

## Unit 4: Further Mechanics, Fields and Particles

WPH14 · Issue 3 · requirements 81–124 · sheets 4.1–4.7

## One chain: force, motion and evidence

- A bumper and a particle detector both use momentum.
- WPH14: requirements 81–124, with Units 1–2 prerequisites.
- Use the seven skill sheets before the authentic sets.

## Momentum and impulse carry signs

### Momentum has direction

Choose a positive axis before using  $p = mv$ . A rebound makes final velocity negative if approach was positive.

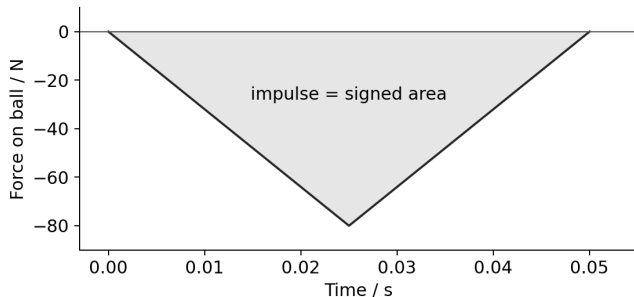
A system's momentum is conserved only when external impulse is negligible during the event.

### Impulse changes momentum

$F_{\text{av}}\Delta t = \Delta p = m(v - u)$ . Subtract initial from final signed momentum.

Signed force–time area gives impulse. For the same change, increasing stopping time reduces average force; it does not remove momentum change.

## A pulse gives signed impulse



- Signed area gives impulse:  $\Delta p = -2.0 \text{ N s}$ .
- Average force is area divided by time, not peak force.

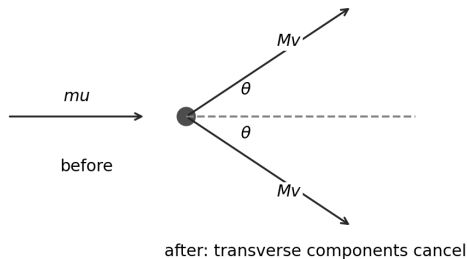
## Your turn: a rebound

- Mass 0.30 kg; approach  $+5.0$  m/s; rebound  $-3.0$  m/s.
- Contact time 0.040 s.
- Find impulse and average force before advancing.

## Rebound: use final minus initial

- 1 Known / model** Known:  $m = 0.30 \text{ kg}$ ,  $u = +5.0 \text{ m s}^{-1}$ ,  $v = -3.0 \text{ m s}^{-1}$ , contact  $0.040 \text{ s}$ . Use signed momentum change.
- 2 Impulse**  $\Delta p = m(v - u) = 0.30 \text{ kg} \times (-3.0 - 5.0) \text{ m/s} = -2.4 \text{ N s}$
- 3 Average force**  $F_{\text{av}} = \Delta p / \Delta t = (-2.4 \text{ N s}) / (0.040 \text{ s}) = -60 \text{ N}$

## Conserve two components, not magnitudes



- In an isolated system, conserve momentum along each perpendicular axis.
- Compare kinetic energies to decide whether the collision is elastic.

## Elasticity and the practical evidence

- 1 Source collision**  $0.10 \text{ kg}$  at  $2.0 \text{ m s}^{-1}$  stops; two equal stationary spheres leave at  $1.2 \text{ m s}^{-1}$  symmetrically.
- 2 Component equation**  $mu = 2Mv \cos \theta$ ;  $\cos \theta = mu/(2Mv) = 0.8333$ . Transverse components cancel.
- 3 Separation**  $2\theta = 2 \cos^{-1}(0.8333) = 67.1^\circ$ .
- 4 Kinetic energies**  $E_{k,i} = \frac{1}{2}mu^2 = 0.20 \text{ J}$ ;  $E_{k,f} = 2 \times \frac{1}{2}Mv^2 = 0.144 \text{ J}$ .
- 5 Classify** Kinetic energy decreases: inelastic. Momentum conservation did not require kinetic-energy conservation.

