

Course Companion

# Pearson Edexcel • International A-Level • Physics

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Every topic handout in one volume

exercises.lol

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# 1 • Mechanics and Materials

Pearson Edexcel • International A-Level • Physics

## What this unit covers

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Edexcel IAL Physics Unit 1 (paper **WPH11**) is **Mechanics and Materials**

: how things move, what forces do, how momentum and energy behave, and how solid and liquid materials respond to forces.

- The paper is 1 hour 30 minutes, 80 marks, every question compulsory. It mixes multiple-choice, short answers, calculations and one or two extended-writing answers marked with an asterisk (\*).
- At least 32 of the 80 marks test Level-2 mathematics: reading graphs, rearranging equations, trigonometry and areas.
- A formula list is printed at the back of the paper. Learn what each symbol means and **when** each equation is allowed – the list gives formulas, not understanding.
- Use  $g = 9.81 \text{ m s}^{-2}$ . The paper requires this value; using  $g = 10$  can lose accuracy marks. Give final answers with a **unit** and a sensible number of significant figures (usually 2–3, matching the data).

A speedometer shows speed at one moment; a journey can still hide stops and bursts. This unit replaces one number with the full picture: graphs, vectors, forces, momentum, energy and the strength of materials.

## Motion: quantities and graphs

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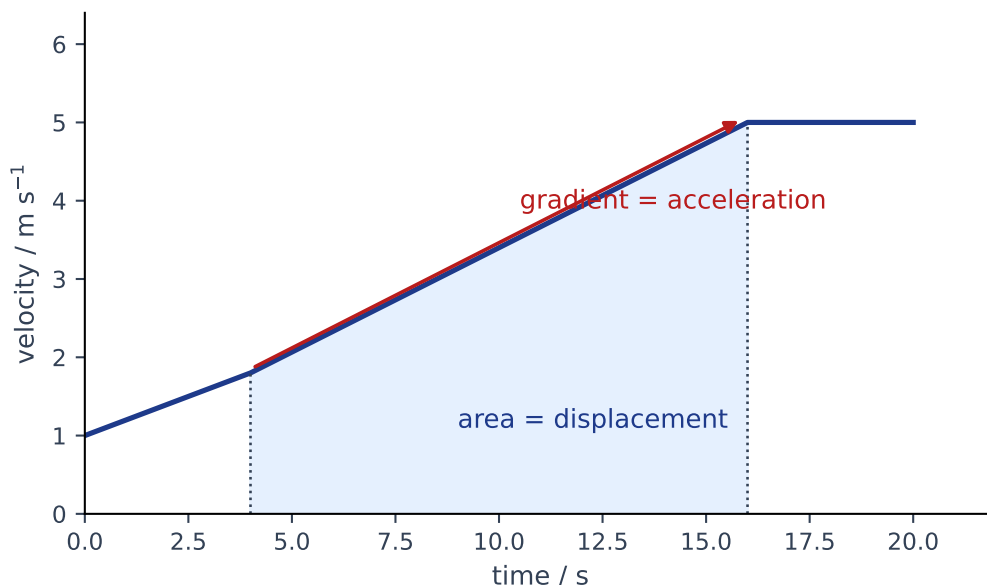
- **displacement** – straight-line change of position, with direction (a vector).  
**distance** – the total path length (a **scalar**, no direction).
- **velocity** – rate of change of displacement. **speed** – rate of change of distance.
- **acceleration** – rate of change of velocity:  $a = \frac{\Delta v}{\Delta t}$ , unit  $\text{m s}^{-2}$ .

Because displacement is the straight line from start to finish, a curved path always gives **distance** > **magnitude of displacement** – the examiner asks for exactly this comparison.

### ■ Reading motion graphs

- On a displacement–time graph, the **gradient** is the velocity. A curved line means changing velocity; take the gradient of a **tangent** at the point.
- On a velocity–time graph, the gradient is the acceleration, and the **area** between the line and the time axis is the displacement. Area below the axis is negative displacement (motion backwards); distance adds the sizes of all the areas.
- On an acceleration–time graph, the area under the line is the **change in velocity**.

- Constant non-zero acceleration bends a displacement–time graph into a parabola ; a straight line on that graph means zero acceleration.



**Worked example (June 2025 style).** A velocity–time line rises straight from  $1.8 \text{ m s}^{-1}$  at  $t = 4 \text{ s}$  to  $5.0 \text{ m s}^{-1}$  at  $t = 16 \text{ s}$ .

- Known: two  $(t, v)$  pairs on the straight part. Why: acceleration is the gradient of a velocity–time graph.

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{5.0 \text{ m s}^{-1} - 1.8 \text{ m s}^{-1}}{16 \text{ s} - 4 \text{ s}} = \frac{3.2 \text{ m s}^{-1}}{12 \text{ s}} = 0.27 \text{ m s}^{-2}$$

- Check: the line rises gently, so a small acceleration is sensible.

### ■ The four suvat equations

For **uniform acceleration** (constant acceleration) in one dimension, with  $s$  displacement,  $u$  initial velocity,  $v$  final velocity,  $a$  acceleration,  $t$  time:

$$v = u + at \quad s = \frac{(u + v)t}{2} \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as$$

- Choose the equation that contains your known quantities and the one you want – and nothing you do not know. No  $t$ ? Use  $v^2 = u^2 + 2as$ . No  $a$ ? Use  $s = \frac{(u+v)}{2}t$ .
- These equations are valid **only** while the acceleration is constant. A curved velocity–time graph shows changing acceleration; a curved displacement–time graph can still represent constant acceleration.
- Free fall is uniform acceleration with  $a = g = 9.81 \text{ m s}^{-2}$  downwards (air resistance negligible).

**Worked example (deduce-and-compare, June 2025 Q13).** A basketball is thrown from 1.7 m above the floor with initial vertical velocity  $5.1 \text{ m s}^{-1}$  upwards, and enters the hoop moving downwards at  $2.1 \text{ m s}^{-1}$ . Is the hoop 3.0 m above the floor?

- Known:  $u = +5.1 \text{ m s}^{-1}$ ,  $v = -2.1 \text{ m s}^{-1}$  (downwards is negative),  $a = -9.81 \text{ m s}^{-2}$ . Wanted: the rise  $\Delta s$ . No time given, so use  $v^2 = u^2 + 2a\Delta s$ .

$$v^2 = u^2 + 2a\Delta s \quad \Rightarrow \quad \Delta s = \frac{v^2 - u^2}{2a}$$

$$\Delta s = \frac{(-2.1 \text{ m s}^{-1})^2 - (5.1 \text{ m s}^{-1})^2}{2 \times (-9.81 \text{ m s}^{-2})} = \frac{4.41 - 26.01}{-19.62} \text{ m} = 1.1 \text{ m}$$

- Hoop height =  $1.7 \text{ m} + 1.1 \text{ m} = 2.8 \text{ m} \neq 3.0 \text{ m}$ , so the hoop was **not** at the regulation height. A "deduce whether" question earns its last mark only when the comparison and conclusion are stated.

### ■ Core practical 1: measuring $g$

Drop an object and measure it: with a light gate or video of known frame rate, get  $s$  and  $t$  (or velocities at two points) and use a suvat equation. Cleaner: plot  $s$  against  $t^2$  for  $s = \frac{1}{2}gt^2$  – a straight line through the origin with gradient  $g/2$ . A straight line that misses the origin reveals a systematic offset, for example a fixed offset in the measured distance. A timing delay can also distort the relationship; automatic timing reduces reaction-time error. In a strobe photograph, the object moves during each flash. A shorter flash produces a sharper image and a smaller range of possible positions, reducing the uncertainty in the distance reading if the scale and image resolution are unchanged.

## Scalars, vectors and projectile motion

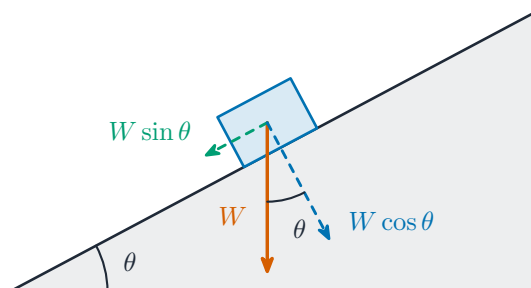
A **vector** has magnitude and direction (displacement, velocity, acceleration, force, momentum); a **scalar** has magnitude only (distance, speed, work, energy). Vector notation in the paper is an arrow above the symbol,  $\vec{F}$ .

### ■ Resolving and combining

- Resolving splits one vector into two perpendicular components: the component of force  $F$  at angle  $\theta$  along a direction is  $F \cos \theta$  if  $\theta$  is measured from that direction,  $F \sin \theta$  from the perpendicular. Choose by drawing the right triangle, not by habit.
- Combining two perpendicular components rebuilds the vector: magnitude  $R = \sqrt{F_x^2 + F_y^2}$ , direction from  $\tan \theta = F_y/F_x$ .
- Combine before squaring.** The resultant of 500 N and 200 N at right angles is  $\sqrt{500^2 + 200^2}$  N, not  $\sqrt{500^2} + \sqrt{200^2}$ .

- At any angle, add vectors by drawing a scale diagram (a vector triangle, nose to tail) or by resolving each vector into components first. In equilibrium, the vector triangle closes:  $\vec{W} + \vec{T}_1 + \vec{T}_2 = 0$ .

**Worked example.** A box sits on a ramp at angle  $\theta$  to the horizontal, weight  $W$  acting vertically down.



- Why: the sliding direction is along the ramp, so resolve  $W$  into components along and perpendicular to the ramp.

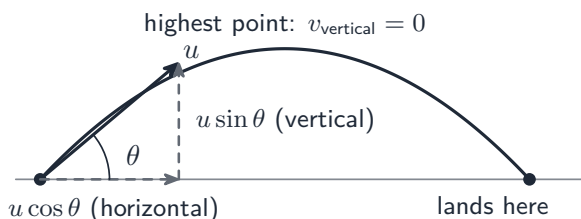
$$W_{\text{along}} = W \sin \theta \quad W_{\text{perp}} = W \cos \theta$$

- At constant speed the resultant force is zero, so the friction  $F = W \sin \theta$  exactly. ( $W \cos \theta$  acts into the ramp and is balanced by the normal contact force.)

## ■ Projectile motion

A projectile moving freely under gravity is **two independent motions**: horizontal velocity constant (no horizontal force), vertical velocity changing under  $g$ .

- The time to fall depends only on the vertical motion. Two balls leaving a table at different speeds – or with different masses – land after the **same** time: mass and horizontal speed do not appear in  $s = ut + \frac{1}{2}gt^2$ .
- Method: resolve the launch velocity into  $u_h = u \cos \theta$  and  $u_v = u \sin \theta$ ; run suvat separately for each direction; time links the two.



**Worked example (January 2025 Q19).** A projectile launches at  $500 \text{ m s}^{-1}$ ,  $22^\circ$  above the horizontal, from  $2.0 \text{ km}$  above the sea. Show it passes over a ship  $15 \text{ km}$  away after about  $30 \text{ s}$ .

- Known:  $u$ ,  $\theta$ , horizontal distance  $s_h$ . Horizontal motion has  $a = 0$ , so use  $s_h = u_h t$ .

$$u_h = u \cos \theta = 500 \text{ m s}^{-1} \times \cos 22^\circ = 464 \text{ m s}^{-1}$$

$$s_h = u_h t \quad \Rightarrow \quad t = \frac{s_h}{u_h} = \frac{15\,000 \text{ m}}{464 \text{ m s}^{-1}} = 32 \text{ s} \approx 30 \text{ s}$$

- The vertical height at that time uses  $s_v = u_v t - \frac{1}{2} g t^2$  with  $u_v = u \sin \theta = 187 \text{ m s}^{-1}$ :

$$s_v = 187 \text{ m s}^{-1} \times 32 \text{ s} - \frac{1}{2} \times 9.81 \text{ m s}^{-2} \times (32 \text{ s})^2 = 5\,984 \text{ m} - 5\,023 \text{ m} = 961 \text{ m}$$

$$h = 2.0 \text{ km} + 0.96 \text{ km} = 3.0 \text{ km above sea level}$$

- Why a maximum range exists: raising the launch angle raises  $u_v$  (more time in the air) but lowers  $u_h$ ; range =  $u_h \times t$  peaks between the two effects.

## Forces and Newton's laws

- A free-body diagram** shows only the forces acting **on** the chosen object – never forces the object applies to other things. On an extended body, the weight acts at the **centre of gravity**.
- Newton's first law**: zero resultant force means no change in motion – at rest or moving at constant velocity. Constant velocity is not “no forces”; it is balanced forces.
- Newton's second law**:  $\sum F = ma$ . The resultant force causes the acceleration; list every force, choose a positive direction, then apply.
- Newton's third law**: when body A exerts a force on body B, B exerts an equal, opposite force **on A**. The pair acts on **different bodies**, is the same type of force, and acts along the same line. A book's weight and the table's normal contact force are **not** a third-law pair: they act on the same body.
- weight**  $W = mg$ ; **gravitational field strength**  $g = \frac{F}{m}$ , so  $g = 9.81 \text{ N kg}^{-1}$ .
- Terminal velocity**: as a falling object speeds up, drag grows until drag (+ upthrust) equals weight; the resultant force is then zero and the velocity stops changing. “Terminal” describes the force balance, not a speed limit printed in nature.

**Worked example (June 2025 Q11).** A resultant force of 4800 N accelerates a boat at  $0.31 \text{ m s}^{-2}$ . Find the boat's weight.

- Known:  $\sum F$ ,  $a$ . Why:  $\sum F = ma$  links them to the mass;  $W = mg$  links mass to weight.

$$\sum F = ma \quad \Rightarrow \quad m = \frac{\sum F}{a} = \frac{4800 \text{ N}}{0.31 \text{ m s}^{-2}} = 15\,500 \text{ kg}$$

$$W = mg = 15\,500 \text{ kg} \times 9.81 \text{ N kg}^{-1} = 1.5 \times 10^5 \text{ N}$$

- Check: the answer needs two equations chained; the unit is newtons, not kilograms.

## Momentum

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- **momentum**  $p = mv$ , a vector, unit  $\text{kg m s}^{-1}$ .
- **Conservation of linear momentum** : when the resultant external force is zero (or its impulse during a short interaction is negligible), total momentum before an event equals total momentum after. Give velocities signs: choose one direction as positive and keep it.
- Momentum conservation follows from Newton's second and third laws: the forces between the two bodies are equal and opposite, act for the same time, so the changes of momentum are equal and opposite.
- Kinetic energy need not survive a collision; momentum always does (in an isolated system).

**Worked example (June 2025 Q10).** A train of mass  $3m$  at speed  $v$  meets a truck of mass  $m$  at speed  $2v$  the other way; they couple. Find the new speed.

- Known: masses, velocities in opposite directions. Why: an isolated pair, so conserve momentum with signs. Take the train's direction as positive.

$$p_{\text{before}} = p_{\text{after}}$$

$$3m \times v + m \times (-2v) = (3m + m) V$$

$$mv = 4mV \quad \Rightarrow \quad V = \frac{v}{4}$$

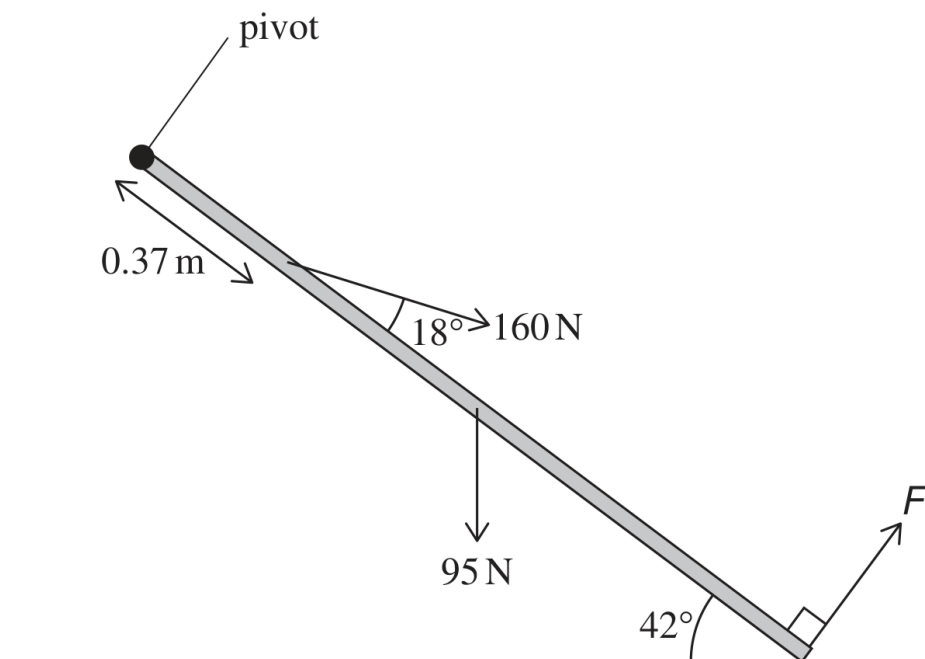
- Check: the momenta nearly cancel, so a small combined speed in the train's direction is sensible.

## Moments and equilibrium

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- The **moment of a force** about a pivot is moment  $= Fx$ , where  $x$  is the **perpendicular distance** from the pivot to the force's line of action. If the force meets the lever at angle  $\theta$ , use the perpendicular component  $F \sin \theta$  or the perpendicular distance – never both.
- **Principle of moments** : in equilibrium, total clockwise moment = total anticlockwise moment (resultant moment is zero).
- Full equilibrium needs **both**: resultant force zero and resultant moment zero. A see-saw with equal children at equal distances has both; the same children at unequal distances still balance forces but not moments.
- For a uniform beam, the weight acts at its centre.

**Worked example (June 2025 Q19).** A uniform rail, length 1.40 m, weight 95 N, lies at  $42^\circ$  to the horizontal, pivoted at one end. A piston pushes with 160 N at  $18^\circ$  to the rail, 0.37 m from the pivot. What perpendicular force  $F$  at the handle end just lifts the rail?



- Known: geometry, both forces, both angles. Why: just lifting means equilibrium at the pivot, so apply the principle of moments with perpendicular parts.

$$\text{moment of piston} = (160 \text{ N} \times \sin 18^\circ) \times 0.37 \text{ m} = 18.3 \text{ N m}$$

$$\text{moment of weight} = (95 \text{ N} \times \cos 42^\circ) \times 0.70 \text{ m} = 49.4 \text{ N m}$$

$$18.3 \text{ N m} + F \times 1.40 \text{ m} = 49.4 \text{ N m} \quad \Rightarrow \quad F = \frac{49.4 \text{ N m} - 18.3 \text{ N m}}{1.40 \text{ m}} = 22 \text{ N}$$

- A vertical pull at the handle instead: its perpendicular distance to the pivot is shorter than the rail's length, so a **larger** force would be needed for the same moment.

## Work, energy, power and efficiency

- **work**  $\Delta W = F\Delta s$  when the force acts along the motion; if the force is at angle  $\theta$  to the motion, only the component along the motion works:  $\Delta W = F\Delta s \cos \theta$ .
- **kinetic energy**  $E_k = \frac{1}{2}mv^2$  – doubling speed quadruples the energy.
- **gravitational potential energy**  $\Delta E_{\text{grav}} = mg\Delta h$  near the Earth's surface.
- **Conservation of energy** : energy is transferred or transformed, never created or destroyed. In a fall with drag,  $\Delta E_{\text{grav}}$  becomes  $E_k$  **plus** work done against air resistance; at terminal velocity  $E_k$  is constant, so *all* of the  $\Delta E_{\text{grav}}$  goes to work against drag.
- **power**  $P = \frac{E}{t} = \frac{W}{t}$  – energy transferred per unit time.

- **efficiency** 
$$= \frac{\text{useful energy output}}{\text{total energy input}} = \frac{\text{useful power output}}{\text{total power input}}.$$

**Worked example (June 2025 Q14).** An electric car of 1800 kg at 14 m s<sup>-1</sup> brakes to rest on a hill, gaining 0.76 m of height; the battery gains 45 kJ. Find the efficiency of the braking.

- Known:  $m$ ,  $v$ ,  $\Delta h$ , battery gain. Why: the total input is the energy the brakes remove – kinetic energy lost **minus** the gravitational energy the car keeps by ending higher.

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 1800 \text{ kg} \times (14 \text{ m s}^{-1})^2 = 1.76 \times 10^5 \text{ J}$$

$$\Delta E_{\text{grav}} = mg\Delta h = 1800 \text{ kg} \times 9.81 \text{ N kg}^{-1} \times 0.76 \text{ m} = 1.34 \times 10^4 \text{ J}$$

$$\text{efficiency} = \frac{\text{useful output}}{\text{total input}} = \frac{45 \times 10^3 \text{ J}}{1.76 \times 10^5 \text{ J} - 1.34 \times 10^4 \text{ J}} = 0.28$$

- Check:  $0.28 < 1$  – required. The common error is dividing by  $E_k$  alone and forgetting the height gain.

## Materials: density, upthrust and viscous drag

- **density**  $\rho = \frac{m}{V}$ . For a sphere,  $V = \frac{4}{3}\pi r^3$ .
- **upthrust** = weight of the fluid displaced – the displaced *weight*, not its volume or its mass.
- **Stokes' law** : viscous drag on a small sphere at low speed in laminar flow is  $F = 6\pi\eta rv$ , with  $\eta$  the viscosity,  $r$  the radius,  $v$  the speed. It is valid only for small spheres moving slowly without turbulence, and  $\eta$  falls as the liquid warms in this experiment; gases do not generally follow this trend.

### ■ Core practical 2: viscosity by the falling-ball method

Drop a ball-bearing into a measuring cylinder of the liquid. Wait until it falls at terminal velocity, then time it over a marked distance. At terminal velocity, the forces balance: weight = upthrust + drag, so

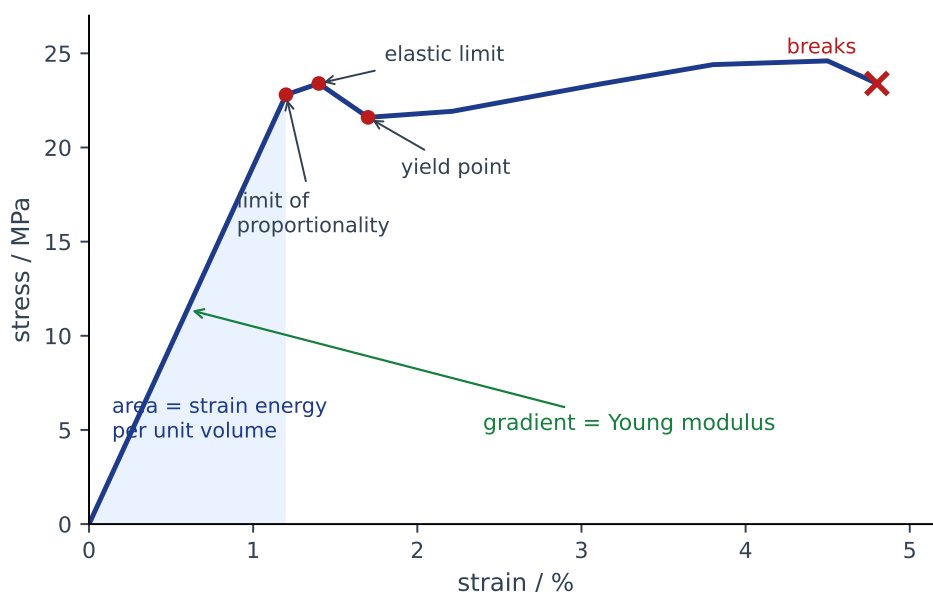
$$6\pi\eta rv = \frac{4}{3}\pi r^3(\rho_{\text{ball}} - \rho_{\text{oil}})g$$

You need the ball's radius and mass (or its density), the oil's density, the fall distance and the time. Warm oil is less viscous, so the ball falls faster and the measured time is shorter.

