

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 Each value passed through the natural log: 2.251, 2.054, 1.841, 1.629, 1.435, 1.224, all to three decimal places and consistent.

2.2 Plot $\lg y$ against $\lg x$; gradient = 3 and intercept = $\lg k$. (Plotting y against x^3 with a line through the origin is an acceptable alternative.)

2.3 $A = e^{1.10} = 3.0$; matching $\ln T = \ln A + B/v$ to $y = c + mx$ gives $B = +12.4 \text{ m s}^{-1}$ – the sign is positive because the exponent is $+B/v$ here.

3.1 (a) $\ln(A/\text{cm})$: 2.251, 2.054, 1.841, 1.629, 1.435, 1.224. (b) Axes labelled $\ln(A/\text{cm})$ and n , sensible scales, points plotted to half a small square, thin best-fit line. (c) Large triangle on the line, e.g. from (5, 2.25) to (30, 1.22): $\lambda = -\text{gradient} = (2.25 - 1.22)/(30 - 5) = 0.041$, accept 0.040–0.043, no unit. (d) Intercepts: extend the line to $n = 0$ and read $\ln A_0 \approx 2.46$, then $A_0 = e^{2.46} = 11.7 \text{ cm}$ (accept 11–12 cm, unit matching the logged quantity). (e) Any paired reasoning: there is no data below $n = 5$, so the line's position at $n = 0$ is an extrapolation that scatter makes uncertain; or a systematic error in judging amplitude shifts the whole line; or the earliest swings may depart from the model if the starting angle was large.

3.2 (a) Taking logs: $\ln V = \ln A - Bw$, so the plot is straight with gradient $-B$. (b) With no glass $V = A$, so the target is $V/A = 0.60$:

$$\ln 0.60 = -Bw \quad \Rightarrow \quad w = \frac{\ln 0.60}{-0.0088 \text{ mm}^{-1}} = 58 \text{ mm}$$

Dividing by the 2.0 mm thickness gives $58/2.0 = 29$, so at least 30 slides: 29 leave the reading marginally above 60%.

3.3 (a) The gross rate is the source's activity plus a constant background; $\ln$ of a sum is not linear in t . Subtracting the background first leaves $R_{\text{corr}} = R_0 e^{-\lambda t}$, whose log is linear. (b) Corrected rates: 450, 347, 268, 207, 159, 123 min^{-1} ; logs to three decimal places: 6.109, 5.849, 5.591, 5.333, 5.069, 4.812; plot $\ln(R_{\text{corr}}/\text{min}^{-1})$ against t , best-fit line, gradient = $-\lambda$. (c) The half-life follows from the decay constant:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{0.026 \text{ min}^{-1}} = 26.7 \text{ min}$$

so the half-life is 26.7 minutes. (d) Interval: $26.7 \times (1 \pm 0.05) = 25.4$ to 28.0 min . The data-sheet 25 minutes lies just below the lower limit, so strictly the measurement does not agree at that uncertainty; alternatively a percentage difference of $\frac{26.7-25}{25} \times 100\% = 6.8\%$ exceeds 5%, again not in agreement. A conclusion that follows the chosen comparison earns the mark.