## CP9: compare independent measurements

Zero the force sensor; record the whole contact pulse with a sufficiently fast sampling rate. Use consistent signs.



Independently measure trolley mass and velocities immediately before/after contact using gates or a motion sensor.



Compare signed force–time area with  $m(v - u)$ . Repeat using different speeds or contact forces.



Impulse versus momentum-change gradient should be near one. Discuss friction, offsets and missed pulse; catch the trolley safely.

## CP10: calibrate a two-dimensional collision

Use a perpendicular overhead camera with a length scale in the collision plane and a known frame rate.



Track sphere centres over several frame intervals before and after contact. Convert positions to metres and times to seconds.



Resolve each velocity into two axes. Compare total momentum components and total kinetic energies separately.



Repeat approach directions; include position, scale and timing uncertainty. Contain spheres. A small mismatch within uncertainty does not refute conservation.

## Radians connect rotation to speed

- 1 **Convert rotations**  $f = N/t = 120/(60 \text{ s}) = 2.0 \text{ Hz}$
- 2 **Angular velocity**  $\omega = 2\pi f = 2\pi \times 2.0 \text{ s}^{-1} = 12.6 \text{ rad/s}$
- 3 **Tangential speed**  $v = \omega r = 4\pi \text{ s}^{-1} \times 0.25 \text{ m} = 3.14 \text{ m/s}$
- 4 **Acceleration**  $a = r\omega^2 = 0.25 \text{ m} \times (4\pi \text{ s}^{-1})^2 = 39.5 \text{ m/s}^2$

## One rigid turntable, different tangential speeds

### One rigid turntable

All points share the same angular speed:

$$\omega = \Delta\theta / \Delta t = 2\pi / T.$$

Use radians:  $\theta_{\text{rad}} = \theta_{\text{deg}}(2\pi/360)$ . One radian subtends an arc equal to the radius.

### Different radii

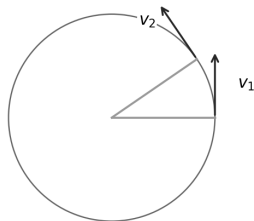
Tangential speed is  $v = \omega r$ : doubling radius doubles speed at fixed  $\omega$ .

Acceleration is  $a = r\omega^2$ : doubling radius also doubles inward acceleration.

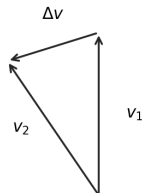
Velocity direction changes continuously.

# Derive the inward acceleration

Tangential velocity vectors



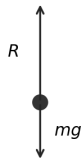
$$\Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$$



- Similar velocity and position triangles give  $\Delta v/v = \Delta s/r$ .
- As  $\Delta t \rightarrow 0$ ,  $a = v^2/r$  points towards the centre.

# Centripetal force is the resultant

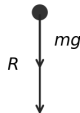
Bottom of drum



$$R - mg = mv^2/r$$

Top of drum

$$R + mg = mv^2/r$$



- Resolve actual forces towards the centre.
- The inward resultant supplies  $mv^2/r$ , do not add an extra force.

## Top and bottom need different force equations

### At the bottom

Inward is upward. Reaction points inward; weight points outward.

$$R - mg = mv^2/r, \text{ hence} \\ R = mv^2/r + mg.$$

### At the top

Inward is downward. Both reaction and weight point inward.

$$R + mg = mv^2/r, \text{ hence} \\ R = mv^2/r - mg. \text{ A negative required} \\ \text{reaction means the assumed contact has} \\ \text{failed.}$$

## Your turn: the friction limit

- A horizontal turntable rotates at  $2.0 \text{ rad/s}$ .
- Maximum friction is  $mg/20$ .
- Find the largest radius and decide whether doubling mass changes it.

## Mass cancels for this friction model

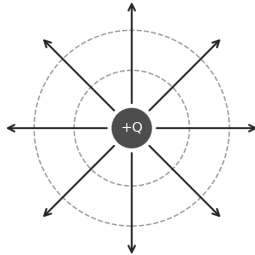
- 1 **Limit**  $mr\omega^2 \leq mg/20 \Rightarrow r_{\max} = g/(20\omega^2)$
- 2 **Substitute**  $r_{\max} = g/(20\omega^2) = (9.81 \text{ m/s}^2)/[20(2.0 \text{ s}^{-1})^2] = 0.123 \text{ m}$
- 3 **Interpret** Required force and maximum friction both scale with mass. Initial release motion is tangent.