## Materials: stretching solids

- **Hooke's law** :  $\Delta F = k \Delta x$  while the material behaves proportionally;  $k$  is the **stiffness** (spring constant), the gradient of the force–extension graph.

- stress**  $\sigma = \frac{F}{A}$  (unit Pa), **strain**  $\varepsilon = \frac{\Delta x}{x}$  (no unit), **Young modulus**  $E = \frac{\sigma}{\varepsilon}$ . Stress and strain describe the *material*; force and extension describe the *sample*.
- On a force–extension or stress–strain graph: the straight part ends at the **limit of proportionality**; beyond the **elastic limit** the sample no longer returns to its original length (**elastic deformation** = it does return; **plastic deformation** = it stays stretched); a material with a distinct **yield point** can stretch substantially with little increase in force at yield; other ductile materials yield gradually. The **yield point** marks the onset of substantial plastic flow; the **breaking stress** is the stress at fracture.
- elastic strain energy** :  $\Delta E_{\text{el}} = \frac{1}{2}F\Delta x$  for a linear spring – the area under the force–extension graph. For a non-linear graph, estimate the area by squares. The area under a loading *stress–strain* graph is the work done **per unit volume**, because  $\frac{F}{A} \times \frac{\Delta x}{x} = \frac{F\Delta x}{V}$ . In the elastic region this is stored recoverably as elastic strain energy; work beyond the elastic limit is not all recoverable.



**Worked example (June 2025 Q17b).** A violin string of original length 0.750 m and radius  $0.85 \times 10^{-3}$  m stretches to 0.752 m under 36 N. Find the Young modulus.

- Known:  $F$ ,  $x$ ,  $\Delta x$ ,  $r$ . Why:  $E = \sigma/\varepsilon$  needs stress and strain; stress needs the cross-sectional area.

$$A = \pi r^2 = \pi \times (0.85 \times 10^{-3} \text{ m})^2 = 2.27 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{F}{A} = \frac{36 \text{ N}}{2.27 \times 10^{-6} \text{ m}^2} = 1.59 \times 10^7 \text{ Pa} \quad \varepsilon = \frac{\Delta x}{x} = \frac{0.002 \text{ m}}{0.750 \text{ m}} = 2.67 \times 10^{-3}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{1.59 \times 10^7 \text{ Pa}}{2.67 \times 10^{-3}} = 6.0 \times 10^9 \text{ Pa}$$

- Check: a metal-wire modulus is  $10^9$ – $10^{12}$  Pa; 6 GPa sits in range.

### ■ Core practical 3: the Young modulus of a wire

Hang known weights on a long wire and measure the extension with a marker against a scale; keep the wire long and the extensions small for precision, add loads in steps within the proportional region, and measure the diameter in several places with a micrometer. Then  $E$  comes from the gradient of the stress–strain line, or from  $\sigma/\varepsilon$  point by point within the proportional region.

## Exam craft for WPH11

- **Show every step.** "Use of  $F = ma$ " earns a mark before any number appears – write the equation in symbols, then substitute.
- **Units and  $g$ .** A missing unit usually loses the final mark;  $g = 10 \text{ m s}^{-2}$  can lose accuracy marks; "show that" questions need one more significant figure than the target value.
- **Command words:** *state* = a short precise answer; *calculate/determine* = numbers with working and a unit; *deduce* = working plus a compared conclusion; *explain* = linked physics reasons; *describe* = what happens, in order.
- The asterisk (\*) answer is marked for linked, logical reasoning as well as content: order the points, then link them.
- The formula list is at the back – know where, so you never memorise what you can look up in two seconds.

## Check yourself

1. On a velocity–time graph, what do the gradient and the area give? – **acceleration; displacement.**
2. Which suvat equation finds a height from two velocities without time? –  $v^2 = u^2 + 2as$ .
3. A projectile's horizontal acceleration in free flight? – **zero** (only gravity acts, vertically).
4. A book rests on a table: are its weight and the table's normal force a Newton's-third-law pair? – **No: they act on the same body.**
5. Two trolleys collide and stick. Which quantity is always conserved? – **total momentum** (kinetic energy need not be).
6. A force at an angle to a lever: which distance goes into the moment? – **the perpendicular distance from the pivot to the line of action.**
7. A car brakes uphill: what is the input energy for the efficiency? – **the kinetic energy lost minus the gravitational energy gained.**

8. Upthrust equals the weight of what? – **the fluid displaced.**
9. Stokes' law needs which three conditions? – **small sphere, low speed, laminar flow** (and  $\eta$  is temperature-dependent).
10. On a stress–strain graph, what is the gradient of the straight part, and what is the area beneath it? – **the Young modulus; the elastic strain energy per unit volume.**

## 2 · Waves and Electricity

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### From a phone signal to its battery

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Your phone receives waves and uses electrical energy. These seem different, but both depend on energy transfer. **Waves** carry energy without carrying matter along with them. A circuit transfers energy through moving charge.

This reference covers **WPH12 Waves and Electricity**, specification statements 33–80. The paper lasts 90 minutes and carries 80 marks. Use the formula list to support your reasoning. Explain why an equation fits the situation, show conversions, and finish comparisons with a conclusion.

### Wave quantities and graphs

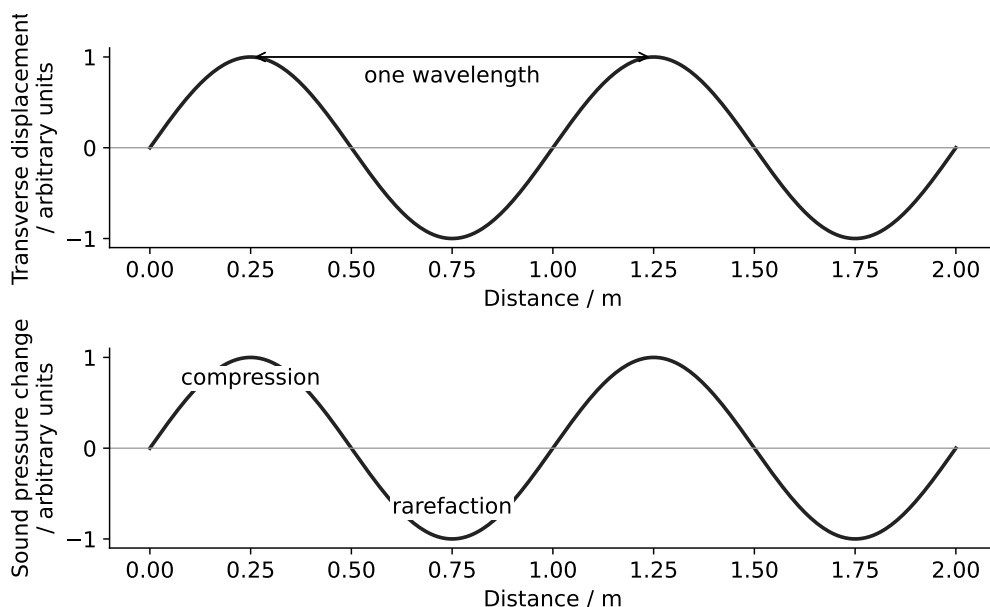
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A vibrating source repeats its motion. The **amplitude** is the greatest displacement from equilibrium. **Equilibrium** means the resting position. The **period**  $T$  is the time for one complete vibration. The **frequency**  $f$  is the number of complete vibrations each second; its unit is hertz, Hz.

$$f = \frac{1}{T} \quad v = f\lambda$$

The **wavelength**  $\lambda$  is the distance between nearest points vibrating in phase. The **wave speed**  $v$  is the speed at which a fixed phase, such as a crest, travels. It is not the speed of a vibrating particle.

- A **transverse wave** has vibrations perpendicular to its direction of travel. A rope moves up and down while its disturbance travels along the rope.
- A **longitudinal wave** has vibrations parallel to its direction of travel. Sound has **compressions**, where pressure and density are higher, and **rarefactions**, where they are lower.
- Air molecules move back and forth about their resting positions. They do not travel from the speaker to your ear with the sound.



*Distance graphs show a whole wave at one instant.*

### ■ Read the horizontal axis first

A displacement–distance graph is a snapshot. The separation of neighbouring crests gives  $\lambda$ . A displacement–time graph follows one particle. The separation of neighbouring peaks gives  $T$ . The same curve shape can represent either graph, but the quantities are different.

For a longitudinal wave, a pressure–distance graph shows compressions at pressure maxima. A molecular displacement–distance graph shows displacement along the direction of travel. Maximum compression occurs where displacement changes most rapidly towards the neighbouring molecules. Pressure maximum and displacement maximum are not at the same place; their patterns are one-quarter wavelength apart in a sinusoidal travelling sound wave.

**Worked example.** A microphone records 12 complete cycles in 0.030 s. Sound travels at  $340 \text{ m s}^{-1}$ . Find its frequency and wavelength.

- Known: cycle count, total time and speed. Frequency counts cycles per second; the wave equation then links speed to wavelength.

$$f = \frac{N}{t} = \frac{12}{0.030 \text{ s}} = 400 \text{ Hz}$$

$$v = f\lambda \quad \Rightarrow \quad \lambda = \frac{v}{f} = \frac{340 \text{ m s}^{-1}}{400 \text{ Hz}} = 0.85 \text{ m}$$

- Check: one cycle takes  $T = 1/f = 0.0025 \text{ s}$ . Twelve cycles take the stated 0.030 s.

## Superposition and standing waves

A **wavefront** joins points at the same phase. **Phase** describes the stage of a vibration cycle. One complete cycle corresponds to  $360^\circ$  or  $2\pi$  radians.

**Superposition** means adding the displacements of overlapping waves at each point. **Interference** is the resulting reinforcement or cancellation. **Coherent sources** have the same frequency and a constant phase difference. Their phase difference need not be zero.

For waves of the same wavelength, the phase difference caused by a **path difference**  $\Delta x$  is:

$$\Delta\phi = \frac{\Delta x}{\lambda} 360^\circ = \frac{\Delta x}{\lambda} 2\pi \text{ rad}$$

For sources initially in phase, path differences  $0, \lambda, 2\lambda, \dots$  give constructive interference. Half-integer wavelength differences give destructive interference. Complete cancellation also needs equal amplitudes. If sources start with a phase difference, include it too.

**Worked example.** Two in-phase speakers produce wavelength 0.80 m. Their paths to a microphone differ by 1.20 m.

- Known: path difference and wavelength. Convert path difference into cycles, then phase.

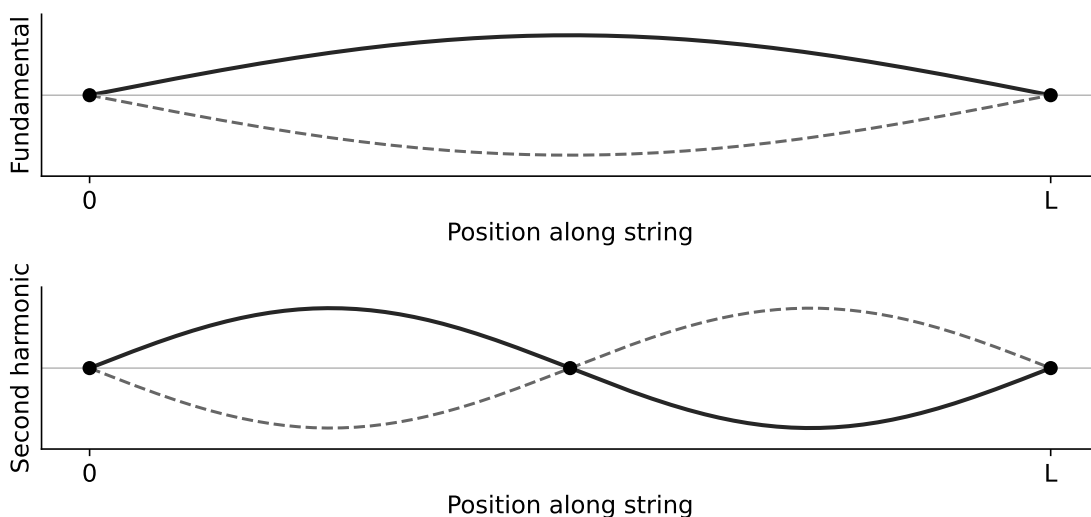
$$\Delta\phi = \frac{\Delta x}{\lambda} 360^\circ = \frac{1.20}{0.80} 360^\circ = 540^\circ$$

- $540^\circ$  is equivalent to  $180^\circ$ . The waves arrive in opposite phase and interfere destructively. The sound need not become silent if the arriving amplitudes differ.

### ■ How a standing wave forms

A **standing wave** forms when two waves of the same frequency and similar amplitude travel in opposite directions and superpose. Reflection at a fixed string end supplies the returning wave.

- A **node** always has zero displacement. An **antinode** has the largest vibration amplitude.
- Adjacent nodes are  $\lambda/2$  apart. A node and its nearest antinode are  $\lambda/4$  apart.
- Points between the same pair of nodes vibrate in phase. Points in neighbouring sections vibrate in opposite phase.
- All points have the same frequency, except that a node does not vibrate. Amplitude depends on position.
- An ideal standing wave has no net energy transfer along it. A travelling wave carries energy along its direction of travel.



The end nodes remain still; the curve changes between the solid and dashed shapes.

For a string fixed at both ends, length  $L$  contains a whole number  $k$  of half-wavelengths:

$$L = \frac{k\lambda}{2} \quad v = \sqrt{\frac{T}{\mu}} \quad f = \frac{k}{2L} \sqrt{\frac{T}{\mu}}$$

Here  $T$  is **tension**, not period;  $\mu$  is **mass per unit length** in  $\text{kg m}^{-1}$ . The fundamental has  $k = 1$ . At fixed mode, frequency increases with  $\sqrt{T}$ , decreases with  $L$ , and decreases with  $\sqrt{\mu}$ .

**Worked example.** A 0.60 m string has  $\mu = 1.5 \times 10^{-3} \text{ kg m}^{-1}$  and tension 24 N. Find its fundamental frequency.

- Known: length, tension and linear density. The fundamental fits half a wavelength between the fixed ends.

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{24}{1.5 \times 10^{-3}}} \text{ m s}^{-1} = 126 \text{ m s}^{-1}$$

$$f = \frac{v}{2L} = \frac{126 \text{ m s}^{-1}}{2 \times 0.60 \text{ m}} = 105 \text{ Hz}$$

- Doubling tension multiplies frequency by  $\sqrt{2}$ , rather than by two.

## ■ Standing sound in a closed tube

In the simplest standing sound wave in a tube closed at one end, the closed end is a molecular displacement node. The open end is approximately a displacement antinode. This fits one-quarter wavelength into the effective tube length, so  $L_{\text{effective}} = \lambda/4$ . The pressure pattern is reversed: the closed end is a pressure antinode. Do not call a pressure node a displacement node. Real tubes have a small open-end correction; use an effective length if the question supplies it.

**Worked example.** A closed tube has effective length 0.20 m and fundamental frequency 425 Hz. The end conditions give  $\lambda = 4L = 0.80$  m; then  $v = f\lambda = 425 \times 0.80 = 340$  m s<sup>-1</sup>. A string fixed at both ends instead fits half a wavelength in its fundamental. Choose from the end conditions, not from the word “standing”.

## ■ Core practical 4: speed of sound

Use a signal generator, speaker, movable microphone and **two-beam oscilloscope**. Connect one oscilloscope channel to the generator reference signal and the other to the microphone output. Both traces have the same frequency.

1. Keep frequency constant. Move the microphone along a measured line away from the speaker. Find a position where the traces are in phase.
2. Move to another in-phase position. Consecutive in-phase positions are one wavelength apart. Measure across several wavelengths and divide by their number.
3. Read the period from the time-base scale: count horizontal divisions across several cycles, then divide. Find  $f = 1/T$  and use  $v = f\lambda$ .
4. Repeat positions and frequencies. Keep the microphone on the speaker axis. Reduce reflected sound by working away from walls. Measure position from a fixed microphone reference point.

Measuring several wavelengths reduces the percentage uncertainty in the distance. Measuring several periods does the same for time. A resonance-tube experiment can also measure sound speed, but it does not replace this specified practical.

## ■ Core practical 5: vibrating string

Use a vibration generator driven by a signal generator. Run the string over a pulley to a hanging mass. The tension is about  $mg$  when pulley friction is small. Increase frequency until a clear standing-wave pattern forms.

- Investigate tension: keep  $L$ ,  $\mu$  and mode fixed. Vary hanging mass. Plot  $f^2$  against  $T$ ; the gradient for the fundamental is  $1/(4L^2\mu)$ .
- Investigate length: keep tension, string type and mode fixed. Plot  $f$  against  $1/L$ .
- Investigate linear density: measure string mass and length,  $\mu = m/l$ . Use different strings with fixed tension, vibrating length and mode. Plot  $f^2$  against  $1/\mu$ .
- Measure length between the end nodes. Find the resonance from both higher and lower frequencies and repeat. Secure the stand and keep clear of falling masses.

## Intensity, refraction and polarisation

---

**Intensity** is power per unit area perpendicular to the energy flow:

$$I = \frac{P}{A}$$

Its unit is  $\text{W m}^{-2}$ . For a receiver, incident power is  $P = IA$ . Useful output is smaller if efficiency is below 100%. Convert area carefully:  $1 \text{ cm}^2 = 10^{-4} \text{ m}^2$ .

**Worked example.** Radiation intensity is  $800 \text{ W m}^{-2}$  over a  $25 \text{ cm}^2$  solar cell. Efficiency is 20%. Can it supply  $0.50 \text{ W}$ ?

- Known: intensity, area, efficiency. First find incident power, then useful power.

$$P_{\text{in}} = IA = 800 \text{ W m}^{-2} \times 25 \times 10^{-4} \text{ m}^2 = 2.0 \text{ W}$$

$$P_{\text{out}} = \eta P_{\text{in}} = 0.20 \times 2.0 \text{ W} = 0.40 \text{ W}$$

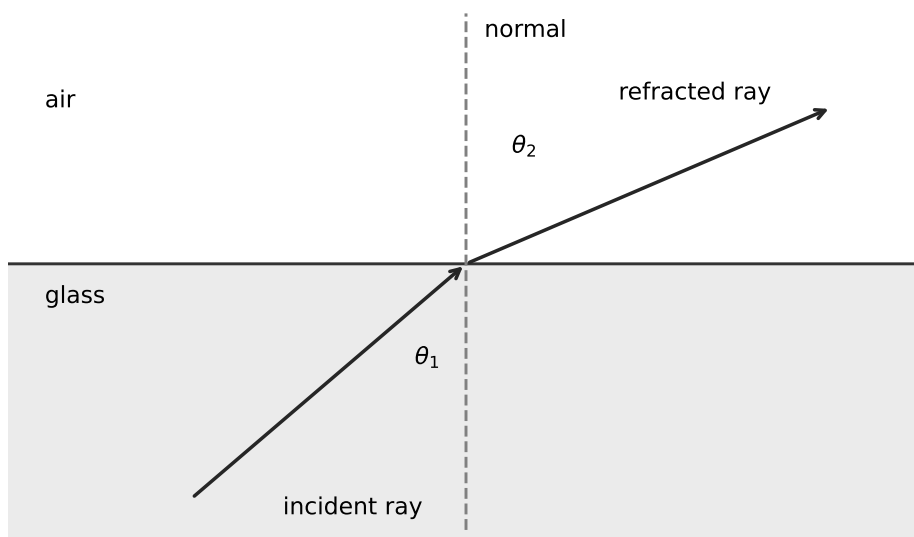
- $0.40 \text{ W} < 0.50 \text{ W}$ , so it cannot supply the required useful power.

### ■ Refraction and total internal reflection

**Refraction** is a change in wave direction when speed changes at an interface. A ray entering along the normal changes speed without changing direction. The **normal** is perpendicular to the boundary. Measure all ray angles from it.

$$n = \frac{c}{v} \quad n_1 \sin \theta_1 = n_2 \sin \theta_2$$

The **refractive index**  $n$  compares vacuum light speed  $c$  with speed in the material. Frequency stays constant across the boundary, so wavelength changes with speed. Entering higher  $n$  bends the ray towards the normal; entering lower  $n$  bends it away.



*The ray bends away from the normal when it enters the lower-index medium.*

**Total internal reflection** requires travel from higher to lower refractive index and incidence greater than the **critical angle**  $C$ .

$$\sin C = \frac{n_2}{n_1} \quad \text{for material to air: } \sin C = \frac{1}{n}$$

At  $C$  the refracted angle is  $90^\circ$ ; call incidence greater than  $C$  total internal reflection. Below  $C$ , some energy is normally reflected and some transmitted. An optical fibre uses a higher-index core and lower-index cladding. Raising the cladding index raises the critical angle and can stop a previously reflected ray being trapped.

**Worked example.** Light travels from glass of index 1.50 into air. Does an incidence angle of  $45^\circ$  produce total internal reflection?

- Known: the two indices and incidence angle. Find the boundary angle before comparing.

$$C = \sin^{-1} \left( \frac{n_2}{n_1} \right) = \sin^{-1} \left( \frac{1.00}{1.50} \right) = 41.8^\circ$$

- The light travels towards lower index and  $45^\circ > 41.8^\circ$ . Both conditions hold, so total internal reflection occurs.

## ■ Measuring refractive index

Place a rectangular transparent block on paper and trace its outline. Use a narrow ray-box beam. Mark two points on the incident ray and two on the emerging ray. Remove the block and join the boundary points to reconstruct the ray inside. Draw the normal at entry and measure incidence  $i$  and refraction  $r$  with a protractor.

Repeat for several incidence angles, keeping the same material and light colour. Plot  $\sin i$  vertically against  $\sin r$  horizontally. For air into the block, the gradient is the block's refractive index relative to air. Use a large triangle on a best-fit line. Narrow beams and widely separated ray marks reduce direction uncertainty. Very small angles give large percentage angle uncertainties.

## ■ Plane polarisation

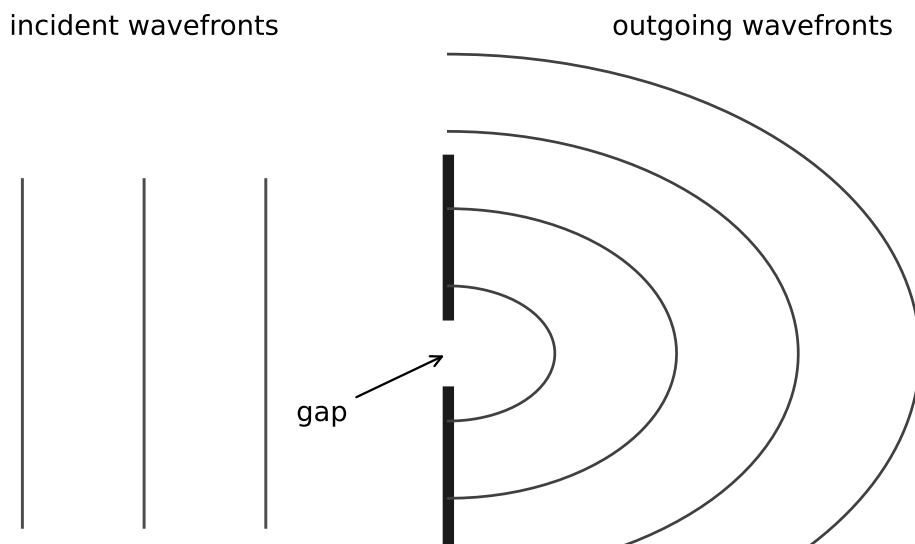
**Plane polarisation** restricts transverse vibrations to one plane containing the propagation direction. Unpolarised light has vibrations in many planes. A polariser transmits one vibration direction. Rotate a second polariser: parallel transmission directions give maximum brightness; perpendicular directions ideally give darkness. The ray still travels forward; its vibration direction changes selection.

