). Its inverse is [[1/2,-1/6],[0,1/3]]. The characteristic equation is (2-$\lambda$)(3-$\lambda$)=0, so the eigenvalues are 2 and 3.



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## Where it goes wrong

- Matrix multiplication is usually not commutative.
- A determinant of zero means the transformation loses a dimension and has no inverse.
- Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.

### ■ Watch out

**Not true:** Every square matrix has an inverse. A square matrix with determinant 0 is singular and has no inverse.

## Applying it

- Find an eigenvector by solving $(A-\lambda I)v=0$ with $v\neq 0$.
- Explain the geometrical meaning: this vector keeps its line direction under the transformation.


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## In your course

- Pearson Edexcel International A Level Pure Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

### ■ Key idea

- **Determinant**: A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
- A 2×2 matrix [[a,b],[c,d]] has determinant ad-bc and inverse [[d,-b],[-c,a]]/(ad-bc) if the determinant is nonzero.
- Watch for: matrix multiplication is usually not commutative.



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