## Field, potential and work are different

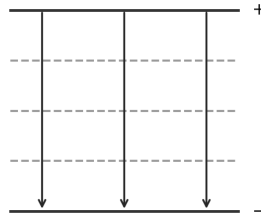
| Quantity                | Relationship                    | Meaning                |
|-------------------------|---------------------------------|------------------------|
| Electric field strength | $\mathbf{E} = \mathbf{F}/q$     | Vector                 |
| Electric potential      | $\Delta U = q\Delta V$          | Signed scalar          |
| Work by field           | $W_{\text{field}} = -q\Delta V$ | Opposite energy change |

# Radial and uniform representations

Radial field and equipotentials



Uniform field between plates



- Field lines point away from a positive source charge.
- Uniform plates give  $E = V/d$  away from edge effects.

## One sphere, two different distances

**1 Known / model** Known: sphere surface  $r = 0.20$  m at 9.0 kV; observation radius 0.30 m. Use  $V = kQ/r$  at the surface first.

**2 Surface charge**

$$Q = Vr/k = (9.0 \times 10^3 \text{ V})(0.20 \text{ m})/(8.99 \times 10^9 \text{ N m}^2/\text{C}^2) = 2.00 \times 10^{-7} \text{ C}$$

**3 Outside field**

$$E = kQ/r^2 = (8.99 \times 10^9 \text{ N m}^2/\text{C}^2)(2.00 \times 10^{-7} \text{ C})/(0.30 \text{ m})^2 = 2.00 \times 10^4 \text{ N/C}$$

## Potential comes from the correct area

### Potential gradient

The signed radial field is  $E_r = -dV/dr$ .  
For a positive source, potential decreases outwards.

Field lines cross equipotentials at right angles. Along an equipotential,  $\Delta U = q\Delta V = 0$ .

### Correct area / reference

With zero at infinity,  $V(r) = \int_r^\infty E_r dr$ .  
Integrate from the observation radius to infinity.

The area from zero to  $r$  is not this potential. Closer equal potential contours indicate stronger field.

## Your turn: hold an oil drop

- $m = 6.52 \times 10^{-15}$  kg; field  $2.00 \times 10^5$  N/C downwards.
- The drop is stationary; neglect other forces.
- Find signed charge and the number of excess or missing electrons.

## Separate magnitude from charge sign

- 1 Known / model** Known:  $m = 6.52 \times 10^{-15} \text{ kg}$ ; field  $2.00 \times 10^5 \text{ N/C}$  downwards.  
Upward electric force balances weight, requiring negative charge.
- 2 Force balance**  
 $|q| = mg/E = [(6.52 \times 10^{-15} \text{ kg})(9.81 \text{ m/s}^2)]/(2.00 \times 10^5 \text{ N/C}) = 3.20 \times 10^{-19} \text{ C}$
- 3 Sign and count**  $q < 0$ ;  $n = |q|/e = (3.20 \times 10^{-19} \text{ C})/(1.60 \times 10^{-19} \text{ C}) = 2$

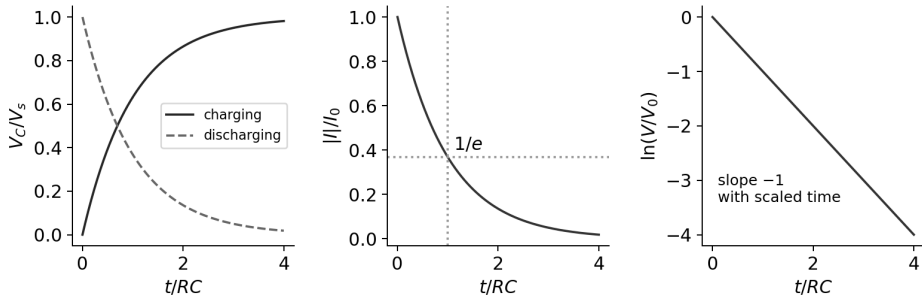
## A dust-lift threshold needs volume and direction