Longitudinal waves cannot be plane polarised because their vibration is already along propagation. Polarisation is therefore evidence that light is transverse.

## Diffraction and pulse-echo

**Diffraction** is the spreading of waves at a gap or edge. Spreading is greater when the gap width is comparable to the wavelength. A wide gap still diffracts at its edges, but the central wave spreads less.

In **Huygens' construction**, each point on a wavefront acts as a source of secondary wavelets. Draw wavelets of equal radius after the same time. Their forward common tangent gives the new wavefront. At a narrow gap only a small part of the original front supplies wavelets, so the emerging fronts spread widely. This explains spreading without inventing a change of frequency at the slit.



*The gap limits which points supply the outgoing wavelets.*

## ■ Gratings and core practical 6

A **diffraction grating** contains many equally spaced slits. Bright maxima occur when contributions from neighbouring slits arrive in phase:

$$k\lambda = d \sin \theta$$

Here  $k$  is the integer **order**,  $d$  is slit spacing, and  $\theta$  is measured from the straight-through direction. The equation assumes normal incidence. If the grating has  $N$  lines per metre,  $d = 1/N$ . Do not confuse grating order with refractive index; printed papers may use  $n$  for either.

**Worked example.** A grating has 600 lines per millimetre. First-order light appears at  $18.0^\circ$ . Find its wavelength.

- Known: line density and first-order angle. Convert line density before taking its reciprocal.

$$d = \frac{1}{N} = \frac{1}{600 \times 10^3 \text{ m}^{-1}} = 1.67 \times 10^{-6} \text{ m}$$
$$\lambda = \frac{d \sin \theta}{k} = \frac{1.67 \times 10^{-6} \text{ m} \times \sin 18.0^\circ}{1} = 5.15 \times 10^{-7} \text{ m}$$

- This is 515 nm. Check each proposed higher order using  $k\lambda/d \leq 1$ .

For CP6, aim a low-power laser normally at a grating and place a screen perpendicular to the central beam. Measure screen distance  $D$  and displacement  $x$  from the central spot to an order. Use  $\tan \theta = x/D$ , then the grating equation. Average corresponding left/right displacements, repeat distances, and use higher visible orders where measurement is clear. Measure from the grating to the screen, not from the laser casing. Do not substitute  $\sin \theta = x/D$  unless a justified small-angle approximation is acceptable. Never look into the beam; keep it below eye level and stop stray beams.

## ■ Electrons also show wave behaviour

An electron beam passing through a thin crystal produces a diffraction pattern. Regular atom spacing provides the diffracting structure. Changing electron momentum changes the pattern spacing. This is evidence of wave behaviour; particles travelling along simple straight paths alone cannot explain the diffraction maxima.

The **de Broglie wavelength** is:

$$\lambda = \frac{h}{p}$$

For a non-relativistic electron,  $p = mv$ . If it gains kinetic energy by crossing a potential difference,  $E_k = eV$  and  $p = \sqrt{2mE_k}$ . Use these links only when the question gives or requires that energy relation.

**Worked example.** An electron has momentum  $2.0 \times 10^{-24} \text{ kg m s}^{-1}$ . With  $h = 6.63 \times 10^{-34} \text{ J s}$ :

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{2.0 \times 10^{-24}} \text{ m} = 3.3 \times 10^{-10} \text{ m}$$

Increasing momentum reduces wavelength. Electrons have both particle and wave properties; diffraction does not mean that an electron becomes a sound wave.

### ■ Reflection, transmission and echoes

At an interface, part of a wave's energy may be **transmitted** and part reflected. A reflected pulse can locate a boundary. In **pulse-echo**, measured delay includes the outward and return journeys:

$$2s = vt \quad \Rightarrow \quad s = \frac{vt}{2}$$

**Worked example.** Ultrasound travels in metal at  $5900 \text{ m s}^{-1}$ . An echo returns after  $12 \mu\text{s}$ .

$$s = \frac{vt}{2} = \frac{5900 \text{ m s}^{-1} \times 12 \times 10^{-6} \text{ s}}{2} = 0.0354 \text{ m}$$

The boundary is 35 mm away, not 71 mm. A nearby echo may overlap the transmitted pulse. Shorter pulse duration reduces this blind region and separates close echoes. Shorter wavelength helps detect or distinguish smaller features. These are related limits, but pulse duration and wavelength are not the same quantity. Use the speed in the actual material, not automatically the speed in air.

## Photons and the photoelectric effect

Wave ideas explain interference, diffraction and polarisation. They did not explain all observations of energy transfer. **Photons** describe discrete packets of electromagnetic energy:

$$E = hf = \frac{hc}{\lambda}$$

Historically, interference and diffraction supported wave models of light. Quantum ideas developed to explain observations including the photoelectric effect and atomic spectra. Modern physics uses both wave and photon descriptions according to the measurement. A successful new model must explain evidence that the older model explains too.

## ■ One photon and one electron

In the **photoelectric effect**, a surface electron absorbs a photon and may escape. The **work function**  $\phi$  is the minimum energy needed to remove an electron from the surface. The **threshold frequency** is  $f_0 = \phi/h$ .

$$hf = \phi + E_{k,\max} = \phi + \frac{1}{2}mv_{\max}^2$$

- Below threshold, increasing intensity does not cause emission in the usual single-photon model.
- Above threshold, higher frequency raises maximum kinetic energy. A graph of  $E_{k,\max}$  against  $f$  has gradient  $h$ , frequency intercept  $f_0$  and extrapolated energy intercept  $-\phi$ .
- At fixed frequency above threshold, higher intensity supplies more photons per second. More electrons can be emitted per second, so the saturation photocurrent rises. Their maximum kinetic energy does not rise.
- Emission starts without the energy-building delay predicted by a simple continuous-wave energy model, even at low intensity above threshold.
- Electrons escape with a range of kinetic energies. Some lose more energy within the material. The equation describes the largest energy, not every electron's energy.

The **electronvolt** is an energy unit:  $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$ . It is not a unit of potential difference. A stopping potential  $V_s$  just prevents even the fastest photoelectrons reaching the collector, so  $eV_s = E_{k,\max}$ .

**Worked example.** Light of frequency  $8.0 \times 10^{14} \text{ Hz}$  reaches a surface with work function  $2.0 \text{ eV}$ .

- Known: photon frequency and work function. Convert the work function into joules before subtracting.

$$\phi = 2.0 \times 1.60 \times 10^{-19} \text{ J} = 3.20 \times 10^{-19} \text{ J}$$

$$E_{k,\max} = hf - \phi = 6.63 \times 10^{-34} \text{ J s} \times 8.0 \times 10^{14} \text{ Hz} - 3.20 \times 10^{-19} \text{ J} = 2.10 \times 10^{-19} \text{ J}$$

$$V_s = \frac{E_{k,\max}}{e} = \frac{2.10 \times 10^{-19} \text{ J}}{1.60 \times 10^{-19} \text{ C}} = 1.3 \text{ V}$$

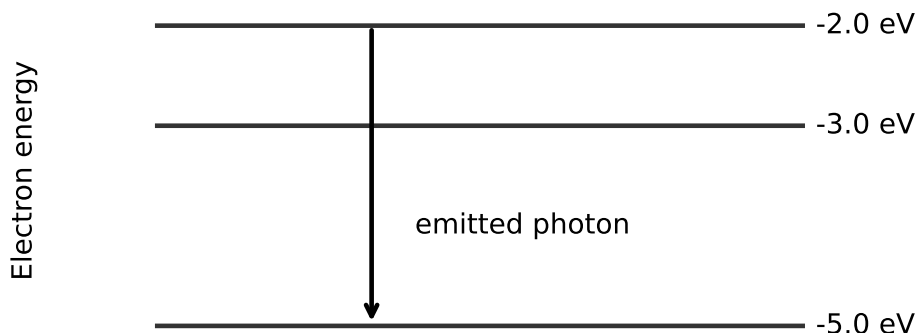
- The positive kinetic energy confirms that this frequency is above threshold. A negative subtraction would mean no emission, not negative kinetic energy.

## ■ Atomic line spectra

Atoms have **discrete energy levels**. An electron moving down between two allowed levels emits a photon. Moving up requires absorption of the matching energy:

$$|\Delta E| = hf = \frac{hc}{\lambda}$$

A line spectrum has particular frequencies because only certain level differences are allowed. The largest downward energy difference gives the highest frequency and shortest wavelength. Count possible transitions by checking level pairs and which upper levels are populated; do not automatically count the number of levels as the number of lines.



*The photon energy equals the difference between the two levels.*

**Worked example.** An electron drops from  $-2.0$  eV to  $-5.0$  eV.

$$E_{\gamma} = E_{\text{upper}} - E_{\text{lower}} = [-2.0 - (-5.0)] \text{ eV} = 3.0 \text{ eV}$$

$$f = \frac{E_{\gamma}}{h} = \frac{3.0 \times 1.60 \times 10^{-19} \text{ J}}{6.63 \times 10^{-34} \text{ J s}} = 7.2 \times 10^{14} \text{ Hz}$$

Photon absorption in a material does not always eject an electron from its surface. Excitation followed by emission, for example in a light-activated material, is not automatically the photoelectric effect.

## Current, resistance and circuits

**Current** is the rate of charge flow. Conventional current follows the direction positive charges would move. In a metal, electrons drift in the opposite direction.

$$I = \frac{\Delta Q}{\Delta t} \quad V = \frac{W}{Q} \quad R = \frac{V}{I}$$

One ampere is one coulomb per second. **Potential difference**  $V$  is energy transferred per unit charge between two points. Resistance  $R$  is defined by the ratio

$V/I$  at an operating point. **Ohm's law** is the extra condition  $I \propto V$  when temperature and other physical conditions stay constant. Not every resistor or component obeys it.

**Worked example.** A current of 0.40 A flows for 30 s. Find the number of electrons passing a point.

$$Q = It = 0.40 \text{ A} \times 30 \text{ s} = 12 \text{ C}$$

$$N = \frac{Q}{e} = \frac{12 \text{ C}}{1.60 \times 10^{-19} \text{ C}} = 7.5 \times 10^{19}$$

## ■ Conservation gives the circuit rules

**Charge conservation** means charge does not build up at a steady-current junction. Total current entering equals total current leaving. Current is not used up by a resistor.

**Energy conservation** means the total energy supplied per coulomb around a complete loop equals the energy transferred per coulomb. The sum of voltage rises equals the sum of voltage drops.

For series resistors, the same current passes through each and their potential differences add:

$$V = V_1 + V_2 = IR_1 + IR_2 \Rightarrow R_{\text{series}} = R_1 + R_2$$

For parallel resistors, each has the same p.d. and the branch currents add:

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} \Rightarrow \frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

A parallel combination has smaller resistance than its smallest branch resistance. A voltmeter connects in parallel across a component; an ideal voltmeter has infinite resistance. An ammeter connects in series; an ideal ammeter has zero resistance.

**Worked example.** A 6.0  $\Omega$  resistor is parallel to 3.0  $\Omega$ . The combination is in series with 4.0  $\Omega$  across 12 V.

- Known: circuit arrangement. Reduce the parallel section first, then apply the series rule.

$$R_p = \left( \frac{1}{6.0} + \frac{1}{3.0} \right)^{-1} \Omega = 2.0 \Omega$$

$$I = \frac{V}{R_p + R_s} = \frac{12 \text{ V}}{(2.0 + 4.0) \Omega} = 2.0 \text{ A}$$

$$V_p = IR_p = 2.0 \text{ A} \times 2.0 \Omega = 4.0 \text{ V}$$

$$I_6 = \frac{V_p}{6.0 \Omega} = 0.67 \text{ A} \quad I_3 = \frac{V_p}{3.0 \Omega} = 1.33 \text{ A}$$

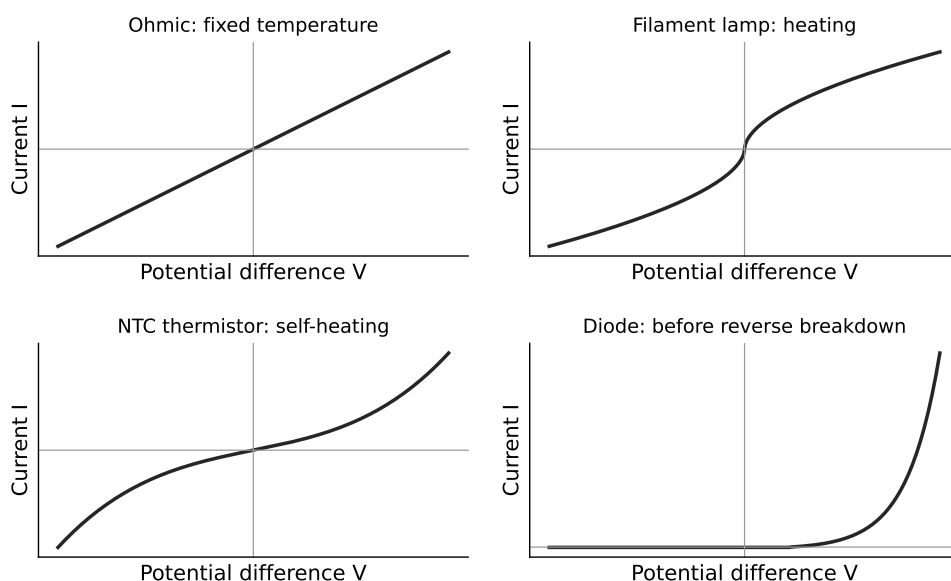
- Check: branch currents add to 2.0 A. The remaining series resistor has an 8.0 V drop.

## ■ Power and component graphs

**Electrical power** is energy transferred each second. Substituting  $V = IR$  into  $P = VI$  gives the other forms:

$$P = VI = I^2R = \frac{V^2}{R} \quad W = Pt = VIt$$

Use voltage across, and current through, the same component. The resistance forms apply to its operating-point resistance; do not assume a heating lamp has constant resistance.



*Axes and temperature conditions matter when interpreting these schematic curves.*

- Ohmic conductor at constant temperature: straight line through the origin. On an  $I$ -vertical,  $V$ -horizontal graph its gradient is  $1/R$ .
- Filament lamp: as voltage magnitude rises, heating increases resistance. The  $I$ - $V$  curve becomes less steep.
- NTC thermistor: heating lowers resistance. If current heats it sufficiently, the curve becomes steeper. At a controlled fixed temperature its behaviour over a suitable range can be approximately ohmic. Do not claim every thermistor curve must bend under every measurement condition.
- Diode: very small reverse current before breakdown; forward current rises rapidly after a characteristic knee. It conducts mainly in one direction. A real diode does not have one constant resistance.

For a curved graph,  $R = V/I$  uses a line from the origin to the operating point. The tangent gradient is not generally  $1/R$ .

## ■ Resistivity and core practical 7

**Resistivity**  $\rho$  is a material property at a stated temperature:

$$R = \frac{\rho l}{A} \quad A = \frac{\pi d^2}{4}$$

Lengthening a uniform wire increases resistance. Increasing cross-sectional area decreases it. Doubling diameter gives four times the area and one-quarter the resistance, if material, length and temperature stay unchanged.

For CP7, connect a uniform wire, ammeter, power supply, switch and current-limiting resistor in series. Connect a voltmeter across the measured wire length. Measure several lengths and find  $R = V/I$  for each. Plot  $R$  against  $l$ : gradient =  $\rho/A$ , so  $\rho = A \times \text{gradient}$ .

Measure diameter with a micrometer at several positions and orientations; check its zero. Measure length between the actual electrical contacts. Use a small current and switch off between readings to limit heating. Keep contacts secure. A non-zero intercept may indicate contact or lead resistance. Repeating readings reduces scatter but does not remove a diameter zero error. Because area depends on  $d^2$ , a small percentage diameter uncertainty contributes about twice that percentage to area uncertainty.

**Worked example.** A wire diameter is 0.40 mm. An  $R$ - $l$  graph has gradient  $3.5 \, \Omega \text{ m}^{-1}$ .

$$A = \frac{\pi d^2}{4} = \frac{\pi(0.40 \times 10^{-3} \text{ m})^2}{4} = 1.26 \times 10^{-7} \text{ m}^2$$

$$\rho = A \times \text{gradient} = 1.26 \times 10^{-7} \text{ m}^2 \times 3.5 \, \Omega \text{ m}^{-1} = 4.4 \times 10^{-7} \, \Omega \text{ m}$$

## ■ The charge-carrier model

The **drift velocity**  $v$  is the small mean directed velocity of charge carriers, not their random thermal speed:

$$I = nqvA$$

Here  $n$  is the number of mobile carriers per unit volume,  $q$  is charge magnitude per carrier, and  $A$  is cross-sectional area. A larger carrier density allows the same current with a smaller drift speed. In series wires the current is the same; changing  $n$  or  $A$  changes  $v$ .

Materials have different resistivities because their carrier densities and the ease of carrier motion differ. More scattering reduces drift speed for the same applied potential gradient. Carrier density alone does not explain every difference between materials.

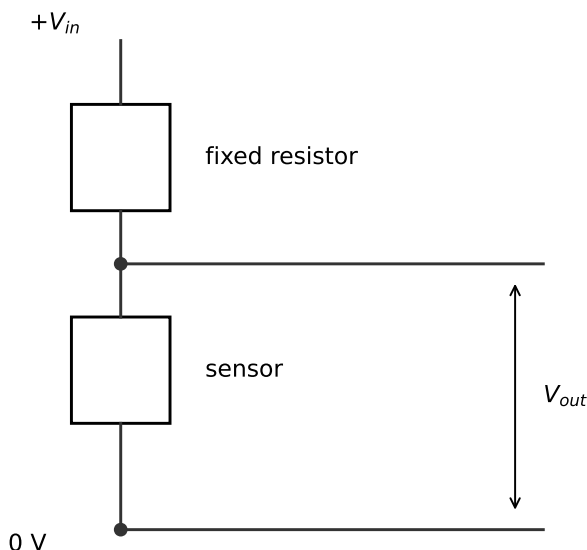
## Potential dividers, sensors and cells

Along a uniform wire carrying steady current at constant temperature, resistance grows in proportion to distance. The potential drop therefore grows in proportion to distance too. A potential–distance graph is straight. This assumes uniform material and area; a tapered or unevenly heated wire need not give a straight line.

A **potential divider** uses series resistances to share a supply voltage:

$$V_{\text{out}} = V_{\text{in}} \frac{R_{\text{across output}}}{R_1 + R_2}$$

The expression assumes the output is unloaded, or the attached load draws negligible current. If a load draws significant current, combine it in parallel with the output resistor first. Always identify which resistor the voltmeter spans.



*Swapping the sensor and fixed resistor reverses the direction of the output change.*

**Worked example.** A 6.0 V supply feeds a 2.0 k $\Omega$  fixed resistor above an LDR. Find the output across the LDR when its resistance changes from 4.0 to 1.0 k $\Omega$ .

$$V_{\text{dark}} = V_{\text{in}} \frac{R_{\text{LDR}}}{R_f + R_{\text{LDR}}} = 6.0 \text{ V} \times \frac{4.0}{2.0 + 4.0} = 4.0 \text{ V}$$

$$V_{\text{bright}} = V_{\text{in}} \frac{R_{\text{LDR}}}{R_f + R_{\text{LDR}}} = 6.0 \text{ V} \times \frac{1.0}{2.0 + 1.0} = 2.0 \text{ V}$$

The output across this LDR falls when illumination rises. The output across the fixed resistor rises. Explain the named output, not just “the voltage changes”.

### ■ Explain sensors using carriers

In a metal, heating increases lattice vibrations. Conduction electrons scatter more often. The mobile electron number stays approximately constant, but drift speed at a given potential gradient decreases, so resistance rises.

In a **negative-temperature-coefficient thermistor**, heating releases more mobile carriers. The carrier-density increase dominates, so resistance falls. Avoid saying heating reduces lattice vibrations.

In a **light-dependent resistor** (LDR), absorbed light releases additional mobile carriers. Increased illumination raises carrier density and lowers resistance. It does not need to work by electrons leaving its surface.

For linked explanations, give the full chain: environmental change → carrier/scattering change → resistance change → divider fraction change → named output change. Keep the circuit arrangement in view throughout.

### ■ E.m.f., internal resistance and core practical 8

**Electromotive force**  $\mathcal{E}$  is energy supplied by the source per unit charge. It is measured in volts, despite its name. **Internal resistance**  $r$  causes energy transfer within the source when current flows. The terminal p.d. is the energy per coulomb available to the external circuit:

$$\mathcal{E} = V + Ir \quad V = \mathcal{E} - Ir \quad I = \frac{\mathcal{E}}{R + r}$$

The source supplies power  $\mathcal{E}I$ , the external circuit receives  $VI$ , and internal heating is  $I^2r$ . These add consistently. With negligible current, terminal p.d. approaches e.m.f. For CP8, connect a cell, ammeter, variable load and switch in series. Connect a high-resistance voltmeter across the cell terminals. Vary the external resistance and record pairs of  $V$  and  $I$ . Plot  $V$  vertically against  $I$  horizontally. The best-fit line has intercept  $\mathcal{E}$  and gradient  $-r$ . Take the magnitude of the gradient for  $r$ .

Use several settings and repeat. Open the switch between readings, avoid very small load resistance, and keep the cell temperature and charge state as steady as possible. Never short-circuit the cell. A graph that curves may show changing internal resistance or e.m.f.; do not force one fixed  $r$  onto it without discussing the limitation.

**Worked example.** A cell graph passes through (0.20 A, 1.40 V) and (0.60 A, 1.20 V).

- Known: two points on the best-fit line. The gradient of  $V = \mathcal{E} - Ir$  is  $-r$ .

$$r = -\frac{\Delta V}{\Delta I} = -\frac{1.20 - 1.40}{0.60 - 0.20} \Omega = 0.50 \Omega$$

$$\mathcal{E} = V + Ir = 1.40 \text{ V} + 0.20 \text{ A} \times 0.50 \Omega = 1.50 \text{ V}$$

- Check using the second point:  $1.20 + 0.60 \times 0.50 = 1.50$  V too. The terminal p.d. falls as current rises.

## Check yourself

Use the matching exercise sheet after each section: 2.1 wave graphs; 2.2 standing waves; 2.3 refraction; 2.4 diffraction; 2.5 photons; 2.6 circuits; 2.7 dividers and cells. Before using a past-paper set, check that you can:

- read an axis and identify whether it gives wavelength, period, pressure or displacement;
- explain a practical with apparatus, measurements, controlled variables, graph and precautions;
- distinguish phase from amplitude, and photon frequency from photon arrival rate;
- derive circuit combinations from charge and energy conservation;
- explain a sensor circuit as a connected causal chain;
- use an actual comparison to finish “deduce whether”, with units and a clear conclusion.

The reviewed January and June 2025 papers supply authentic examples, but do not test every requirement. Original practice fills the teaching gaps. Completing a sampled paper alone does not prove full syllabus coverage.

# 3 · Practical Skills in Physics I

Pearson Edexcel · International A-Level · Physics

## A number needs a method

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Two groups measure the same wire. Their values differ. Which should you trust? You need more than the final number: the instrument, method, repeated readings and uncertainty all matter.

**WPH13 Practical Skills in Physics I** is a written paper about practical work from Units 1–2. It lasts 80 minutes and has 50 marks. All questions are compulsory. At least 20 marks assess mathematics at Level 2 or above. This guide supports real laboratory work; reading it does not replace making measurements yourself.

The three skill sheets follow the process: plan a valid investigation, collect and record measurements, then process results and justify conclusions. The official specification gives skill lists rather than numbered knowledge statements for this unit.

## Planning a valid investigation

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A **valid measurement** measures what you intend to measure. A precise timer does not help if it times the wrong event.

Start by naming the **independent variable** that you change and the **dependent variable** that you measure. List **control variables** that could also affect the dependent variable. Say how you will keep each important one steady.

### ■ Turn a description into a usable method

A method needs:

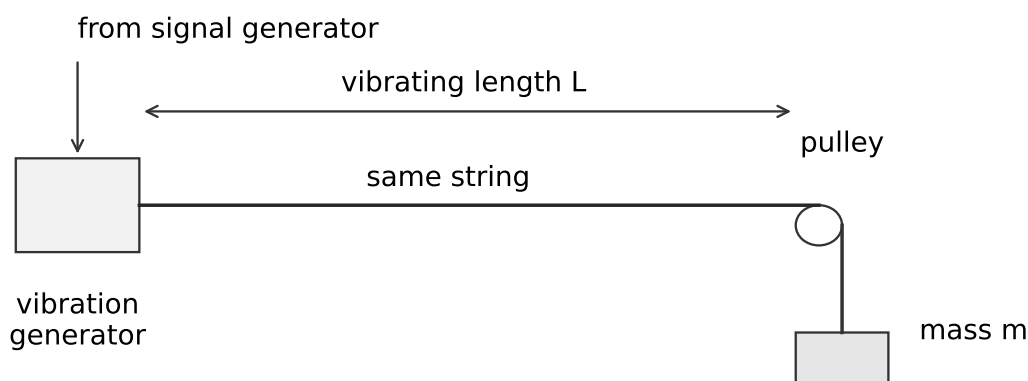
- named apparatus, with a suitable measuring range and **resolution** ;
- an arrangement showing where measurements are taken;
- steps that another student can follow, including the start and end events;
- several independent-variable settings over a useful range;
- repeats where they test stability and estimate variation;
- a calculation or graph that answers the investigation question;
- specific sources of uncertainty and improvements;
- realistic hazards with a practical way to reduce each risk.

Do not write only “take readings” or “use better equipment”. Name the reading, the instrument and what the improvement changes.

### ■ Worked planning example: string frequency and tension

Investigate how fundamental frequency depends on tension in a string.

- Use a vibration generator and signal generator. Pass the string over a pulley to a mass hanger.
- Change hanging mass  $m$ . With a freely moving pulley, tension is approximately  $T = mg$ .
- Measure vibrating length between the end nodes. Use the same string and keep that length fixed.
- Adjust frequency for the fundamental each time. The same mode is a control, not an optional detail.
- Measure the string's mass and total length to find linear density  $\mu$ . Keep it unchanged in this investigation.
- Record several tensions and their resonant frequencies. Repeat the tuning near each resonance.



The model for the fundamental is:

$$f = \frac{1}{2L} \sqrt{\frac{T}{\mu}} \quad \Rightarrow \quad f^2 = \frac{T}{4L^2\mu}$$

A graph of  $f^2$  vertically against  $T$  horizontally should be straight through the origin. A large triangle gives the gradient. Secure the stand and keep your feet away from falling masses. This is stronger than saying “be careful”.

## ■ Calibration and zero checks

**Calibration** checks an instrument against known reference values. A **zero error** occurs when it does not read zero at the correct zero condition. Check closed micrometer jaws or an empty balance before measuring.

If a micrometer reads  $+0.03$  mm with its jaws correctly closed, subtract  $0.03$  mm from each subsequent reading. Repeating the measurement alone does not remove that offset. A zero check cannot prove that every point on the scale is calibrated correctly.

## ■ Range, spacing and repeats

Choose a range wide enough to reveal the relationship. Several measurements crowded near one setting may give a poor gradient. Use more settings near a turning point if you need to locate a maximum accurately.

Repeats are useful when conditions can be reproduced. They show scatter and help identify unstable readings. However, repeated readings of a discharging cell or a heating wire may drift because the conditions change. Reduce the current, switch off between readings, control temperature or restore conditions. Do not average a drift as though it were random variation.

## ■ Control the actual cause

When varying the angle of a lamp above a solar cell, keep lamp–cell distance and lamp output fixed. Control background light, for example with shielding or a darkened room. Keep the cell and its load the same. Otherwise a change in measured power may come from distance, illumination or circuit resistance instead of angle.

Discuss context using a physical cause. A tracking solar panel can stay closer to normal incidence and collect more energy over a day. Its motor also uses energy and adds cost or maintenance. “Better for the environment” alone does not explain either effect.

## Measuring and recording

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**Accuracy** describes closeness to the true value. **Precision** describes how closely repeated values cluster. Closely clustered results can still all share a systematic offset.

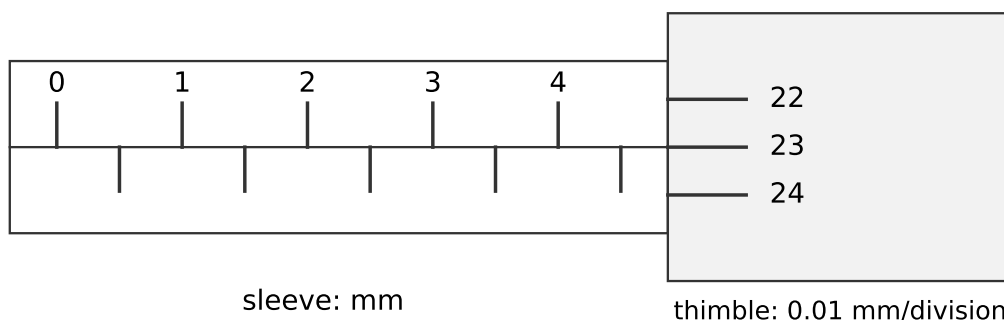
- **Repeatability** concerns similar results using the same operator and method over a short time.
- **Reproducibility** concerns similar results from different operators, apparatus or methods.
- **Random effects** cause unpredictable variation. Repeating and taking a mean can reduce their influence.
- **Systematic error** shifts results in a consistent way. Correct the cause or known offset; repeating does not remove it.
- **Uncertainty** is a reasonable interval associated with a measurement. It is not automatically a mistake or the known size of the error.

## ■ Read instruments at their resolution

A millimetre ruler has a smallest interval of 1 mm. The specification's standard Vernier calipers resolve 0.1 mm, and its standard micrometer resolves 0.01 mm. Check the instrument shown: a digital display or a different scale may have a different resolution.

For a micrometer, add the visible sleeve reading and the thimble reading. Include a visible half-millimetre sleeve mark when appropriate. Use the ratchet for consistent contact pressure, rather than overtightening the jaws.

For Vernier calipers, read the main scale just before the Vernier zero, then add the aligned Vernier division times its resolution. Check that the jaws contact the intended surfaces without tilting. Do not infer a missing scale from an extracted text description: read the instrument diagram itself.



*Read the sleeve before adding the aligned thimble division.*

**Worked example.** The last visible sleeve mark is 4.5 mm. Thimble division 23 aligns with the reference line. Resolution is 0.01 mm and zero error is +0.02 mm.

$$d_{\text{indicated}} = d_{\text{sleeve}} + d_{\text{thimble}} = 4.5 \text{ mm} + 23 \times 0.01 \text{ mm} = 4.73 \text{ mm}$$

$$d_{\text{corrected}} = d_{\text{indicated}} - d_{\text{zero}} = 4.73 \text{ mm} - 0.02 \text{ mm} = 4.71 \text{ mm}$$

## ■ Geometry and timing techniques

Avoid **parallax** by viewing a scale along the correct line of sight. Use a set square to transfer a height or position onto a ruler. Keep the ruler parallel to the distance being measured; a sloping ruler measures a different distance.

A light gate measures the blocking time of an interrupting object. With known interrupting length  $l$ , speed is  $v = l/t$ . That gives speed during passage, not acceleration by itself. To find acceleration, measure a change of speed over known time, or use a justified motion equation with additional measured quantities. Do not call  $l/t$  acceleration.

A video with known frame rate can reduce reaction-time uncertainty. Count frames between clearly defined start and end events, then use  $t = N/f_{\text{frame}}$ . The frame interval limits timing resolution. Blurred images or unclear event positions can still limit the result.

## ■ Tables and significant figures

Put quantity and unit together in each column heading, for example  $l/\text{mm}$ . Record raw measurements with decimal places matching instrument resolution. Do not add extra digits that the instrument cannot resolve.

| Reading | Time / s |
|---------|----------|
| 1       | 5.12     |
| 2       | 5.24     |
| 3       | 5.18     |
| 4       | 5.20     |

These all have the same two decimal places. A calculated mean can be kept with extra digits during working, then reported sensibly. Processed values for plotting are commonly given to three significant figures, unless the question or data requires otherwise. Do not mix 5.1, 5.24 and 5.180 as if they came from one unchanged display.

An **anomalous reading** does not fit the pattern. Check the reading, method and repeat measurement if possible. Do not silently delete the least convenient value. A suspected anomaly needs a reason and a recorded decision.

## Uncertainty in this unit

For one reading, this specification uses half the instrument resolution as the basic absolute uncertainty estimate. This does not mean resolution is the only source of uncertainty. A poorly defined endpoint, reaction time or alignment may make the uncertainty larger.

$$\text{percentage uncertainty} = \frac{\text{absolute uncertainty}}{\text{measured value}} \times 100\%$$

■ **Worked example.** A balance displays 135.0 g with resolution 0.1 g.

$$\Delta m = \frac{0.1 \text{ g}}{2} = 0.05 \text{ g}$$

$$\text{percentage uncertainty} = \frac{0.05 \text{ g}}{135.0 \text{ g}} \times 100\% = 0.037\%$$

For repeated values, the specification uses **half range** as an uncertainty estimate:

$$\bar{x} = \frac{\sum x}{N} \quad \Delta x = \frac{x_{\max} - x_{\min}}{2}$$

**Worked example.** Repeated times are 5.12, 5.24, 5.18, 5.20 s.

$$\bar{t} = \frac{5.12 + 5.24 + 5.18 + 5.20}{4} \text{ s} = 5.185 \text{ s}$$

$$\Delta t = \frac{5.24 - 5.12}{2} \text{ s} = 0.06 \text{ s}$$

$$\text{percentage uncertainty} = \frac{\Delta t}{\bar{t}} \times 100\% = \frac{0.06}{5.185} \times 100\% = 1.2\%$$

Report about  $(5.19 \pm 0.06)$  s. Keep the unrounded mean in the percentage calculation. The repeat spread is much larger than half a hundredth of a second, so quoting display resolution alone would miss the observed variation.

Unit 3 does not require compounding percentage uncertainties in a calculated quantity. If an uncertainty for that quantity is supplied, use it directly. Do not introduce an advanced propagation rule as a requirement here.

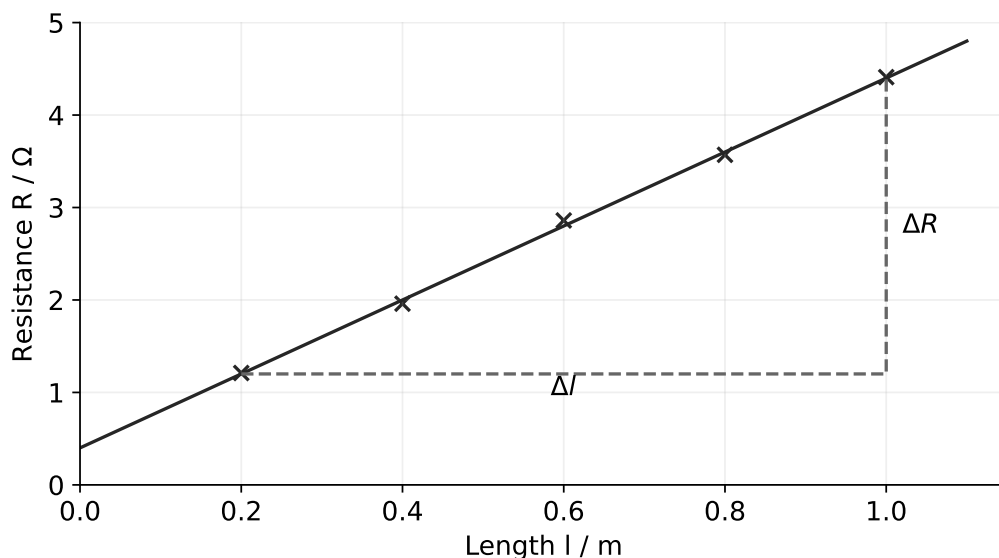
## Graphs and processing results

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### ■ Choose scales that show the data

Read which quantity belongs on each axis. Use clear quantity/unit labels and regular scales. Use a large part of the available grid; avoid awkward intervals that are hard to subdivide. Neither axis must always start at zero, but do not hide whether a proposed proportional relationship passes through the origin.

Plot small crosses accurately. Draw a thin best-fit line or smooth curve according to the relationship. Do not join noisy points with a zigzag unless the question requests it. A best-fit straight line balances scatter rather than passing through every point.



Use distant points on the best-fit line, not a tiny pair of neighbouring data points.

### ■ Turn a gradient into a physical constant

For a line  $y = mx + c$ , identify which part of the physical equation matches  $m$  and  $c$ . A straight line does not prove direct proportionality if its intercept is non-zero.

**Worked example.** A wire follows  $R = (\rho/A)l + R_0$ . A best-fit line passes through (0.20 m, 1.20 Ω) and (1.00 m, 4.40 Ω).

- Known: two widely separated points on the fitted line. Why: the slope equals  $\rho/A$ .

$$m = \frac{\Delta R}{\Delta l} = \frac{4.40 - 1.20}{1.00 - 0.20} \Omega \text{ m}^{-1} = 4.00 \Omega \text{ m}^{-1}$$

$$R_0 = R - ml = 1.20 \Omega - 4.00 \Omega \text{ m}^{-1} \times 0.20 \text{ m} = 0.40 \Omega$$

If  $A = 1.0 \times 10^{-7} \text{ m}^2$ , the resistivity is:

$$\rho = mA = 4.00 \Omega \text{ m}^{-1} \times 1.0 \times 10^{-7} \text{ m}^2 = 4.0 \times 10^{-7} \Omega \text{ m}$$

The fixed intercept may represent contact or lead resistance. Dividing one measured  $R$  by  $l$  would include that offset and give the wrong slope.

### ■ Scale factors belong in the gradient

Suppose an axis is labelled  $f/\text{MHz}$ . A gradient read from that graph has MHz in its units. Convert to Hz before using SI constants. A graph of  $f$  against  $\sin \theta$  has a dimensionless horizontal axis; its gradient therefore has frequency units. Use degree mode if the measured angle is in degrees.

If a supplied equation is  $f = (v/\lambda) \sin \theta$ , a graph of  $f$  against  $\sin \theta$  has gradient  $v/\lambda$ . Rearrange  $v = \lambda \times \text{gradient}$ . Do not assume every frequency graph has gradient equal to speed.

### ■ Curves, maxima and specific improvements

A broad range first locates a maximum. Then take more closely spaced readings around that region. Repeating one point does not locate the maximum between existing points. Keep the other conditions steady, and consider scatter when quoting the best angle or resistance.

For a motion experiment, a set square can improve a horizontal-distance measurement. For a lamp experiment, a dark enclosure can reduce changing background illumination. For a resonance experiment, approach the loudest sound from both higher and lower frequencies and repeat. Each improvement targets a named limitation.

## Conclusions supported by uncertainty

A value with uncertainty describes an interval. Compare that interval with the proposed value, then state what the evidence supports. Agreement within uncertainty does not prove that a material or model is uniquely identified.

**Worked example.** A measured density is  $8.90 \text{ g cm}^{-3}$  with percentage uncertainty 0.9%. Could it agree with a proposed value  $8.94 \text{ g cm}^{-3}$ ?

$$\Delta\rho = \frac{0.9}{100} \times 8.90 \text{ g cm}^{-3} = 0.080 \text{ g cm}^{-3}$$

The interval is about  $8.82$  to  $8.98 \text{ g cm}^{-3}$ . The proposed value lies inside it, so the measurement is consistent with that value. It does not prove the object has that composition.

If two model predictions both lie inside the interval, the data does not distinguish them. Reduce the dominant uncertainty or add a different measurement before claiming a unique identification.

### Check yourself

Before attempting a written practical problem, check that you can:

- turn apparatus names into a usable method with controls and a processing route;
- read resolution and correct a known zero offset;
- distinguish accuracy, precision, repeatability and reproducibility;
- calculate a mean, half range and percentage uncertainty;
- plot data, use a large gradient triangle and interpret an intercept;
- turn a supplied uncertainty into an interval and give a justified conclusion;
- link each improvement to the measurement problem it actually reduces.

# 4 · Further Mechanics, Fields and Particles

Pearson Edexcel · International A-Level · Physics

## From a collision to a particle accelerator

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A car bumper and a particle detector seem very different. Both use momentum and forces. This unit follows those ideas into circular motion, electric fields, magnetic fields and particle interactions.

WPH14 assesses requirements 81–124 of the Issue 3 specification. It can also use knowledge from Units 1–2. Use the seven skill sheets in order. Check units, directions and the assumptions behind each equation. A familiar equation can give a wrong answer when its model does not fit.

## Impulse and two-dimensional collisions

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### ■ Force changes momentum

**Momentum** is the vector  $\mathbf{p} = m\mathbf{v}$ . Choose a positive direction before using signs. **Impulse** is the change in momentum. For constant force, or the average force over an interval:

$$F_{\text{average}}\Delta t = \Delta p = m(v - u)$$

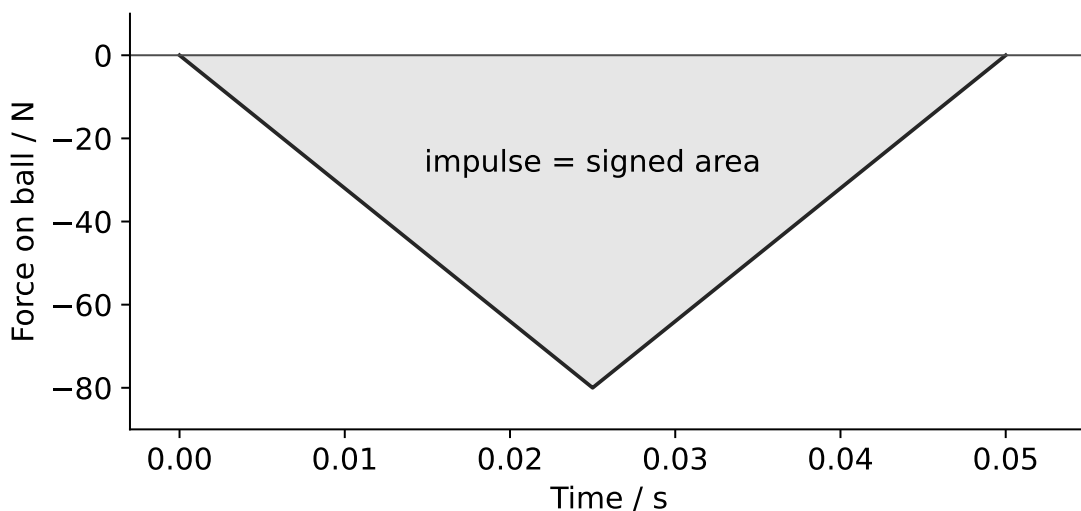
A changing force gives an impulse equal to the signed area under its force–time graph. A triangular pulse has area  $\frac{1}{2} \times \text{base} \times \text{height}$ . An area below the time axis is negative. The maximum force is not automatically the average force.

**Worked example.** A 0.20 kg ball approaches a wall at  $+6.0 \text{ m s}^{-1}$ . It rebounds at  $-4.0 \text{ m s}^{-1}$ . Contact lasts 0.050 s. Find average force on the ball.

- Known: mass, signed initial/final velocities and contact time.
- Why: force impulse changes the ball's momentum; rebound reverses the sign.

$$\Delta p = m(v - u) = 0.20 \text{ kg} \times (-4.0 - 6.0) \text{ m s}^{-1} = -2.0 \text{ kg m s}^{-1}$$

$$F_{\text{average}} = \frac{\Delta p}{\Delta t} = \frac{-2.0 \text{ kg m s}^{-1}}{0.050 \text{ s}} = -40 \text{ N}$$



The force is opposite the chosen positive direction. The wall feels an opposite force. A deforming bumper or helmet increases stopping time. For the same momentum change, this reduces average force. It does not remove the momentum change.

### ■ Conserve each momentum component

An **isolated system** has no significant external impulse during the event. Its total momentum is conserved. Internal forces cancel in opposite pairs. Momentum conservation does not require kinetic energy conservation.

For a collision in a plane, resolve every momentum into two perpendicular axes:

$$\sum p_{x,\text{before}} = \sum p_{x,\text{after}} \quad \sum p_{y,\text{before}} = \sum p_{y,\text{after}}$$

Use the angle measured from the chosen axis. Include signs; a downward component is negative if upwards is positive. Do not add momentum magnitudes when directions differ.

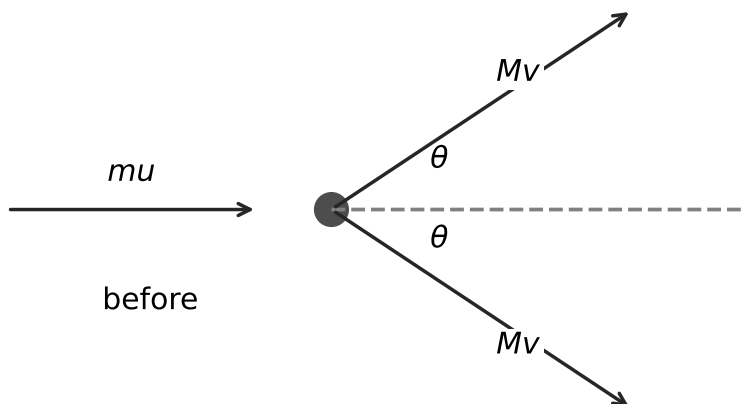
**Worked example.** A 0.10 kg sphere moving at  $2.0 \text{ m s}^{-1}$  strikes two stationary spheres, each 0.10 kg. It stops. The two others leave symmetrically at  $1.2 \text{ m s}^{-1}$ , at angles  $+\theta$  and  $-\theta$  to the original direction. Find their separation angle and classify the collision.

- Why: transverse momenta cancel by symmetry; longitudinal momentum gives  $\theta$ .

$$mu = 2Mv \cos \theta$$

$$\cos \theta = \frac{mu}{2Mv} = \frac{0.10 \text{ kg} \times 2.0 \text{ m s}^{-1}}{2 \times 0.10 \text{ kg} \times 1.2 \text{ m s}^{-1}} = 0.833$$

$$2\theta = 2 \cos^{-1} \left( \frac{mu}{2Mv} \right) = 2 \cos^{-1}(0.833) = 67.1^\circ$$



An **elastic collision**

conserves total kinetic energy as well as momentum.