- 1 **Model / geometry** A spherical grain of density  $\rho$  and radius  $r$  has  $m = \rho(4\pi r^3/3)$ . A listed diameter is  $2r$ .
- 2 **Horizontal threshold** For an upward electric force, detachment requires  $|q|E \geq mg$ . At the limiting size use equality.
- 3 **Tilted panel** If the field is normal to a panel inclined by  $\theta$ , balance the normal weight component:  $|q|E = mg \cos \theta$ .
- 4 **Decision** Convert the calculated radius into diameter. Select the largest listed diameter below the threshold; do not round up beyond it.

## Capacitance and the half in energy

- 1 Charge definition**  $C = Q/V$ : quoted  $Q$  is the magnitude on either plate, not the sum of their charge magnitudes.
- 2 Energy model** For constant  $C$ , the  $V$ - $Q$  graph is straight. Work is its triangular area because voltage rises during charging.
- 3 Equivalent forms**  $W = \frac{1}{2} QV = \frac{1}{2} CV^2 = Q^2/(2C)$ .
- 4 Energy change**  $\Delta W = \frac{1}{2} C(V_i^2 - V_f^2)$ . Subtract stored energies; squaring the voltage difference gives a different, incorrect result.

## Different quantities need different RC curves



- Discharge voltage decays exponentially towards zero.
- A plot of  $\ln(V/V_0)$  against time has gradient  $-1/(RC)$ .

## Your turn: discharge and lost energy

- $C = 100 \mu\text{F}$ ,  $R = 20.0 \text{ k}\Omega$ ,  $V_0 = 12.0 \text{ V}$ .
- Find voltage after 3.0 s and stored energy lost.
- Convert prefixes; retain the unrounded voltage for energy.

## Exponential voltage; difference of energies

**1 Known / model** Known:  $C = 100 \mu\text{F}$ ,  $R = 20.0 \text{ k}\Omega$ ,  $V_0 = 12.0 \text{ V}$ ,  $t = 3.0 \text{ s}$ .

Use fixed- $RC$  discharge.

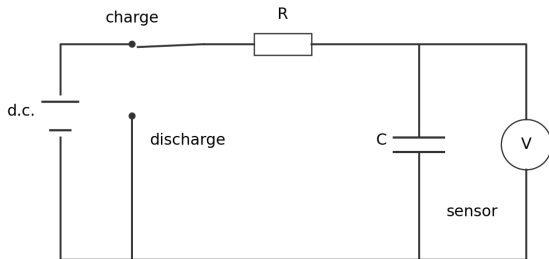
**2 Time constant**  $\tau = RC = (20.0 \times 10^3 \Omega)(100 \times 10^{-6} \text{ F}) = 2.00 \text{ s}$

**3 Voltage**  $V = V_0 e^{-t/RC} = 12.0 \text{ V} \times e^{-3.0/2.00} = 2.68 \text{ V}$

**4 Energy lost**

$$\Delta W = \frac{1}{2} C (V_0^2 - V^2) = \frac{1}{2} (100 \times 10^{-6} \text{ F}) (12.0^2 - 2.68^2) \text{ V}^2 = 6.84 \text{ mJ}$$

## CP11: measure the capacitor, not the supply



- Connect the voltmeter across the capacitor.
- Isolate the supply before timing discharge through the resistor.

## Use a log fit to test resistance

- 1 Discharge model**  $V = V_0 e^{-t/RC}$ , hence  $\ln(V/V_0) = -t/(RC)$ .
- 2 Log fit** Plot the dimensionless log ratio vertically against time: gradient  $m = -1/(RC)$ , so  $R = -1/(Cm)$ .
- 3 Two readings** Within one uninterrupted discharge,  $R = -\Delta t/[C \ln(V_2/V_1)]$ . Restore the same initial charge for repeats.
- 4 Test / limits** Compare calculated  $R$  with both permitted tolerance limits. Avoid near-zero voltages; sensor input resistance can load the discharge path.