An **inelastic collision**

other forms of energy.

does not conserve kinetic energy; some becomes

$$E_{k,\text{before}} = \frac{1}{2}mu^2 = \frac{1}{2} \times 0.10 \times 2.0^2 \text{ J} = 0.20 \text{ J}$$

$$E_{k,\text{after}} = 2 \times \frac{1}{2}Mv^2 = 2 \times \frac{1}{2} \times 0.10 \times 1.2^2 \text{ J} = 0.144 \text{ J}$$

This collision is inelastic. Total energy is still conserved. In an ordinary passive collision, kinetic energy cannot increase without an additional energy source.

For a **non-relativistic particle**

, use  $p = mv$  and  $E_k = \frac{1}{2}mv^2$ . Substi-

tuting  $v = p/m$  gives:

$$E_k = \frac{1}{2}m \left( \frac{p}{m} \right)^2 = \frac{p^2}{2m}$$

At equal kinetic energy,  $p = \sqrt{2mE_k}$ : a larger mass has larger momentum. At equal momentum, a larger mass has smaller kinetic energy. These equations are not valid for a particle moving close to light speed.

## ■ Core practicals 9 and 10

For **core practical 9**, use a trolley, force sensor and data logger. Measure trolley mass and velocities just before/after contact using light gates or a motion sensor. Record the force–time pulse at a sampling rate fast enough to resolve it. Zero the force sensor; use consistent axes for force and velocity. Integrate the graph area and compare it with  $m(v - u)$ . Repeat for different contact forces or speeds. A graph of impulse against momentum change should have gradient close to one. Friction, sensor offsets and missed parts of the pulse can cause disagreement. Secure the track and catch the trolley safely.

For **core practical 10**, record small spheres colliding on a level surface using an overhead camera. Include a length scale in the collision plane and a known frame rate. Keep the camera perpendicular to reduce perspective error. Track centres over several frames before and after the short collision. Convert pixel displacements to metres and frame intervals to seconds. Determine both velocity components, then compare total momentum components and kinetic energies. Repeat with different approach directions. Include uncertainty from positions, timing and scale calibration. A small mismatch within uncertainty is not evidence that momentum fails. Keep spheres contained so they cannot fall or become a slipping hazard.

## Circular motion

### ■ Angle, angular speed and tangential speed

An angle of one **radian** subtends an arc equal to the radius. Hence  $\theta = s/r$ , with angle in radians. One revolution is  $2\pi$  radians or  $360^\circ$ .

$$\theta_{\text{rad}} = \theta_{\text{degrees}} \frac{2\pi}{360} \quad \omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} \quad v = \omega r$$

**Angular velocity** describes the rate of angular displacement. In the plane problems here, use its signed rotational rate or magnitude as appropriate. Points on one rigid turntable have the same angular speed. Points farther from its centre have larger **tangential speed**. Their velocity directions also vary around the circle.

**Worked example.** A wheel rotates at 120 revolutions per minute. Find angular speed and acceleration of a point 0.25 m from its centre.

$$f = \frac{N}{t} = \frac{120}{60 \text{ s}} = 2.0 \text{ Hz}$$

$$\omega = 2\pi f = 2\pi \times 2.0 \text{ s}^{-1} = 12.6 \text{ rad s}^{-1}$$

$$a = r\omega^2 = 0.25 \text{ m} \times (4\pi \text{ s}^{-1})^2 = 39.5 \text{ m s}^{-2}$$

Keep the unrounded angular speed during the calculation. Use radius, not diameter.

## ■ Why acceleration points inward

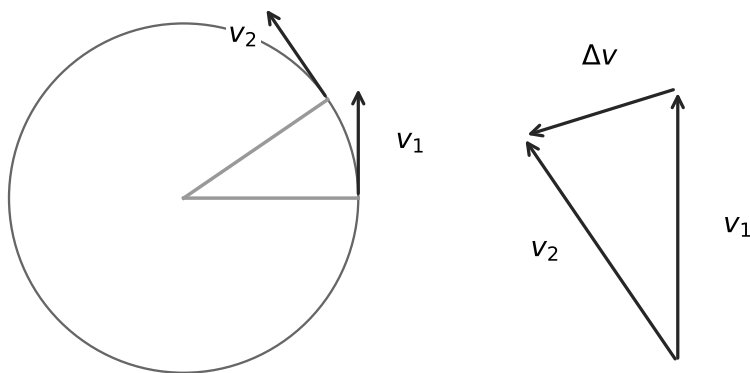
Uniform circular motion has constant speed but changing velocity. Draw the initial and final tangential velocity vectors. The change is  $\Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$ , not their sum. For a small angular change, the velocity triangle and radius triangle are similar:

$$\frac{\Delta v}{v} \approx \frac{\Delta s}{r} \quad \Delta s \approx v \Delta t$$

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{r} = r\omega^2$$

Tangential velocity vectors

$$\Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$$



The change in velocity points towards the centre in this limit. A resultant **centripetal force** is therefore needed:

$$\sum F_{\text{inward}} = ma = \frac{mv^2}{r} = mr\omega^2$$

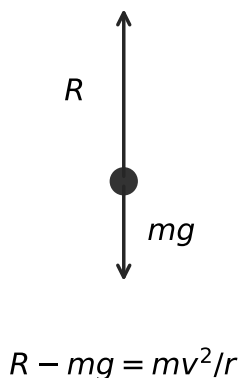
Centripetal force names the resultant of real forces. It is not an extra force to add to a force diagram. Friction can provide it on a turntable. Tension can provide it for a ball on a string.

## ■ Write the real-force equation

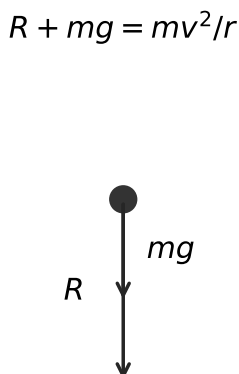
For clothing against the inside of a vertical drum, weight always points down. The drum's normal reaction points towards the centre. At constant speed:

$$\text{bottom: } R - mg = \frac{mv^2}{r} \quad \text{top: } R + mg = \frac{mv^2}{r}$$

Bottom of drum



Top of drum



The required resultant has the same magnitude, but the reaction is larger at the bottom. If contact is lost, the surface cannot provide a pulling normal reaction. Check this before using a circular-path model.

For a horizontal turntable, the largest available friction must be at least  $mr\omega^2$ . If maximum friction is  $mg/25$ :

$$mr\omega^2 \leq \frac{mg}{25} \Rightarrow r \leq \frac{g}{25\omega^2}$$

Mass cancels because both required and available forces scale with mass. If the inward force disappears, motion initially follows the tangent. Gravity can then curve the later path; “tangent” describes the release direction.

## Electric fields and potential

### ■ Force and potential describe different things

An **electric field** is a region where a charged particle experiences a force. **Electric field strength** is force per unit positive test charge:

$$\mathbf{E} = \frac{\mathbf{F}}{q} \quad \mathbf{F} = q\mathbf{E}$$

Its units are  $\text{N C}^{-1}$  or  $\text{V m}^{-1}$ . A positive charge feels force along the field; a negative charge feels force opposite it.

**Electric potential** is potential energy per unit charge relative to a chosen reference. For a charge  $q$  moved through a potential change:

$$\Delta U = q\Delta V \quad W_{\text{field}} = -q\Delta V$$

Potential is a scalar; field is a vector. A point with zero potential need not have zero field. Adding signed potentials is different from adding field vectors.

## ■ Radial and uniform fields

For point charges, or outside an isolated charged conducting sphere, with  $k = 1/(4\pi\epsilon_0)$ :

$$|F| = \frac{k|Q_1Q_2|}{r^2} \quad |E| = \frac{k|Q|}{r^2} \quad V = \frac{kQ}{r}$$

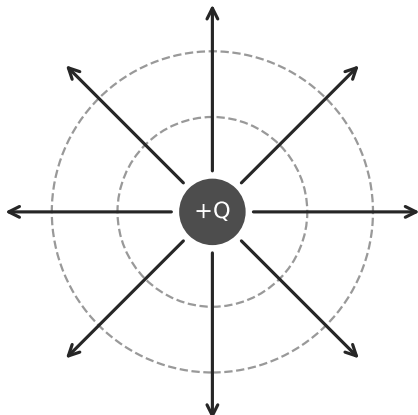
Here  $r$  is separation of charge centres; sphere distances are measured from its centre. Like charges repel, unlike charges attract. Potential takes the sign of  $Q$  when zero is at infinity.

Between large parallel plates, away from edges, the field is approximately uniform:

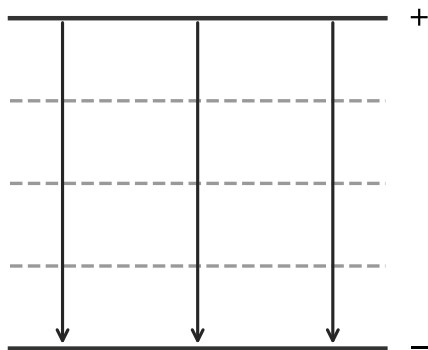
$$|E| = \frac{|\Delta V|}{d}$$

The field points from higher to lower potential. Plate separation  $d$  is perpendicular to the plates. An **equipotential** has the same potential everywhere. Field lines cross equipotentials at right angles. Moving along one involves no change in electric potential energy.

Radial field and equipotentials



Uniform field between plates



The radial field is the negative potential gradient:  $E_r = -dV/dr$ . For a positive source, the signed area under its  $E$ - $r$  curve from  $r$  to infinity equals  $V(r)$ . The area from zero to  $r$  is not that potential. In a uniform field, closer equally spaced potential levels mean stronger field.

**Worked example.** A sphere of radius 0.20 m has potential 9.0 kV. Find charge and field at 0.30 m from its centre. Use  $k = 8.99 \times 10^9 \text{ N m}^2\text{C}^{-2}$ .

$$Q = \frac{Vr}{k} = \frac{9.0 \times 10^3 \text{ V} \times 0.20 \text{ m}}{8.99 \times 10^9 \text{ N m}^2\text{C}^{-2}} = 2.00 \times 10^{-7} \text{ C}$$

$$E = \frac{kQ}{r^2} = \frac{8.99 \times 10^9 \times 2.00 \times 10^{-7}}{0.30^2} \text{ N C}^{-1} = 2.00 \times 10^4 \text{ N C}^{-1}$$

Use the surface radius to find charge, then the new centre-distance to find field. Do not reuse the surface distance for the second step.

### ■ Compare electrical and gravitational forces

A stationary charged oil drop can satisfy  $|q|E = mg$ . Use  $m = \rho\mathcal{V}$  if density and volume are given. Divide the charge magnitude by  $e = 1.60 \times 10^{-19} \text{ C}$  to test whether it is close to an integer multiple. Charge sign follows the direction of force required, not its magnitude alone.

To lift a spherical grain, compare  $qE$  with  $\rho(4\pi r^3/3)g$ . Radius is half the listed diameter. A calculated maximum diameter is a threshold: choose the largest listed diameter below it. On an inclined panel, the normal component of weight is  $mg \cos \theta$ . The electric force perpendicular to the panel need overcome that component to detach a grain; gravity also has a downslope component.

## Capacitors and RC circuits

### ■ Charge storage and energy

A **capacitor** stores separated charge on two conductors. They carry equal and opposite charges in the ideal two-plate model. The quoted charge  $Q$  is the magnitude on either plate, not the sum of both magnitudes. **Capacitance** is charge stored per unit p.d.:

$$C = \frac{Q}{V}$$

One farad is one coulomb per volt. For constant capacitance, a  $V$ - $Q$  graph is straight through the origin with gradient  $1/C$ . Its area gives the work needed to transfer charge onto the capacitor:

$$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

The factor one half appears because p.d. rises from zero while charge is transferred. It is not  $QV$  at the final voltage throughout charging. For discharge between non-zero voltages, subtract the two stored energies:

$$\Delta W = \frac{1}{2}C(V_{\text{initial}}^2 - V_{\text{final}}^2)$$

This is different from  $\frac{1}{2}C(V_{\text{initial}} - V_{\text{final}})^2$ .

## ■ Charging and discharging

In an ideal series resistor–capacitor charging circuit, initially uncharged, the capacitor p.d. is zero. The resistor initially has the full supply p.d., so current is maximum. As charge builds, capacitor p.d. rises. Resistor p.d. and current fall. At full charge, current is zero and capacitor p.d. equals supply p.d.

For discharge through a fixed resistance, use signed current consistently or work with its magnitude:

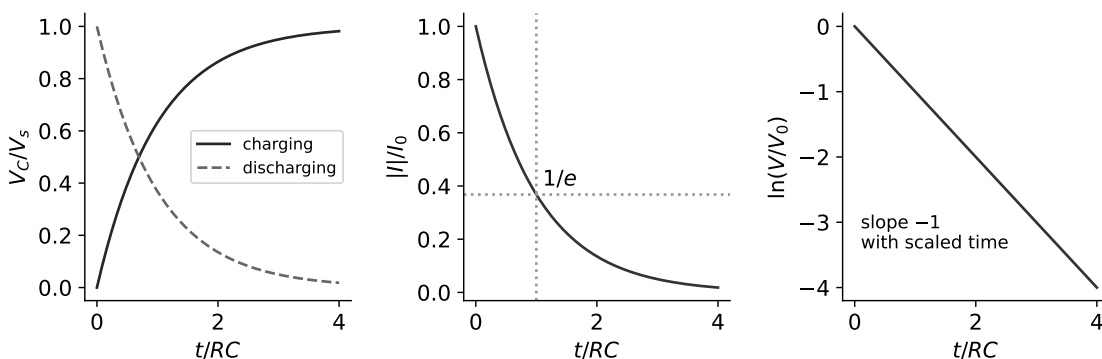
$$Q = Q_0 e^{-t/RC} \quad V = V_0 e^{-t/RC} \quad |I| = I_0 e^{-t/RC}$$

The voltage relation follows from  $Q = CV$  with constant  $C$ . Current magnitude follows from  $|I| = V/R$  with constant  $R$ . The **time constant** is  $\tau = RC$ . After one time constant,  $Q$ ,  $V$  and current magnitude are  $e^{-1} \approx 0.368$  of their initial values. After three time constants they are about 5%, not zero. Stored energy falls faster because it depends on  $V^2$ .

For charging from an ideal constant supply  $V_s$ :

$$V_C = V_s(1 - e^{-t/RC}) \quad V_R = V_s e^{-t/RC} \quad I = \frac{V_s}{R} e^{-t/RC}$$

A charging capacitor voltage rises towards  $V_s$ ; charging current falls towards zero. Do not sketch the same curve for both quantities. Equal  $V_C$  and  $V_R$  means each is  $V_s/2$ , reached at  $t = RC \ln 2$ .



Taking natural logs of the discharge equation gives:

$$\ln V = \ln V_0 - \frac{t}{RC}$$

Thus a graph of  $\ln V$  against  $t$  has gradient  $-1/(RC)$  and intercept  $\ln V_0$ . The same forms hold for  $Q$  and current magnitude. Use consistent measurement units inside the logged numerical values.

**Worked example.** A  $100\ \mu\text{F}$  capacitor discharges from  $12.0\ \text{V}$  through  $20.0\ \text{k}\Omega$ . Find p.d. after  $3.0\ \text{s}$  and energy lost.

$$\tau = RC = 20.0 \times 10^3\ \Omega \times 100 \times 10^{-6}\ \text{F} = 2.00\ \text{s}$$

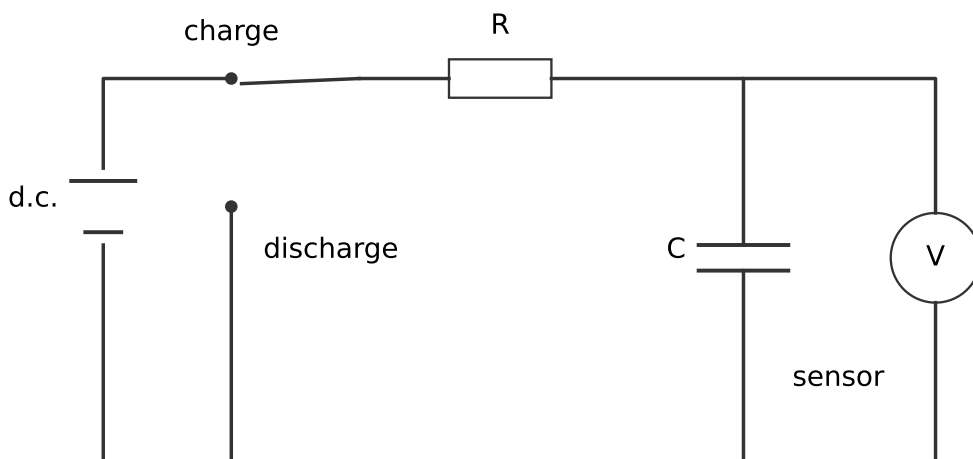
$$V = V_0 e^{-t/RC} = 12.0\ \text{V} \times e^{-3.0/2.00} = 2.68\ \text{V}$$

$$\Delta W = \frac{1}{2}C(V_0^2 - V^2) = \frac{1}{2} \times 100 \times 10^{-6}\ \text{F} \times (12.0^2 - 2.68^2)\ \text{V}^2 = 6.84 \times 10^{-3}\ \text{J}$$

The lost stored energy becomes mainly heat in the discharge resistance. Use unrounded voltage in the final calculation.

### ■ Core practical 11 and useful extensions

Use a low-voltage d.c. supply, resistor, capacitor and switch. Charge through the resistor, then disconnect the supply and discharge through the known resistance. Connect a voltage sensor, data logger or oscilloscope across the capacitor. Record time and p.d.; choose a sampling interval short compared with  $RC$ . Use an instrument input resistance large compared with the discharge resistance, otherwise it changes the discharge path.



Keep  $R$  and  $C$  fixed, repeat after restoring the same initial charge, and compare measured curves with exponential predictions. Determine  $RC$  from the  $1/e$  level or a log-fit gradient. Avoid taking logs of readings near zero where relative uncertainty is large. For a resistance-tolerance test, calculate  $R = -\Delta t / [C \ln(V_2/V_1)]$  during one uninterrupted discharge and compare with both limits of the permitted interval.

Use correct polarity for an electrolytic capacitor and remain below its voltage rating. Discharge it safely through a resistor before altering connections; do not short a charged capacitor. Leakage, sensor loading and resistor heating can change the measured curve.

A smoothing capacitor charges near supply peaks and discharges into the load between them. A smaller load resistance gives a smaller time constant, faster discharge and less smoothing. For two identical series capacitors, the same magnitude of charge appears on each. Their equal p.d.s share the supply, so each stores half the charge of one identical capacitor connected alone. This sharing is a useful bridge to the selected paper; it does not replace the main capacitance model.

## Magnetic forces and induction

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### ■ Directions, flux and force

**Magnetic flux density**  $B$  describes the field's force effect. For a straight wire of length  $l$  in a uniform field:

$$F = BIl \sin \theta$$

Here  $\theta$  is between conventional current and field. Only the length inside the field counts. For one charged particle:

$$F = |q|vB \sin \theta$$

Use **Fleming's left-hand rule** : first finger along field, second along conventional current, thumb gives force. A positive charge moves with conventional current. Reverse the result for an electron's motion. Dot symbols mean out of the page; crosses mean into it.

Magnetic force is perpendicular to velocity, so it changes direction without doing work. It cannot, alone, increase the particle's kinetic energy. If  $\mathbf{v}$  is parallel to  $\mathbf{B}$ , magnetic force is zero.

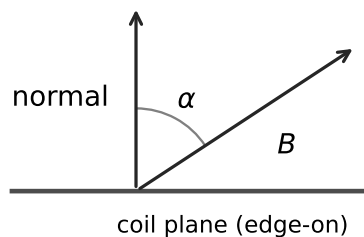
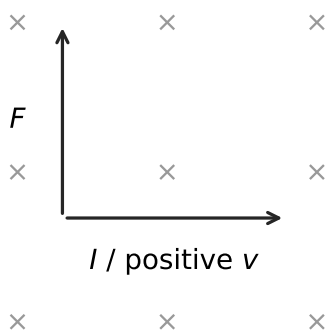
**Magnetic flux** through a flat area in a uniform field is:

$$\phi = BA \cos \alpha$$

Here  $\alpha$  is between field and the area's normal, not the plane itself. The unit is the weber. **Flux linkage** is  $N\phi$  when the same flux links each of  $N$  turns.

Field into page; force upwards

Flux uses angle to area normal



$$\phi = BA \cos \alpha$$

**Worked example.** A coil has 40 turns and radius 0.015 m. A uniform 0.020 T field is perpendicular to its plane. Find flux linkage.

$$A = \pi r^2 = \pi(0.015 \text{ m})^2 = 7.07 \times 10^{-4} \text{ m}^2$$

$$N\phi = NBA = 40 \times 0.020 \text{ T} \times 7.07 \times 10^{-4} \text{ m}^2 = 5.65 \times 10^{-4} \text{ Wb}$$

Use radius, not diameter, and multiply by turns only when finding linkage. A balance under magnets measures the opposite reaction to a force on a separately supported wire. Convert a balance reading change  $\Delta m$  to force  $\Delta mg$ , not  $\Delta m$  itself.

### ■ A changing linkage induces an e.m.f.

**Electromagnetic induction** occurs when flux linkage changes. Relative movement of a coil and magnet can cause it. Rotating the coil or changing current in another linked coil can also cause it. A stationary coil in a constant field has no induced e.m.f. merely because flux is present.

**Faraday's law** relates e.m.f. magnitude to rate of flux-linkage change. **Lenz's law** gives the opposing direction:

$$\mathcal{E} = -\frac{d(N\phi)}{dt} \quad |\mathcal{E}_{\text{average}}| = \frac{|\Delta(N\phi)|}{\Delta t}$$

Induced current, if the circuit is closed, produces a magnetic effect opposing the change that caused it. It does not always oppose the existing field: when that field decreases, the induced effect helps maintain it. This opposition is consistent with energy conservation.

**Worked example.** Flux through each of 50 turns decreases from  $6.0 \times 10^{-5}$  to  $2.0 \times 10^{-5}$  Wb in 0.020 s. Find average induced e.m.f. magnitude.

$$|\mathcal{E}| = \frac{N|\phi_2 - \phi_1|}{\Delta t} = \frac{50 \times |2.0 - 6.0| \times 10^{-5} \text{ Wb}}{0.020 \text{ s}} = 0.10 \text{ V}$$

Faster movement, stronger field, more turns or larger linked area can increase the rate. Explain which quantity actually changes. In a magnetic-strip reader, reversing pole orientation reverses signal direction. For a straight boundary swept across an area of width  $L$  at speed  $v$ ,  $\Delta A/\Delta t = Lv$ , giving  $|\mathcal{E}| = NBLv$  under that uniform-field model.

Wireless charging uses alternating current in one coil, producing changing magnetic field and changing linkage in the phone coil. That induces an e.m.f. The receiving circuit can then charge the battery. Equal turn counts do not ensure equal voltages: some flux may fail to link the other coil, especially with separation or without an iron core.

## Nuclear structure, accelerators and tracks

### ■ The nucleus and scattering evidence

In  ${}_Z^AX$ , **proton number**  $Z$  counts protons and **nucleon number**  $A$  counts protons plus neutrons. The neutron count is  $A - Z$ . A neutral atom has  $Z$  electrons; an ion need not.

The older distributed-positive-charge model could explain small deflections but not the observed rare large-angle alpha scattering. Most alpha particles pass nearly straight through thin foil, showing that most of the atom is empty space. A small fraction experience very large deflections. Positive alpha particles feel strong electrostatic repulsion near a small, dense, positive nucleus containing most atomic mass. Few large-angle events show that the nucleus occupies a small fraction of the atom's volume. Keep each observation linked to the particular conclusion it supports.

### ■ Make, accelerate and steer a beam

**Thermionic emission** releases electrons from a heated metal. Heating supplies energy so some electrons can leave the surface. A positive accelerating electrode attracts them through a vacuum. A potential difference changes kinetic energy by the electric work:

$$\Delta E_k = q(V_{\text{initial}} - V_{\text{final}})$$

For speed calculations use the positive energy gain  $|q||\Delta V|$  only when the particle is accelerated through the field in the appropriate direction. Starting from rest, well below light speed:

$$\frac{1}{2}mv^2 = |q||\Delta V|$$

A **linear accelerator** uses alternating p.d. across gaps between drift tubes. Particles accelerate in gaps and are shielded inside the conducting tubes. The

polarity reverses while particles are inside, so the next gap accelerates them again. At constant alternating frequency, increasing speed requires longer tubes to maintain the time between successive gaps. Across  $n$  equal accelerating gaps, total kinetic-energy gain is  $n|q|V_{\text{gap}}$ .

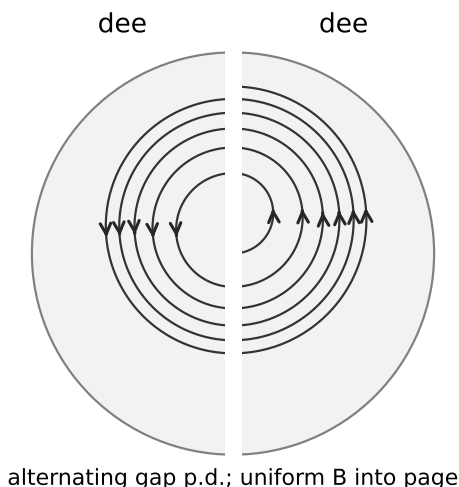
A **cyclotron** uses a magnetic field perpendicular to two hollow dees. Electric field accelerates particles across the gap. Inside a dee, magnetic force curves the path without increasing speed. Polarity reverses during each half-turn, allowing another energy gain at the next crossing. With perpendicular entry:

$$|q|vB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{B|q|} = \frac{p}{B|q|}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{B|q|} \quad f = \frac{B|q|}{2\pi m}$$

In the non-relativistic model, the period is independent of speed. Increasing speed increases radius. Time in one dee is  $T/2$ , not  $T$ . The formula assumes constant field and fixed mass in this model; synchronisation fails as relativistic effects become important.

### Cyclotron: radius grows after each gap



**Worked example.** A proton moves perpendicular to  $B = 0.40 \text{ T}$  with momentum  $2.0 \times 10^{-20} \text{ kg m s}^{-1}$ . Use proton mass  $1.67 \times 10^{-27} \text{ kg}$ .

$$r = \frac{p}{B|q|} = \frac{2.0 \times 10^{-20}}{0.40 \times 1.60 \times 10^{-19}} \text{ m} = 0.313 \text{ m}$$

$$t_{\text{dee}} = \frac{T}{2} = \frac{\pi m}{B|q|} = \frac{\pi \times 1.67 \times 10^{-27}}{0.40 \times 1.60 \times 10^{-19}} \text{ s} = 8.20 \times 10^{-8} \text{ s}$$

When starting instead from a de Broglie wavelength, use  $p = h/\lambda$ , then  $E_k = p^2/(2m)$  and  $V = E_k/|q|$ . Check that the non-relativistic assumption remains suitable.

### ■ Read detector evidence carefully

Charged particles ionise matter along their paths, leaving detectable tracks. A neutral particle does not make a direct ionisation track in this model. Its presence can be inferred from missing momentum or from charged products it later produces.

In a known perpendicular magnetic field, curvature direction together with travel direction can identify charge sign. Without field direction, curvature alone cannot give sign. Radius gives momentum through  $p = B|q|r$ ; comparing radii requires knowledge of charge magnitude. A tightening track often shows momentum being lost. Do not infer travel direction from a constant-radius arc alone.

Conserve charge, total energy and vector momentum at an interaction. If visible outgoing momenta do not add to the incoming momentum, a neutral product may carry the missing component. Draw a vector balance rather than guessing from the number of tracks.

High energies probe nucleon structure in two ways. Large momentum gives a short de Broglie wavelength, allowing small structure to be resolved. High collision energy also allows new massive particles to be created. Quarks are not observed as isolated free particles; do not describe a high-energy collision simply as removing a free quark.

Fast muons can reach the ground because their average lifetime measured in the Earth's frame is increased at speeds close to  $c$ . This does not mean they exceed light speed. Unit 4 requires the significance of this lifetime increase, not use of relativistic equations.

## Particles, antiparticles and conservation

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### ■ Families and antiparticles

In the **quark–lepton model** — , baryons and mesons contain quarks. Leptons are fundamental particles in the model.

- A **baryon** contains three quarks, for example a proton  $uud$  or neutron  $udd$ . An antibaryon contains three antiquarks.
- A **meson** contains one quark and one antiquark, for example a pion.
- A **lepton** is fundamental, for example an electron or neutrino. A positron is an antilepton, not a meson.
- A photon is the quantum of electromagnetic radiation; it is neither a baryon nor a lepton.

Up-type quarks have charge  $+2e/3$  and down-type quarks  $-e/3$ . Antiquarks have opposite charges. For example,  $uud$  sums to  $+e$  and  $udd$  to zero. The symmetry of the quark families predicted a top quark before it was observed. Classification models make testable predictions, not just lists of known particles.

An **antiparticle** has the same rest mass as its particle and opposite electric

charge when charged. Other relevant additive quantum numbers also reverse. An electron has lepton number +1; its positron has −1. Both have baryon number zero. A neutral antiparticle can differ in quantum numbers despite having no electric charge.

■ **Check conserved totals**

Assign baryon number +1 to a baryon, −1 to an antibaryon, and zero to mesons/leptons/photons. Quarks carry +1/3 and antiquarks −1/3. Assign lepton number +1 to a lepton and −1 to an antilepton. Compare total charge, baryon number and lepton number on both sides of a proposed reaction. Also check energy and momentum. Passing the listed conservation checks alone does not prove that an interaction will occur; it establishes that those laws do not forbid it.

└ **Worked example.** Test  $n \rightarrow p + e^- + \bar{\nu}_e$ .

| Quantity      | Before | After            |
|---------------|--------|------------------|
| Charge / $e$  | 0      | $+1 - 1 + 0 = 0$ |
| Baryon number | 1      | $1 + 0 + 0 = 1$  |
| Lepton number | 0      | $0 + 1 - 1 = 0$  |

The antineutrino balances the electron’s lepton number. Changing it to a neutrino would make the final total +2, so that proposed equation would fail this check. Preserve bars and charge signs when interpreting supplied particle symbols.

■ **Rest energy, creation and annihilation**

**Mass–energy equivalence** relates a rest-mass change to energy:

$$\Delta E = c^2 \Delta m$$

In **pair production** , energy creates a particle–antiparticle pair. The energy must at least supply both rest masses; extra energy can become kinetic energy or recoil. Momentum must also be conserved, so the surrounding interaction matters. In **annihilation** , particle and antiparticle can turn into photons. A pair initially at rest producing two photons gives equal photon energies and opposite momenta. Each photon carries one particle’s rest energy for an equal-mass pair; the total is twice that value.

One electronvolt is  $e$  joules:  $1\text{ eV} = 1.60 \times 10^{-19}\text{ J}$ . MeV means  $10^6$  eV and GeV means  $10^9$  eV. A quoted mass in MeV/ $c^2$  is a mass unit, not an energy unit. Multiply by  $c^2$  to get its corresponding rest energy.

└ **Worked example.** Convert a mass of 140 MeV/ $c^2$  to kilograms.

$$mc^2 = 140 \times 10^6 \times 1.60 \times 10^{-19}\text{ J} = 2.24 \times 10^{-11}\text{ J}$$

$$m = \frac{E}{c^2} = \frac{2.24 \times 10^{-11}\text{ J}}{(3.00 \times 10^8\text{ m s}^{-1})^2} = 2.49 \times 10^{-28}\text{ kg}$$

Use the same units before comparing masses. Percentage difference from an accepted value is  $100 \times |m_{\text{predicted}} - m_{\text{accepted}}|/m_{\text{accepted}}$ . State the reference value. In threshold questions, distinguish rest-energy supply from kinetic energy; do not use  $p = mv$  for a photon or a particle close to  $c$ .

## Check yourself

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- Can you use signed impulse and two momentum components, then test kinetic energy separately?
- Can you derive centripetal acceleration and draw only actual forces?
- Can you distinguish field vectors, signed potential and energy changes?
- Can you choose the correct capacitor curve and use log slope or tolerance evidence?
- Can you explain induction as a change in linkage, including the opposing direction?
- Can you separate electric acceleration from magnetic steering and halve a cyclotron period correctly?
- Can you preserve particle symbols, conservation totals and energy/mass units?

The skill sheets develop these methods before authentic past-paper tasks. Original practice also covers requirements not sampled in the two recent papers. A short sample of authentic questions does not define the whole syllabus.

# 5 • Thermodynamics, Radiation, Oscillations and Cosmology

Pearson Edexcel • International A-Level • Physics

## From an ice pack to a star

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An ice pack, a swinging bridge and a distant star all involve energy. This unit links molecular motion, nuclear changes, oscillations and astronomical observations. The task is often to test a claim, not just calculate a number: state the model, carry units through the calculation, then compare your result with the claim.

WPH15 covers statements 125–171 of Pearson Issue 3. It also uses Units 1, 2 and 4. Work through sheets 5.1–5.9 before their matching authentic sets. Use the supplied constants consistently:  $g = 9.81 \text{ m s}^{-2}$ ,  $k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$  and  $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$  unless the question gives a different local value. Here  $k_B$  is the Boltzmann constant;  $k_s$  denotes a spring constant.

## Heating, phase change and thermistor calibration

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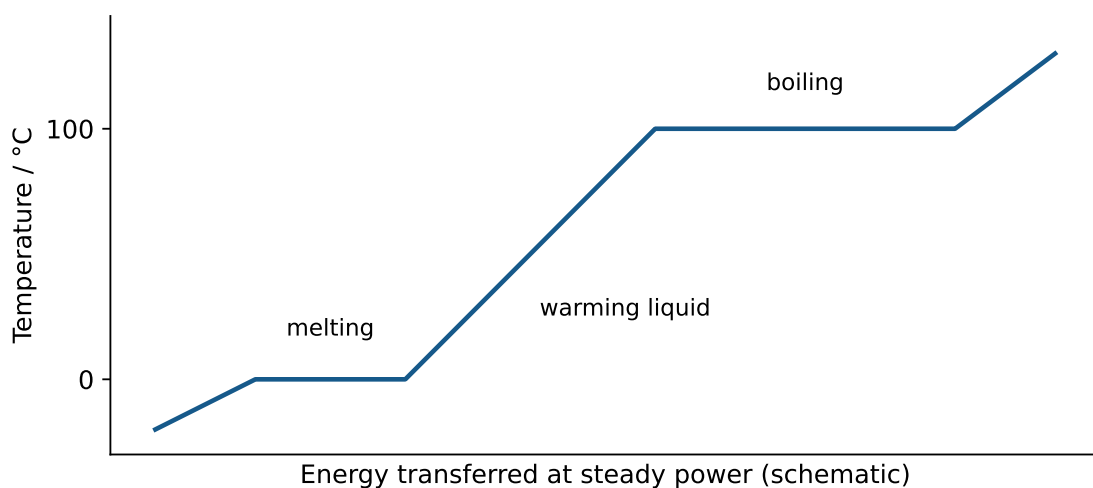
### ■ Follow the energy, not only the temperature

**Internal energy** is the total energy in the random motion and interactions of molecules: random kinetic energy plus intermolecular potential energy. It excludes the kinetic energy of the whole object moving together. Temperature relates to average random molecular kinetic energy, not total internal energy. Two samples at the same temperature can have different internal energies because their amounts and states differ.

**Specific heat capacity**  $c$  is the energy needed per kilogram for a one-kelvin temperature rise without a change of state. **Specific latent heat**  $L$  is the energy transferred per kilogram during a particular change of state at constant temperature.

$$\Delta E = mc\Delta\theta \quad \Delta E = L\Delta m$$

A temperature difference of  $1^\circ\text{C}$  equals a difference of 1 K. Absolute temperatures in gas and radiation laws must be kelvin. During melting or boiling, energy changes molecular arrangements and potential energy while temperature remains constant in the ideal constant-pressure phase change. An ice pack absorbs energy from the food/surroundings as it melts. It does not send “cold energy” into the food.



At constant heating power, steeper temperature–time gradients mean smaller  $mc$ , provided heat loss is negligible. A flat section does not mean that energy transfer has stopped. For a heater, input energy is  $E_{\text{in}} = Pt = VIt$ ; useful thermal energy can be smaller because energy escapes to surroundings.

**Worked example.** A 0.40 kg block of ice at  $-10\text{ }^{\circ}\text{C}$  melts to water at  $0\text{ }^{\circ}\text{C}$ . Use  $c_{\text{ice}} = 2100\text{ J kg}^{-1}\text{ K}^{-1}$  and  $L_f = 3.4 \times 10^5\text{ J kg}^{-1}$ .

Known: mass, initial/final temperatures and material constants. Why: warming the solid and melting it are separate energy transfers.

$$E_{\text{warm}} = mc\Delta\theta = 0.40\text{ kg} \times 2100\text{ J kg}^{-1}\text{ K}^{-1} \times 10\text{ K} = 8.40 \times 10^3\text{ J}$$

$$E_{\text{melt}} = mL_f = 0.40\text{ kg} \times 3.4 \times 10^5\text{ J kg}^{-1} = 1.36 \times 10^5\text{ J}$$

$$E_{\text{total}} = E_{\text{warm}} + E_{\text{melt}} = (8.40 \times 10^3 + 1.36 \times 10^5)\text{ J} = 1.44 \times 10^5\text{ J}$$

**Worked example.** A 1000 W heater boils away 0.20 kg of water in 600 s. Find efficiency, using  $L_v = 2.26 \times 10^6\text{ J kg}^{-1}$ .

$$E_{\text{useful}} = mL_v = 0.20\text{ kg} \times 2.26 \times 10^6\text{ J kg}^{-1} = 4.52 \times 10^5\text{ J}$$

$$E_{\text{input}} = Pt = 1000\text{ W} \times 600\text{ s} = 6.00 \times 10^5\text{ J}$$

$$\eta = \frac{E_{\text{useful}}}{E_{\text{input}}} = \frac{4.52 \times 10^5\text{ J}}{6.00 \times 10^5\text{ J}} = 0.753 \approx 75\%$$

For repeated kettle loads, calculate total water volume, number of loads and total operating time. Compare energies over the same stated interval. Standby loss over a day and input needed to boil water are different quantities; state what a particular comparison actually establishes.

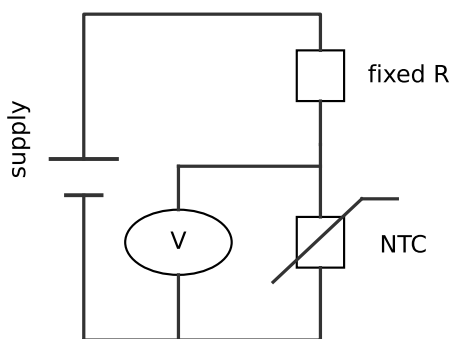
## ■ CP12: calibrate a thermostat

A **thermistor** has resistance that changes with temperature. The usual negative-temperature-coefficient (NTC) thermistor decreases in resistance when heated. In a **potential divider** :

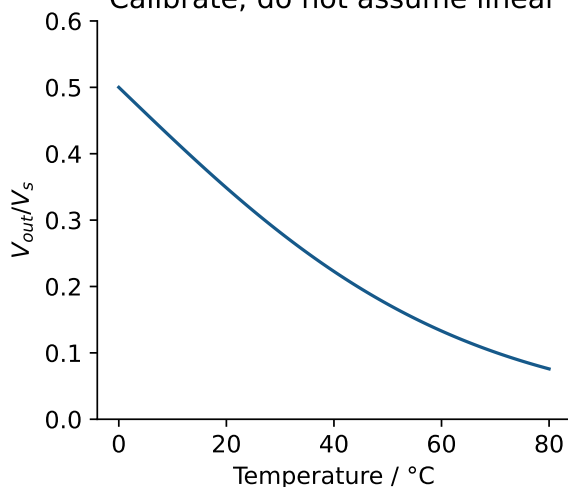
$$V_{\text{out}} = V_s \frac{R_{\text{lower}}}{R_{\text{upper}} + R_{\text{lower}}}$$

With the thermistor as the lower resistor and output measured across it, warming reduces output voltage. Swapping resistor positions reverses that trend. A thermostat switches at a chosen output voltage threshold; the divider alone is not a complete switching device.

Output across lower thermistor



Calibrate; do not assume linear



For core practical 12, place the thermistor and a reference thermometer close together in a stirred water bath. Keep electrical contacts dry and insulated. At several temperatures across the intended range, wait for thermal equilibrium and record temperature and divider voltage. Keep supply voltage and fixed resistance unchanged. Use a high-input-resistance voltmeter to reduce loading. Plot output against temperature and interpolate the voltage for the chosen switching temperature. Repeat readings or compare warming/cooling to reveal lag. Do not assume a linear calibration without evidence. Use low-voltage electricity and take care with hot water.

### ■ CP13: measure latent heat

Supply measured electrical power to a material during its phase change. Measure mass melted or evaporated over a timed interval after conditions stabilise. A balance measures mass change; do not confuse collected liquid volume with mass. Without loss correction,  $L = VIt/\Delta m$  assumes all supplied energy causes the phase change.

A useful correction is to compare two powers at the same steady phase-change temperature. If heat-loss power is approximately unchanged, subtracting the two measurements removes it:

$$L = \frac{P_2 - P_1}{\dot{m}_2 - \dot{m}_1}$$

State that constant-loss assumption. A control measurement of melting without the heater is another possible correction in a suitable ice experiment. Reduce heat loss with insulation where safe; avoid splashing, hot surfaces and unsafe mains connections.

## Ideal gases and molecular kinetic energy

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### ■ Kelvin and the ideal-gas model

**Absolute zero** is 0 K, about  $-273\text{ }^{\circ}\text{C}$ . It is the lower limit of thermodynamic temperature. In the classical ideal-gas model, average translational kinetic energy tends to zero as temperature tends to zero. Real gases condense before that extrapolation is reached; do not claim molecules in every real material lose all possible energy.

**Absolute temperature** is measured in kelvin:  $T = \theta_{\text{C}} + 273$  to the precision normally used here. An **ideal gas** consists of particles with negligible volume, no intermolecular forces except during elastic collisions, and random motion. Pressure comes from momentum changes at walls.

$$pV = Nk_B T$$

Here  $N$  is the number of molecules, not the number of moles. Use pressure in pascals, volume in cubic metres and temperature in kelvin. For a fixed number of molecules:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

At fixed volume, pressure is proportional to kelvin temperature. At fixed temperature,  $pV$  is constant. Include external pressure if a gauge reports pressure relative to the atmosphere.

**Worked example.** A rigid  $2.0 \times 10^{-3}\text{ m}^3$  vessel contains gas at  $1.0 \times 10^5\text{ Pa}$  and  $300\text{ K}$ . Find  $N$  and pressure after warming to  $360\text{ K}$ .

$$N = \frac{pV}{k_B T} = \frac{(1.0 \times 10^5 \text{ Pa})(2.0 \times 10^{-3} \text{ m}^3)}{(1.38 \times 10^{-23} \text{ J K}^{-1})(300 \text{ K})} = 4.83 \times 10^{22}$$

$$p_2 = p_1 \frac{T_2}{T_1} = 1.0 \times 10^5 \text{ Pa} \times \frac{360 \text{ K}}{300 \text{ K}} = 1.2 \times 10^5 \text{ Pa}$$

If one molecule has mass  $m_0$ , total gas mass is  $Nm_0$ . For a sphere, first find  $V = 4\pi r^3/3$ ; radius is half the diameter.

### ■ Derive the molecular energy relation

The kinetic-theory pressure relation is  $pV = \frac{1}{3}Nm_0\langle c^2 \rangle$ , where  $\langle c^2 \rangle$  is **mean square speed**. It follows from elastic momentum changes at a wall and equal mean squared motion in the three perpendicular directions. The **root mean square speed** is  $c_{\text{rms}} = \sqrt{\langle c^2 \rangle}$ ; it is not the square of mean speed.

For a cube of side  $l$ , one molecule with velocity component  $c_x$  changes wall momentum by  $2m_0c_x$  at each elastic collision. Its return time to that same wall is  $2l/c_x$ . Hence its average force is  $m_0c_x^2/l$ . Sum over molecules and divide by wall area  $l^2$ :

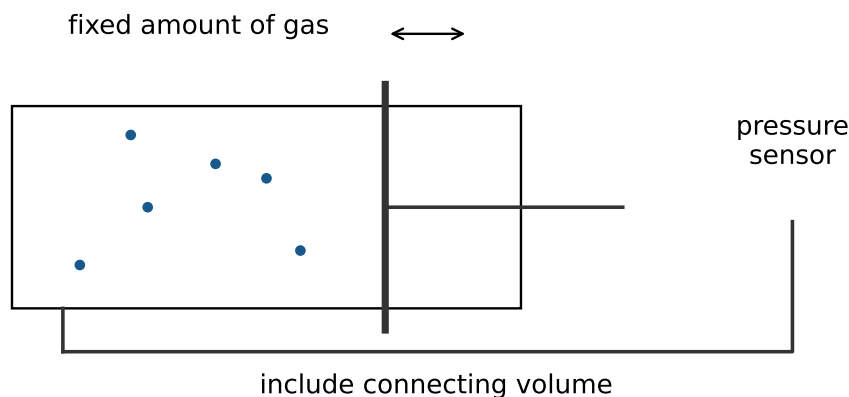
$$p = \frac{Nm_0\langle c_x^2 \rangle}{l^3} \quad V = l^3$$

Random motion is equally distributed among perpendicular directions, so  $\langle c^2 \rangle = 3\langle c_x^2 \rangle$ . This gives the pressure relation above. Equate the two expressions for  $pV$ , cancel  $N$ , then rearrange:

$$\frac{1}{3}Nm_0\langle c^2 \rangle = Nk_B T$$

$$\frac{1}{2}m_0\langle c^2 \rangle = \frac{3}{2}k_B T \quad c_{\text{rms}} = \sqrt{\frac{3k_B T}{m_0}}$$

At one temperature, different gas species have equal mean translational kinetic energies, but lighter molecules have greater RMS speeds. If speed doubles at the same temperature, molecular mass is one quarter. Heating increases average molecular kinetic energy; it does not give every molecule the same speed.