## RC extensions retain the same charge model

### Smoothing a varying supply

The capacitor charges near supply peaks and discharges through the load between them.

Smaller load resistance makes  $RC$  smaller: voltage falls faster between peaks, giving poorer smoothing.

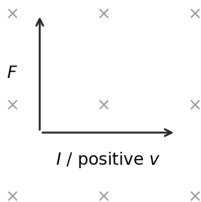
### Identical series capacitors

The same charge magnitude appears on each capacitor; equal capacitances share the supply voltage equally.

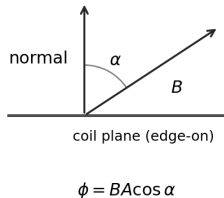
Each has  $Q = C(V_s/2)$ , half the charge of one identical capacitor alone across  $V_s$ . Keep voltage rating and polarity within limits.

# Force angle and flux angle are different

Field into page; force upwards



Flux uses angle to area normal



- Magnetic flux density  $B$ : force uses the angle between velocity and field.
- Magnetic flux  $\phi$ : use the angle between field and surface normal.
- Fleming's left-hand rule gives force for conventional current.

## Flux linkage counts every linked turn

- 1 **Known / model** Known: 40 turns, radius 0.015 m, field 0.020 T normal to the coil plane. Thus  $\phi = BA$ .
- 2 **Coil area**  $A = \pi r^2 = \pi(0.015 \text{ m})^2 = 7.07 \times 10^{-4} \text{ m}^2$
- 3 **Flux linkage**  $N\phi = NBA = 40(0.020 \text{ T})(7.07 \times 10^{-4} \text{ m}^2) = 5.65 \times 10^{-4} \text{ Wb}$

## A balance reads the reaction on the magnets

A separately supported current-carrying wire in the field experiences  $F = BIl \sin \theta$ .  
Only the wire length inside the field counts.



The magnets experience an equal, opposite force. The balance reads this reaction, not the wire's own mass.



Convert the reading change to force magnitude:  $|F| = |\Delta m|g$ . Convert grams to kilograms before using SI units.



Use field and conventional-current directions to determine the reaction direction. Reverse the current to test the sign change.

## Your turn: a changing flux

- 50 turns; flux per turn falls from  $6.0 \times 10^{-5} \text{ Wb}$  to  $2.0 \times 10^{-5} \text{ Wb}$ .
- The change takes 0.020 s.
- Find average e.m.f. magnitude and describe the opposing magnetic effect.

## Faraday magnitude; Lenz direction

- 1 **Faraday's law**  $|\mathcal{E}| = N|\Delta\phi|/\Delta t = 50|2.0 - 6.0| \times 10^{-5} \text{ Wb}/(0.020 \text{ s}) = 0.10 \text{ V}$
- 2 **Lenz's law** Linked flux decreases; induced current, if a closed circuit exists, helps maintain it.

## Moving boundaries turn speed into a flux rate

- 1 Geometry** A straight field boundary sweeps width  $l$  at speed  $v$  in a uniform perpendicular field.
- 2 Area rate**  $\Delta A / \Delta t = lv$ .
- 3 Flux-linkage rate**  $N\phi = NBA$ , so  $|\mathcal{E}| = NB|\Delta A| / \Delta t = NBlv$ .
- 4 Interpret** Faster motion increases the rate, not merely the existing flux. Reversing a magnetic strip's pole orientation reverses the signal sign.

## Wireless charging needs changing linkage

Alternating current in the transmitting coil creates a magnetic field that changes with time.



Some changing flux links the receiving phone coil. Faraday's law gives an induced e.m.f. from the linkage-change rate.



A receiving circuit uses that induced supply to charge the battery. A constant linked field alone would not induce e.m.f.



Separation and incomplete magnetic coupling mean some flux misses the receiver. Equal turn counts do not guarantee equal voltages.