### ■ CP14: pressure and volume at fixed temperature

Trap a fixed amount of gas in a syringe or calibrated cylinder connected to a pressure sensor. Check calibration and use absolute pressure. Include connecting-tube dead volume where significant. Change volume gradually, wait after each change for temperature to return to the surroundings, then record several pressure–volume pairs. Do not let gas escape. A plot of  $p$  against  $1/V$  should be straight through the origin within uncertainty. A  $p$ – $V$  curve is not straight. Avoid excessive pressure and secure connections. Rapid compression heats the gas and breaks the intended constant-temperature condition.

## Binding energy, fission and fusion

### ■ Mass deficit is an energy difference

A nucleus contains  $Z$  protons and  $A - Z$  neutrons. Its mass is smaller than the total mass of these free nucleons. This **mass deficit** corresponds to the **binding energy** needed to separate the nucleus completely into free nucleons:

$$\Delta m = Zm_p + (A - Z)m_n - m_{\text{nucleus}} \quad E_b = \Delta mc_0^2$$

Here  $c_0$  is the speed of light, distinguished from specific heat capacity. The **unified atomic mass unit** is  $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$ . Convert all masses to the same unit before subtracting. Nuclear masses and neutral-atom masses are not interchangeable without accounting consistently for electrons. Use the masses the question actually supplies.

$$1 \text{ MeV} = 1.60 \times 10^{-13} \text{ J} \quad \text{binding energy per nucleon} = E_b/A$$

**Worked example.** A nucleus has mass deficit 0.030 u and  $A = 4$ . Find binding energy per nucleon.

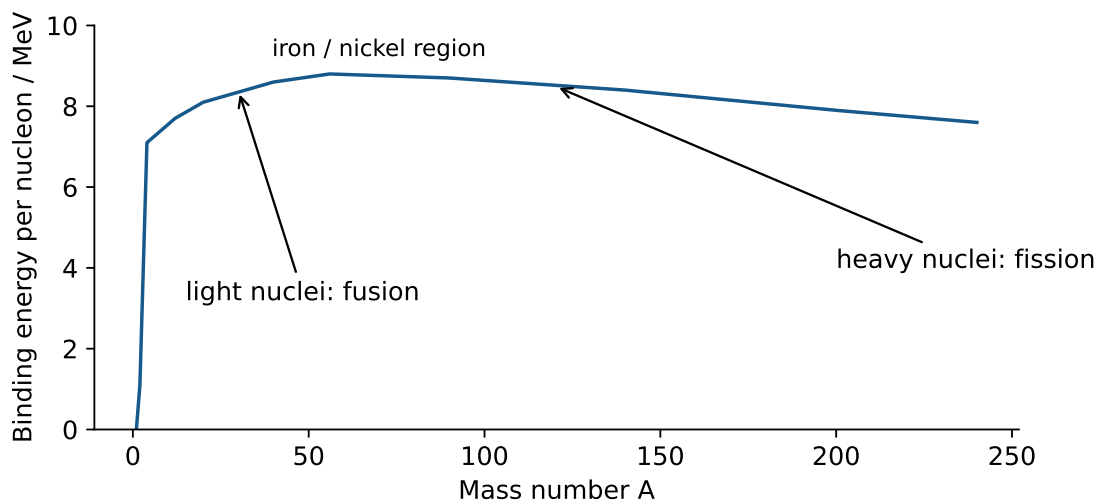
$$\Delta m_{\text{kg}} = \Delta m_{\text{u}}(1.66 \times 10^{-27} \text{ kg/u}) = 0.030 \text{ u} \times 1.66 \times 10^{-27} \text{ kg/u} = 4.98 \times 10^{-29} \text{ kg}$$

$$E_b = \Delta mc_0^2 = 4.98 \times 10^{-29} \text{ kg} \times (3.00 \times 10^8 \text{ m s}^{-1})^2 = 4.48 \times 10^{-12} \text{ J}$$

$$\frac{E_b}{A} = \frac{4.48 \times 10^{-12} \text{ J}}{4 \times 1.60 \times 10^{-13} \text{ J/MeV}} = 7.00 \text{ MeV per nucleon}$$

### ■ Why fusion and fission can both release energy

The binding-energy-per-nucleon curve rises steeply for light nuclei, peaks near iron/nickel, then decreases gradually for heavy nuclei. Greater binding energy per nucleon generally means nucleons are more tightly bound. It is the vertical value, not just mass number, that matters when comparing points.



**Nuclear fusion** combines light nuclei. **Nuclear fission** splits a heavy nucleus into smaller products, often with neutrons. Energy is released when the products have greater total binding energy and smaller total rest mass than the reactants. For a reaction, calculate the total mass difference, not merely a difference of two per-nucleon values. Total energy is conserved.

Fusion needs very high temperature to give nuclei enough kinetic energy to approach despite electrostatic repulsion, and high density/confinement to make enough close encounters and sustain energy release. Temperature and density have different roles. Merely naming “high pressure” does not explain overcoming repulsion.

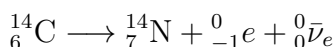
# Radiation, background and decay

## ■ Choose radiation by its interaction

**Alpha radiation** consists of helium nuclei: strongly ionising, short range and stopped by paper or a few centimetres of air. **Beta radiation** consists of electrons or positrons: less ionising, more penetrating, typically stopped by a few millimetres of aluminium. **Gamma radiation** is electromagnetic radiation: weakly ionising and highly penetrating; lead or concrete reduces its intensity but does not provide a sharp stopping thickness.

In a cloud chamber, alpha tracks are thick mainly because of strong ionisation, and relatively straight because the particles have large mass compared with beta particles. Beta tracks are thinner and more easily deflected. Do not swap these explanations. Beta radiation can monitor paper thickness because some passes through and some is absorbed; a change in thickness changes the transmitted count rate.

In **nuclear equations**, conserve nucleon number and charge number. Beta-minus decay converts a neutron to a proton; the mass number stays the same and atomic number increases by one. Include the antineutrino where required, as in Unit 4:



Alpha decay reduces  $A$  by four and  $Z$  by two. Gamma emission changes neither  $A$  nor  $Z$ . Nuclear equations conserve more than these two numbers: energy and momentum still matter.

## ■ Correct background before applying a model

**Background radiation** comes from sources such as rocks, cosmic rays and the environment. Measure it for a sufficiently long time with the test source absent. Subtract its count rate from every source-plus-background reading. Keep counting-time units consistent. Corrected detector count rate is proportional to source activity only when geometry and detection efficiency remain unchanged; it is not automatically the activity in becquerels.

**Worked example.** A detector reads  $62 \text{ min}^{-1}$  at distance  $r$ , including background  $14 \text{ min}^{-1}$ . Predict total rate at  $2r$  for a small isotropic gamma source with negligible absorption.

$$R_{s,1} = R_{\text{total},1} - R_b = (62 - 14) \text{ min}^{-1} = 48 \text{ min}^{-1}$$

$$R_{s,2} = R_{s,1} \left( \frac{r}{2r} \right)^2 = 48 \text{ min}^{-1} \times \frac{1}{4} = 12 \text{ min}^{-1}$$

$$R_{\text{total},2} = R_{s,2} + R_b = (12 + 14) \text{ min}^{-1} = 26 \text{ min}^{-1}$$

Do not divide the background by four: it is not all coming from the test source.

## ■ Random events, predictable populations

**Radioactive decay** is spontaneous: an unstable nucleus decays without an external trigger. It is random: the exact time for an individual nucleus cannot be predicted. A large population has a predictable statistical decay pattern. **Activity**

$A$  is the number of decays per second, measured in **becquerels**,  $1 \text{ Bq} = 1 \text{ s}^{-1}$ .

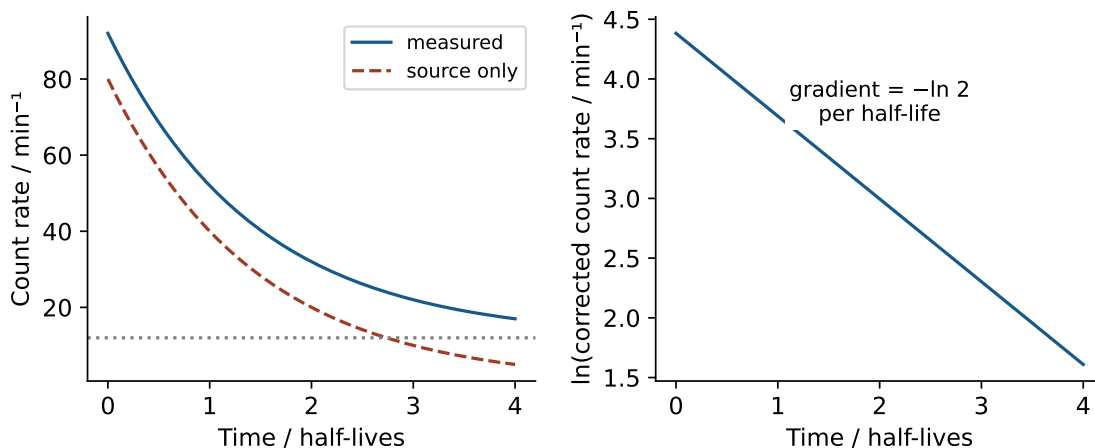
$$A = \lambda N \quad \frac{dN}{dt} = -\lambda N \quad N = N_0 e^{-\lambda t} \quad A = A_0 e^{-\lambda t}$$

The **decay constant**  $\lambda$  is the probability per unit time of decay for a nucleus. The **half-life**  $t_{1/2}$  is the time for the undecayed population, or activity, to halve on average.

At one half-life,  $N/N_0 = 1/2 = e^{-\lambda t_{1/2}}$ . Taking logs gives:

$$\lambda = \frac{\ln 2}{t_{1/2}} \quad \ln N = \ln N_0 - \lambda t \quad \ln(A/A_0) = -\lambda t$$

A log-activity-time line has gradient  $-\lambda$ . Take logarithms of corrected rates, not source-plus-background readings. Read several half-life intervals from a graph and average; measure from the corrected activity level, not from zero total counts.



**Worked example.** A source has half-life 3.0 years. When will activity fall to 2.0% of its initial value?

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{3.0 \text{ years}} = 0.231 \text{ year}^{-1}$$

$$t = -\frac{\ln(A/A_0)}{\lambda} = -\frac{\ln(0.020)}{0.231 \text{ year}^{-1}} = 16.9 \text{ years}$$

Use a fraction, not 2.0 inside the logarithm. If calculating power from a radioactive source,  $P = AE_{\text{decay}}$ , with energy per decay in joules. This is released nuclear power; useful electrical output may be smaller. Convert the half-life to seconds when activity must be in becquerels.

### ■ CP15: absorption of gamma radiation by lead

Keep source, absorber and detector in fixed positions. Measure background, then counts over equal known intervals for several total lead thicknesses. Repeat or extend the counting time because counts fluctuate. Subtract background rate before comparing transmission. If each equal thickness gives the same fractional reduction, the attenuation is exponential; a graph of log corrected rate against thickness is approximately straight. Three half-value thicknesses transmit  $1/8$ , not  $1/3$ .

The material-absorption constant is not the time-decay constant of the source. Avoid readings too close to background, where subtraction gives large relative uncertainty. Handle sources with the specified tools under supervision, keep distance, minimise exposure time and return them to shielding. Never touch a source or direct it at people. Keep lead handling clean and wash hands.

## Simple harmonic motion, graphs and energy

### ■ The restoring condition

**Simple harmonic motion** (SHM) occurs when resultant acceleration is proportional to displacement from a fixed equilibrium position and directed towards it:

$$F = -k_s x \quad a = -\omega^2 x \quad \omega^2 = k_s/m$$

The negative sign is the restoring direction. Constant speed, repetition alone or a force merely pointing towards a centre does not establish SHM. For a vertically hanging spring, measure  $x$  from the loaded equilibrium position. Weight is already balanced there; the resultant for displacement is  $-k_s x$ .

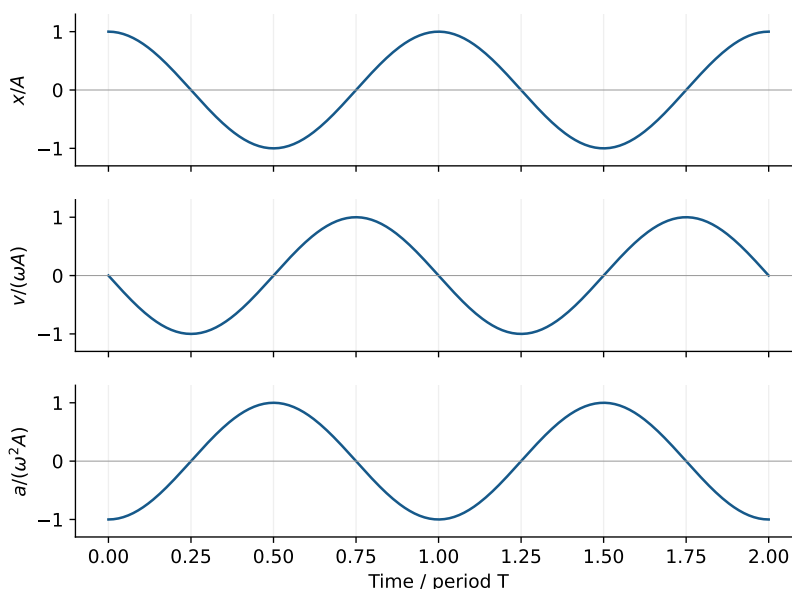
For release from positive maximum displacement at  $t = 0$ :

$$x = A_0 \cos \omega t \quad v = -A_0 \omega \sin \omega t \quad a = -A_0 \omega^2 \cos \omega t$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f} \quad v_{\text{max}} = A_0 \omega \quad |a|_{\text{max}} = A_0 \omega^2$$

Here  $A_0$  is amplitude, not radioactive activity. The phase depends on the chosen starting time; a sine displacement can describe the same motion with another origin. Differentiate graphically: the displacement–time gradient is velocity, and velocity–time gradient is acceleration. At maximum displacement, speed is zero and acceleration

points back towards equilibrium. At equilibrium, speed is greatest and acceleration is zero.



**Worked example.** A graph gives amplitude 0.030 m and period 0.50 s. Find maximum speed and acceleration.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.50 \text{ s}} = 12.6 \text{ rad s}^{-1}$$

$$v_{\text{max}} = A_0\omega = 0.030 \text{ m} \times \frac{2\pi}{0.50 \text{ s}} = 0.377 \text{ m s}^{-1}$$

$$|a|_{\text{max}} = A_0\omega^2 = 0.030 \text{ m} \times \left(\frac{2\pi}{0.50 \text{ s}}\right)^2 = 4.74 \text{ m s}^{-2}$$

Alternatively find maximum speed from a tangent to the displacement graph at equilibrium. Convert centimetres to metres. A negative straight  $a$ - $x$  gradient is  $-\omega^2$ , so take its negative before the square root.

## ■ Periods and energy

For a mass on an ideal spring and for a simple pendulum at small angles:

$$T_{\text{spring}} = 2\pi\sqrt{m/k_s} \quad T_{\text{pendulum}} = 2\pi\sqrt{l/g}$$

Pendulum length is pivot to bob centre. The small-angle period does not depend on bob mass. A period ratio for one unchanged spring gives  $T_2/T_1 = \sqrt{m_2/m_1}$ . Use total new mass before finding added mass.

**Worked example.** A 0.50 kg mass has period 0.80 s. An extra mass increases the period to 1.00 s on the same spring.

$$m_2 = m_1 \left( \frac{T_2}{T_1} \right)^2 = 0.50 \text{ kg} \left( \frac{1.00 \text{ s}}{0.80 \text{ s}} \right)^2 = 0.781 \text{ kg}$$

$$m_{\text{added}} = m_2 - m_1 = (0.781 - 0.50) \text{ kg} = 0.281 \text{ kg}$$

For an undamped spring oscillator, energy transfers between elastic potential and kinetic stores:

$$E = \frac{1}{2}k_s A_0^2 = \frac{1}{2}m\omega^2 A_0^2 \quad E_p = \frac{1}{2}k_s x^2 \quad E_k = \frac{1}{2}k_s (A_0^2 - x^2)$$

Thus the  $E_k$ - $x$  graph is an inverted parabola: zero at  $\pm A_0$ , positive maximum at  $x = 0$ . The  $E_p$ - $x$  graph opens upwards. Their sum is constant in the undamped model. In a damped system energy is transferred out, so amplitude decreases. Total energy including surroundings remains conserved.

### ■ CP16: infer an unknown mass from resonance

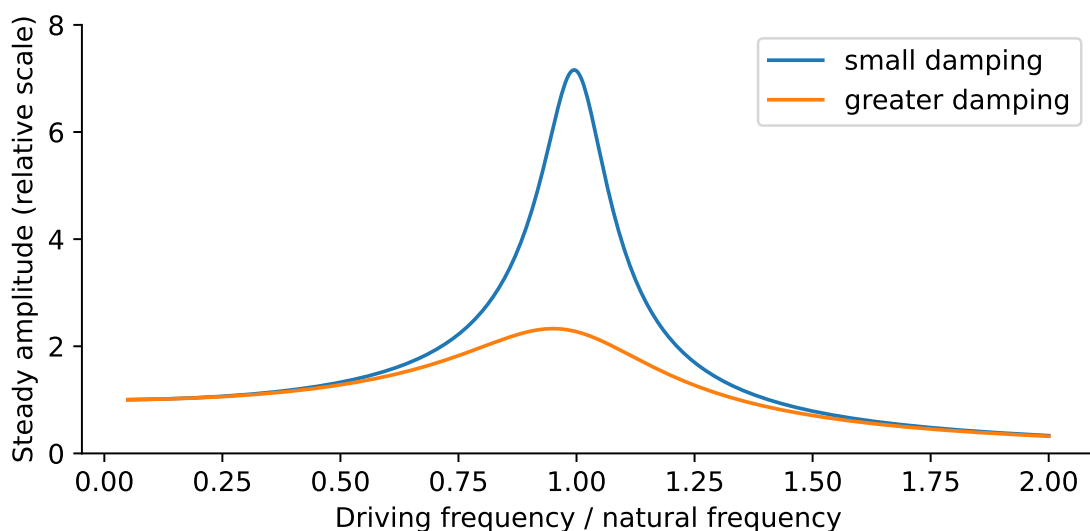
Keep the same spring/support system. Attach several known masses, drive with small oscillations and vary frequency to find the largest steady amplitude for each mass. Record the resonant frequency, repeating slowly around each peak. With light damping it approximates natural frequency. Plot  $1/f^2$  against total known mass; the spring model gives gradient  $4\pi^2/k_s$ . Use the calibration to infer an unknown mass from its resonant frequency.

Account for a hanger and, where significant, the effective moving mass of the spring. A nonzero intercept may represent this contribution; do not force the line through the origin without justification. Keep amplitudes small and below the elastic limit. Secure masses and stand, and keep feet clear. Measuring free periods can provide a useful independent comparison, but the specified practical uses resonant frequencies.

## Forced oscillations, resonance and damping

A **free oscillation** follows an initial disturbance without continued periodic driving. A **forced oscillation** is maintained by a periodic driving force; its steady frequency is the driving frequency. **Natural frequency** is the frequency of free oscillation for the system under the stated conditions.

**Resonance** occurs when driving frequency is at or near natural frequency, giving efficient energy transfer and a large steady amplitude. It does not mean “maximum frequency”. For a lightly damped system, the amplitude peak is near the undamped natural frequency. Greater damping lowers and broadens the peak; do not present every damped peak as exactly at the undamped natural frequency.



**Damping** transfers energy away from oscillation, for example through resistive work that increases the thermal energy of dampers and surroundings. Stronger damping can reduce dangerous bridge/building motion. **Plastic deformation**

of a ductile material also absorbs mechanical energy irreversibly; purely elastic deformation returns stored energy and is not the same mechanism.

**Worked explanation.** People walking supply a periodic driving force to a bridge. If its frequency is near a natural frequency, energy transfers efficiently into the bridge and amplitude increases. Dampers do work against motion, transferring oscillation energy to thermal stores, so the steady amplitude is limited. Link cause and effect in this order rather than listing “resonance, energy, damping”.

For the same shape and amplitude, adding mass to a light pendulum increases its stored mechanical energy while the small-angle ideal period remains unchanged. Under comparable resistive losses it can lose a smaller fraction of energy per cycle. That contextual damping comparison does not prove that every heavier oscillator always damps more slowly.

## Gravitational fields and orbits

### ■ Field and potential have different meanings

A **gravitational field** is a region where a mass experiences a force. **Gravitational field strength**  $g = F/m$  is force per unit test mass. It is a vector pointing towards an isolated source mass. Newton’s law for point masses, or outside a spherical symmetric source, is:

$$F = \frac{GMm}{r^2} \quad g = \frac{F}{m} = \frac{GM}{r^2}$$

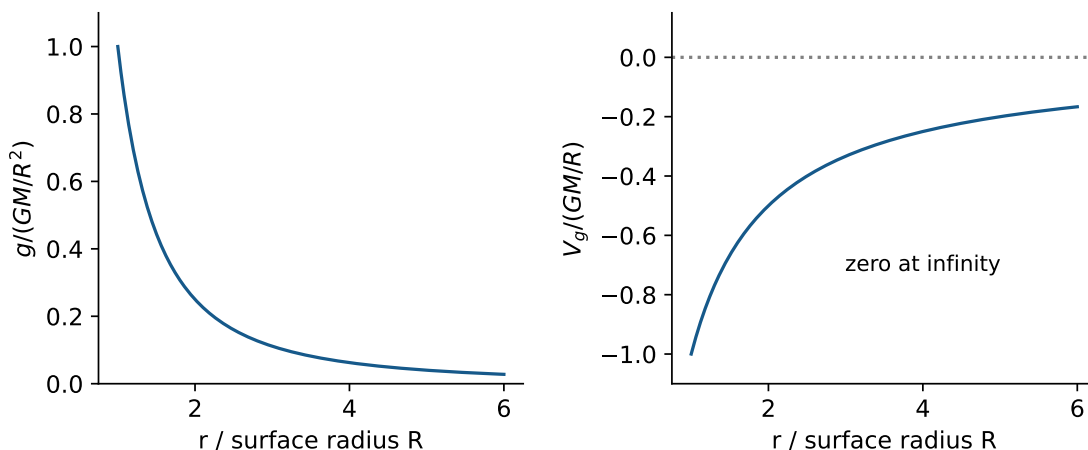
$r$  is centre-to-centre distance, not height above the surface. For near-Earth local

motion, constant  $g$  can be suitable. Over large radial distances it is not constant.

**Gravitational potential** is potential energy per unit mass, with zero at infinity:

$$V_{\text{grav}} = -\frac{GM}{r} \quad E_{\text{grav}} = mV_{\text{grav}} = -\frac{GMm}{r}$$

Potential is a scalar and negative at finite distance. Moving outward makes it less negative, so potential energy increases while attractive force weakens. A potential graph value and its corresponding radius give  $M = -Vr/G$ .



Gravity and electric fields are both radial for isolated point sources and their force magnitudes obey inverse-square laws. Gravity acts on mass and is attractive in this model. Electric force acts on charge and can attract or repel. Electric field direction follows the force on positive test charge; a negative charge is forced oppositely.

**Worked example.** A 1000 kg satellite moves from radius  $r_1 = 6.4 \times 10^6$  m to  $r_2 = 1.28 \times 10^7$  m around Earth,  $M = 6.0 \times 10^{24}$  kg. Find its potential-energy change.

$$\Delta E_{\text{grav}} = GMm \left( \frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$\Delta E_{\text{grav}} = (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(6.0 \times 10^{24} \text{ kg})(1000 \text{ kg}) \left( \frac{1}{6.4 \times 10^6 \text{ m}} - \frac{1}{1.28 \times 10^7 \text{ m}} \right) = +3.13 \times 10^{10} \text{ J}$$

The positive sign is final minus initial. This is not the total launch energy, since kinetic energy may also change.  $mg\Delta h$  with constant surface  $g$  is unsuitable over this distance.