## Scattering links observation to a model

| Observation / symbol      | Supported conclusion                                      |
|---------------------------|-----------------------------------------------------------|
| Most pass nearly straight | Most atomic volume is empty                               |
| Rare large deflections    | Tiny, dense, positive nucleus                             |
| ${}^A_ZX$                 | Proton number $Z$ ; nucleon number $A$ ; neutrons $A - Z$ |

## Why a distributed positive charge was inadequate

### Distributed positive charge

Positive charge spread throughout an atom can give predominantly small alpha-particle deflections.

It cannot account for the rare very large deflections observed from a thin foil.

### Concentrated nucleus

Strong repulsion in a close encounter requires a small, concentrated positive region containing most mass.

Most particles pass nearly straight: most volume is empty. Few close encounters imply that the dense region occupies very little volume.

## Thermionic emission supplies electrons

Heating the metal gives some electrons enough energy to leave its surface: thermionic emission supplies the beam.



A positive electrode attracts electrons through a vacuum. Electric work changes their kinetic energy:  $\Delta E_k = q(V_i - V_f)$ .



For acceleration from rest well below light speed,  $\frac{1}{2}mv^2 = |q||\Delta V|$ . Use the actual potential direction.



Magnetic steering then changes velocity direction without increasing kinetic energy. It is a different role from electric acceleration.

## From wavelength to acceleration voltage

- 1 Given / model** Starting with an electron de Broglie wavelength, use  $p = h/\lambda$ . The kinetic-energy relation below is non-relativistic.
- 2 Energy**  $E_k = p^2/(2m_e) = h^2/(2m_e\lambda^2)$ .
- 3 Accelerating p.d.** If the electron starts from rest,  $eV = E_k$ , so  $V = h^2/(2m_e e\lambda^2)$ .
- 4 Validity** Check that the implied speed remains well below light speed. Starting with initial kinetic energy would require an energy change, not the final energy alone.

## Linear accelerator: timing determines tube length

An alternating p.d. creates accelerating electric fields across gaps between conducting drift tubes.



Inside a tube, the particle is shielded while the polarity reverses; the next gap must accelerate it again.



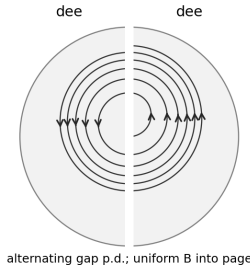
At fixed alternating frequency, the time between gaps stays fixed. As speed rises, later tubes must be longer.



Across  $n$  equal accelerating gaps, energy gain is  $n|q|V_{\text{gap}}$ . Count gaps, not simply the number of tubes.

## Cyclotron: electric acceleration, magnetic steering

Cyclotron: radius grows after each gap



- The gap electric field increases kinetic energy.
- The magnetic field bends motion without doing work.

## Your turn: radius and half-turn time

- Proton:  $p = 2.0 \times 10^{-20} \text{ kg m/s}$ ;  $B = 0.40 \text{ T}$ .
- $m_p = 1.67 \times 10^{-27} \text{ kg}$ ;  $e = 1.60 \times 10^{-19} \text{ C}$ .
- Find radius and time in one dee; distinguish half period from full period.

## Radius grows; the classical period does not

**1 Known / model** Known: proton momentum  $2.0 \times 10^{-20} \text{ kg m/s}$  perpendicular to  $0.40 \text{ T}$ . Magnetic force supplies circular acceleration.

**2 Radius**

$$r = p/(Be) = (2.0 \times 10^{-20} \text{ kg m/s})/[(0.40 \text{ T})(1.60 \times 10^{-19} \text{ C})] = 0.313 \text{ m}$$

**3 Period**  $T = 2\pi r/v = 2\pi m/(Be)$

**4 One dee**

$$t_{\text{dee}} = \pi m/(Be) = \pi(1.67 \times 10^{-27} \text{ kg})/[(0.40 \text{ T})(1.60 \times 10^{-19} \text{ C})] = 8.20 \times 10^{-8} \text{ s}$$

## Tracks provide evidence with limits

| Detector evidence          | Supported inference / limit                                                                     |
|----------------------------|-------------------------------------------------------------------------------------------------|
| Ionisation track           | A charged particle travels through matter; a neutral product need not leave a direct track.     |
| Perpendicular-field radius | Use $p = B q r$ . Comparing radii requires the charge magnitudes.                               |
| Missing vector momentum    | A neutral product may carry the missing component; draw a vector balance.                       |
| High collision energy      | Short de Broglie wavelength probes smaller structure; energy can also create massive particles. |

## Curvature does not supply every missing direction

### Curvature direction

Charge sign requires magnetic-field direction and the particle's travel direction. A constant-radius arc alone supplies no travel arrow.

A tightening track often indicates falling momentum as the particle loses energy.

### Fast muons

Their mean lifetime measured in the Earth frame increases at speeds close to  $c$ , so more reach the ground.

They remain below light speed. This unit requires the significance of the observation, not relativistic calculations.

## The quark–lepton model organises particles

| Family              | Constituents / example   |
|---------------------|--------------------------|
| Baryon              | Three quarks; proton uud |
| Meson               | Quark + antiquark; pion  |
| Lepton / antilepton | Electron / positron      |
| Photon              | Electromagnetic quantum  |

## Quark charges make testable predictions

- 1 Quark charges** Up-type quarks carry  $+2e/3$ ; down-type quarks carry  $-e/3$ . Antiquarks reverse these signs.
- 2 Proton**  $Q_{uud} = 2e/3 + 2e/3 - e/3 = +e$ .
- 3 Neutron**  $Q_{udd} = 2e/3 - e/3 - e/3 = 0$ .
- 4 Prediction** Symmetry between quark families predicted the top quark before observation: the classification made a testable prediction.

## Antiparticle: more than opposite charge

| Property          | Electron versus positron                                                                                                        |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------|
| Rest mass         | Equal rest masses; antiparticles are not negative-mass particles.                                                               |
| Electric charge   | Electron $-e$ ; positron $+e$ .                                                                                                 |
| Lepton number     | Electron $+1$ ; positron $-1$ . Both have baryon number zero.                                                                   |
| Neutral particles | Zero electric charge alone cannot establish that a particle is identical to its antiparticle. Other quantum numbers can differ. |

## Your turn: repair a decay equation

- Candidate:  $n \rightarrow p + e^- + \nu_e$ .
- Compare charge, baryon and lepton totals.
- If one total fails, choose the corrected neutrino symbol.

# The bar changes lepton number

| <b>Total</b> | $n$ | $p + e^- + \bar{\nu}_e$ |
|--------------|-----|-------------------------|
| Charge / e   | 0   | $+1-1+0 = 0$            |
| Baryon       | 1   | $1+0+0 = 1$             |
| Lepton       | 0   | $0+1-1 = 0$             |

## Pair production and annihilation

### Pair production

Energy creates a particle–antiparticle pair; at least both rest energies must be supplied.

$\Delta E = c^2 \Delta m$ . Additional energy can supply kinetic energy or recoil; momentum conservation also constrains the interaction.

### Annihilation at rest

An equal-mass particle pair initially at rest can produce two photons with equal energies and opposite momenta.

Each photon carries one particle's rest energy; total energy is twice that value. Do not use  $p = mv$  for a photon.

## MeV/c squared is a mass unit

- 1 **Rest energy**  $mc^2 = 140 \times 10^6 \times 1.60 \times 10^{-19} \text{ J} = 2.24 \times 10^{-11} \text{ J}$
- 2 **Mass**  $m = E/c^2 = (2.24 \times 10^{-11} \text{ J}) / (3.00 \times 10^8 \text{ m/s})^2 = 2.49 \times 10^{-28} \text{ kg}$
- 3 **Compare** % difference =  $100|m_{\text{predicted}} - m_{\text{reference}}|/m_{\text{reference}}$

## Check yourself

- At the top of a vertical circle, which actual forces supply the inward resultant?
- During RC discharge, what does a log-voltage gradient measure?
- Can a magnetic field alone increase a particle's kinetic energy? Explain.
- A proposed decay conserves charge: what else must you test?