## ■ Derive a circular orbit

The gravitational force supplies the inward resultant:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} = m\omega^2 r$$
$$v = \sqrt{GM/r} \quad T = 2\pi\sqrt{\frac{r^3}{GM}} \quad M = \frac{4\pi^2 r^3}{GT^2}$$

Orbiting mass cancels. The period grows with radius. To appear stationary above one point on a rotating planet, a satellite needs a circular equatorial orbit, the same rotational direction and matching angular velocity/period. Matching period alone is insufficient.

**Worked example.** For  $GM = 4.00 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$  and period  $T = 8.64 \times 10^4 \text{ s}$ , find stationary-orbit radius.

$$r = \left( \frac{GMT^2}{4\pi^2} \right)^{1/3} = \left[ \frac{(4.00 \times 10^{14} \text{ m}^3 \text{ s}^{-2})(8.64 \times 10^4 \text{ s})^2}{4\pi^2} \right]^{1/3} = 4.23 \times 10^7 \text{ m}$$

Subtract planet radius only if asked for height above the surface. For a star's surface field, use its radius, not its listed diameter; test a multiple-of-Earth-field claim by calculating the ratio.

## Stellar radiation, distances and evolution

### ■ Temperature, luminosity and received intensity

A **black body** absorbs all incident electromagnetic radiation and is an ideal thermal emitter. Its continuous spectrum has a characteristic shape. Increasing temperature increases total emitted power per unit area and moves the wavelength peak to shorter wavelength. The area under a spectral-intensity curve represents total intensity only with the appropriate spectral-axis definition.

**Luminosity**  $L_\star$  is total power emitted. **Intensity**  $I$  received at distance  $d$  is power per unit receiving area. For a spherical black-body star of radius  $R$  radiating equally in all directions:

$$L_\star = \sigma(4\pi R^2)T^4 \quad \lambda_{\max}T = 2.898 \times 10^{-3} \text{ m K} \quad I = \frac{L_\star}{4\pi d^2}$$

The first area is the emitting surface; the second is the expanding receiving sphere. Do not substitute observer distance into the star's surface area. Real emitters may not be perfect black bodies, and intervening material can absorb radiation.

Wien's stated constant describes a spectrum per unit wavelength. A maximum of a spectrum plotted per unit frequency does not transform into that wavelength maximum simply by replacing  $\lambda$  with  $c_0/f$ . If an exam gives the frequency corresponding

to its specified wavelength peak, convert that given wavelength using  $c_0 = f\lambda$ . Do not turn that particular task convention into a general rule for every spectral plot.

**Worked example.** A star's wavelength spectrum peaks at 580 nm and its radius is  $7.0 \times 10^8$  m. Find temperature, luminosity and intensity at  $1.5 \times 10^{11}$  m.

$$T = \frac{2.898 \times 10^{-3} \text{ m K}}{\lambda_{\text{max}}} = \frac{2.898 \times 10^{-3} \text{ m K}}{580 \times 10^{-9} \text{ m}} = 5.00 \times 10^3 \text{ K}$$

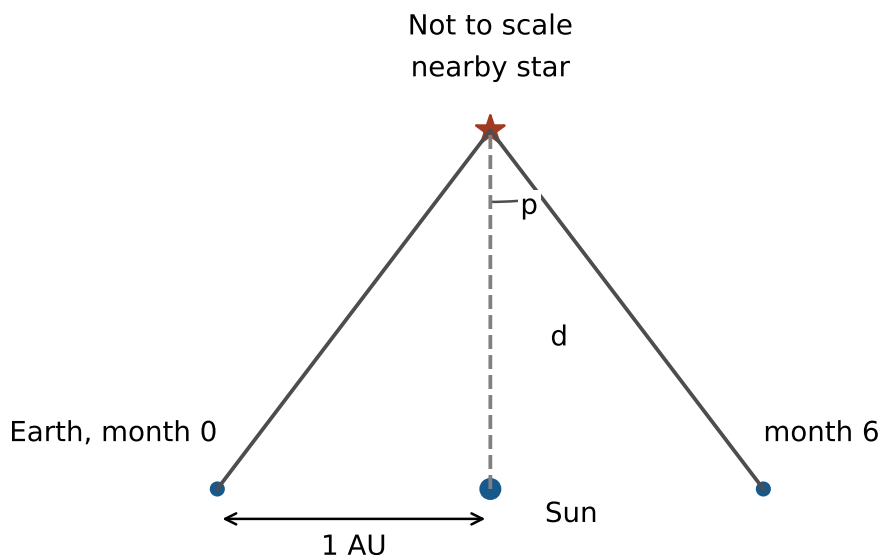
$$L_{\star} = 4\pi R^2 \sigma T^4 = 4\pi (7.0 \times 10^8 \text{ m})^2 (5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}) (4.997 \times 10^3 \text{ K})^4 = 2.18 \times 10^{26} \text{ W}$$

$$I = \frac{L_{\star}}{4\pi d^2} = \frac{2.18 \times 10^{26} \text{ W}}{4\pi (1.5 \times 10^{11} \text{ m})^2} = 7.70 \times 10^2 \text{ W m}^{-2}$$

Keep unrounded temperature because it is raised to the fourth power. Compare the received intensity with the stated reference before accepting a planetary-temperature or power claim.

## ■ Two distance methods

**Trigonometric parallax** measures a nearby star's apparent angular shift against distant background stars as Earth moves around the Sun. Observations six months apart use a baseline of two Earth-orbit radii. The parallax angle  $p$  is half the total shift. For small  $p$  in radians, distance is  $d \approx r_{\text{orbit}}/p$ . A **parsec** is the distance giving parallax 1 arcsecond;  $d_{\text{pc}} = 1/p_{\text{arcsec}}$ . Do not use the diameter of Earth or Sun as the orbital baseline.



At large distance,  $p$  becomes too small for the instrument to measure accurately. The limitation is angular resolution/relative uncertainty, not necessarily that the star is invisible.

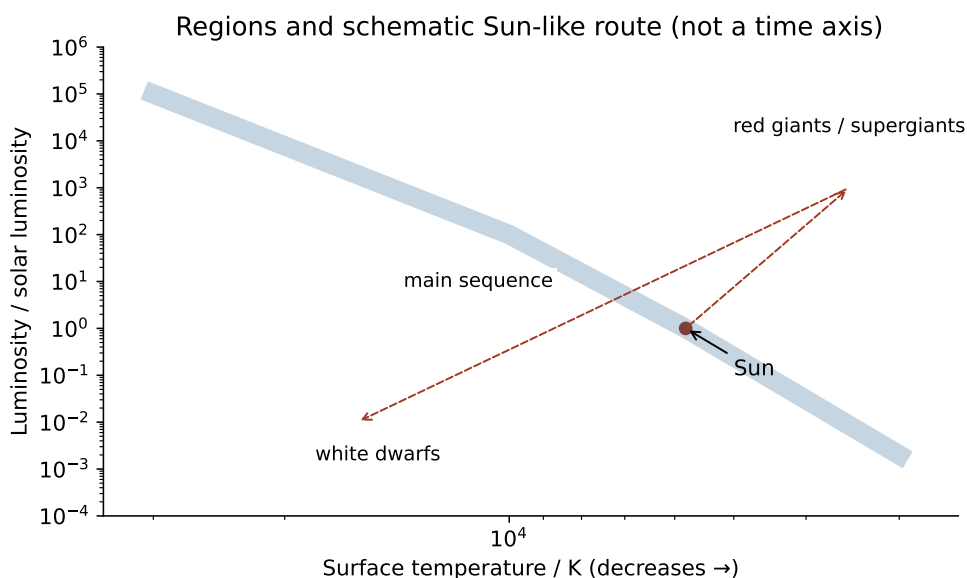
A **standard candle** is an object of known luminosity. Identify one in a cluster, measure received intensity and use  $d = \sqrt{L_*/(4\pi I)}$ . Its apparent brightness is not assumed known beforehand. Cepheid variable stars have a calibrated relationship between period and luminosity. Measure several complete light-curve periods, divide by their number, read the calibration carefully and convert any solar-luminosity units. A straight line on log axes is not automatically a linear relationship between the raw quantities.

**Worked example.** A standard candle has  $L_* = 4.0 \times 10^{28} \text{ W}$  and received intensity  $2.0 \times 10^{-10} \text{ W m}^{-2}$ .

$$d = \sqrt{\frac{L_*}{4\pi I}} = \sqrt{\frac{4.0 \times 10^{28} \text{ W}}{4\pi(2.0 \times 10^{-10} \text{ W m}^{-2})}} = 3.99 \times 10^{18} \text{ m}$$

## ■ Read the Hertzsprung–Russell diagram

A **Hertzsprung–Russell diagram** plots luminosity vertically against surface temperature horizontally, usually with temperature decreasing to the right and logarithmic scales. The **main sequence** runs from hot, bright stars at upper left to cool, faint stars at lower right. **Red giants** are cool but luminous because of their large surface areas. **White dwarfs** are hot but faint because of their small surface areas. Mark the Sun near one solar luminosity and about 5800 K on the main sequence.



Stars form when gravity contracts a cloud of gas and dust. The core heats until

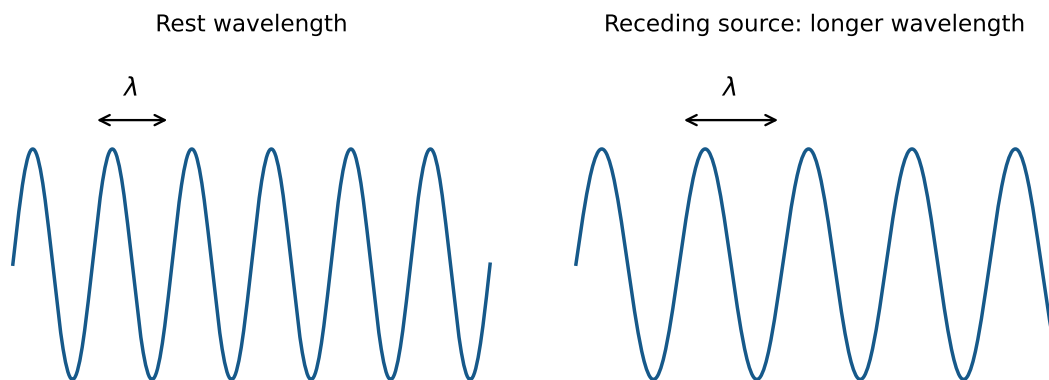
hydrogen fusion can sustain a main-sequence star; outward pressure balances gravity. In a Sun-like star, core hydrogen eventually runs low, fusion there decreases and gravity contracts the core. Core temperature rises; shell hydrogen burning and later helium fusion are associated with expansion to a red giant. Outer layers are lost, leaving a white-dwarf core with no sustained fusion. It cools over a very long time. The Sun does not become a supernova or a neutron star.

A much more massive star can fuse heavier elements through later stages. When its core can no longer gain energy from fusion, collapse and a supernova can leave a neutron star or black hole, depending on the remnant. A massive main-sequence star has more fuel but consumes it much faster because of its hotter core. Its main-sequence lifetime can therefore be shorter. Explain fusion rate, not just fuel amount. An HR track shows changing temperature/luminosity, not a star travelling across space.

## Doppler shifts and cosmology

### ■ Compare the same spectral line

The **Doppler effect** is a change in observed frequency/wavelength caused by relative motion along the line of sight. Successive wavefronts arrive farther apart from a receding source and closer together from an approaching source. Recession gives longer wavelengths and lower frequencies; approach gives shorter wavelengths and higher frequencies. Motion purely perpendicular to the line of sight is not the recession speed in the simple model.



For electromagnetic radiation at low recession speeds:

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} \approx \frac{v}{c_0}$$

**Redshift**  $z$  is positive for recession. For small shifts, its magnitude also approximately equals the fractional decrease of observed frequency relative to emitted frequency. State the sign convention:  $(f_{\text{observed}} - f_{\text{rest}})/f_{\text{rest}}$  is negative for recession.

Do not equate a positive redshift to a signed frequency increase. The approximation  $v/c_0$  is not a general high-speed relativistic formula.

■ **Worked example.** A line emitted at 500 nm is observed at 505 nm.

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = \frac{(505 - 500) \text{ nm}}{500 \text{ nm}} = 0.010$$

$$v \approx z c_0 = 0.010 \times 3.00 \times 10^8 \text{ m s}^{-1} = 3.00 \times 10^6 \text{ m s}^{-1}$$

To use a star's absorption lines, recall Unit 2: photons are absorbed only when their energies match allowed atomic-level differences. Compare the same identified spectral line, not two different elements.

For opposite limbs of a rotating star, one approaches while the other recedes. If their shifts are equal and opposite, the separation of the two observed wavelengths is twice the shift of either limb. Halve that separation before finding the equatorial speed. Then use  $T = 2\pi R/v$ . This assumes the observed line-of-sight limb speed represents the equatorial rotation speed under the stated viewing geometry.

## ■ Hubble law and its limits

On large cosmological scales, recession speed approximately follows **Hubble's law** :

$$v = H_0 d$$

The **Hubble constant**  $H_0$  has units of inverse time. In  $\text{km s}^{-1} \text{Mpc}^{-1}$ , convert kilometres to metres and megaparsecs to metres before taking its reciprocal in seconds. Nearby objects may have local motions that do not follow the large-scale relationship.

■ **Worked example.** Use  $H_0 = 70 \text{ km s}^{-1} \text{Mpc}^{-1}$  and  $1 \text{ Mpc} = 3.1 \times 10^{22} \text{ m}$  to estimate an expansion timescale.

$$H_0 = \frac{70 \times 10^3 \text{ m s}^{-1}}{3.1 \times 10^{22} \text{ m}} = 2.26 \times 10^{-18} \text{ s}^{-1}$$

$$t_H = \frac{1}{H_0} = \frac{1}{2.26 \times 10^{-18} \text{ s}^{-1}} = 4.43 \times 10^{17} \text{ s}$$

The reciprocal is the **Hubble time** . Interpreting it as an age assumes a model for the past expansion rate; it is not an exact model-independent age. A larger  $H_0$  gives a smaller reciprocal timescale. Evidence for expansion supports an earlier denser, hotter universe; it does not describe an explosion into an already empty centre-surrounding space.

**Dark matter** is inferred from gravitational effects that visible matter alone does not explain, such as galaxy rotation and motion. It need not emit detectable light.

The amount of gravitating matter affects how expansion changes, so uncertainty in matter content and the expansion model affects predictions of the universe's fate. Do not infer that one measurement of  $H_0$  alone proves perpetual expansion or future collapse. Modern models also consider dark energy; outcome 171 specifically requires understanding the controversy concerning  $H_0$ , dark matter, age and fate, rather than memorising one unqualified prediction.

## Check yourself

1. Why can internal energy increase without temperature increasing during a phase change?
2. A fixed amount of gas warms in a rigid vessel. Which temperature scale must be used to compare pressures?
3. Does greater binding energy per nucleon mean that less or more energy is needed to separate each nucleon on average?
4. Why must background be subtracted before using a count-rate ratio or logarithm?
5. An  $a$ - $x$  line has gradient  $-25 \text{ s}^{-2}$ . Find angular frequency.
6. Why is maximum SHM speed at equilibrium although acceleration is zero there?
7. Why can a damped bridge be driven at the same frequency but oscillate with a smaller amplitude?
8. Does lifting a satellite to a larger radius increase or decrease its signed gravitational potential energy?
9. A star is hot but faint. Which HR region is plausible, and what can explain its low luminosity?
10. When using the wavelength separation of opposite rotating limbs, why is a factor of two needed?

Answers: (1) molecular potential energy changes; (2) kelvin; (3) more; (4) background does not follow the source's decay/distance law; (5)  $\omega = \sqrt{25 \text{ s}^{-2}} = 5.0 \text{ rad s}^{-1}$ ; (6) restoring resultant is zero there but energy is mostly kinetic; (7) greater energy loss limits steady amplitude; (8) increases towards zero; (9) white dwarf, small emitting surface; (10) one limb is blueshifted while the other is redshifted.

# 6 · Practical Skills in Physics II

Pearson Edexcel · International A-Level · Physics

## From a reading to a result

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Two students measure the same damped pendulum. One records amplitudes; the other records amplitudes, processes them with logarithms, draws one straight line and reads off a decay constant and an initial amplitude. The second student has turned raw readings into a result. Unit 6 tests exactly that step.

**WPH16 Practical Skills in Physics II** is a written paper about the experiments you met while studying Units 4 and 5: gas laws, capacitors, damped oscillations, radioactive decay, light intensity. It lasts 80 minutes and has 50 marks. All questions are compulsory, and at least 20 marks use Level-2 mathematics. A written guide supports real laboratory work; it does not replace carrying out experiments yourself.

The four skill sheets follow the paper's own flow: plan the investigation, carry it out and judge the measurements, then process the data with uncertainties and graphs.

## Planning an A2 investigation

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A **valid measurement** still means measuring what you intend to measure. At A2 the plan has one extra demand: decide how the data will be processed **before** writing the method. The graph you intend to draw decides how many readings you need and where you take them.

### ■ Choose the graph before the method

Suppose you must test a predicted relationship. Work backwards:

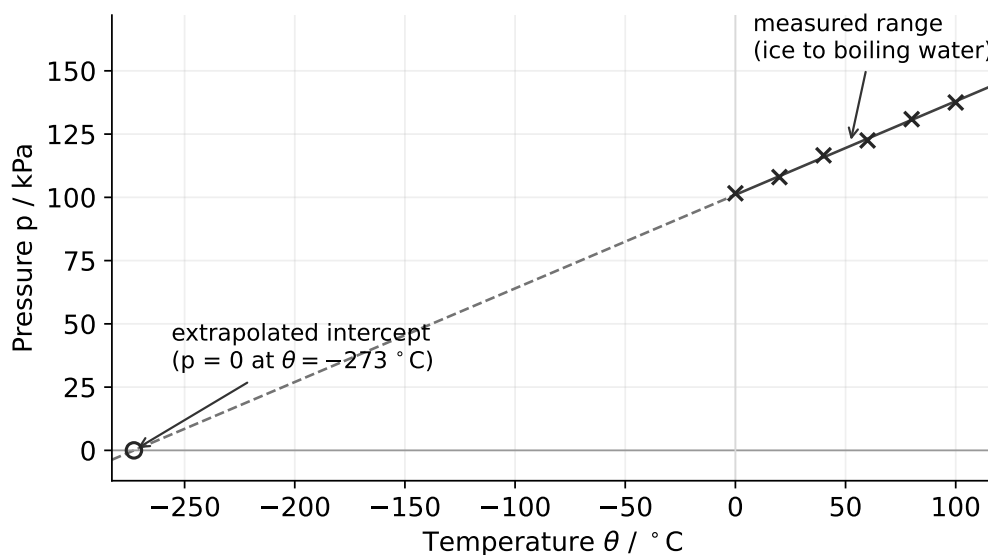
- Write the model, for example  $P = kX^4$  or  $A = A_0e^{-\lambda n}$ .
- Rearrange it into the straight-line form  $y = mx + c$ . This step is **linearisation**: choose a plot such as  $P$  against  $X^4$ ,  $\ln A$  against  $n$ , or  $\lg P$  against  $\lg X$ .
- State what the graph proves: a straight line through the origin for  $P$  against  $X^4$ , or **gradient** 4 on the **log graph** of  $\lg P$  against  $\lg X$ .
- Then choose the readings: at least five well-spread settings of the quantity you change.

A plan without its graph is an unfinished plan. The paper's six-mark "devise a method" questions expect the graph to appear in the answer.

### ■ Worked planning example: absolute zero from a gas flask

A flask of fixed air volume carries a pressure gauge and a thermometer. Predict how to estimate absolute zero.

- Known aim: the pressure of a fixed mass of gas falls linearly with temperature and reaches zero at absolute zero. Why this route: the zero-pressure point cannot be reached in water baths, but a straight line can be extended to it.
- Use a beaker of water with ice for  $0\text{ }^{\circ}\text{C}$  and a hot plate (or Bunsen burner) to raise the temperature to boiling. Stir so the water shares one temperature.
- Place the thermometer close to the flask, on the side away from the heater, and read the scale perpendicular to it.
- Record paired pressure and temperature readings at a minimum of five temperatures, spaced no more than about  $20\text{ }^{\circ}\text{C}$  apart. These are new settings, not repeats of one setting.
- Plot pressure against temperature in  $^{\circ}\text{C}$ , draw the **best-fit line** and **extrapolate** it to  $p = 0$ . The **intercept** on the temperature axis estimates  $-273\text{ }^{\circ}\text{C}$ .



*Extrapolation extends a straight line beyond the measured range; the measured points themselves stay between ice and boiling water.*

The extrapolated answer depends on the fitted line, so scatter matters more here than in a single reading. That is the price of a point you cannot measure directly.

### ■ Worked planning example: testing a power law

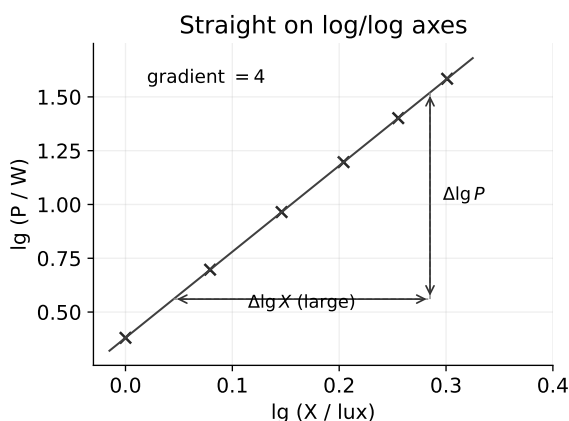
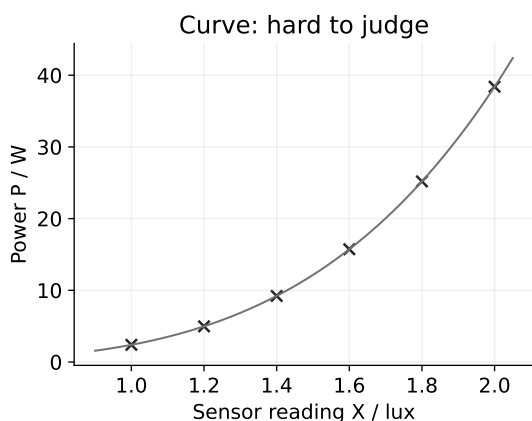
A light sensor a fixed distance from a filament bulb reads  $X$ ; the electrical power is predicted to follow  $P = kX^4$ .

- Control what else changes  $X$ : keep the bulb-sensor distance and alignment fixed, and work in a darkened space so background light does not add to the reading.

- Measure the current  $I$  and potential difference  $V$  at each setting, and calculate  $P = IV$ .
- Take at least five settings by changing the current, then choose one of two graphs:

$$\lg P = \lg k + 4 \lg X \quad \text{or} \quad P = k(X^4)$$

- A graph of  $\lg P$  against  $\lg X$  should be straight with gradient 4. A graph of  $P$  against  $X^4$  should be straight through the origin. Either answers the question; name the expected gradient or origin when you claim the test.



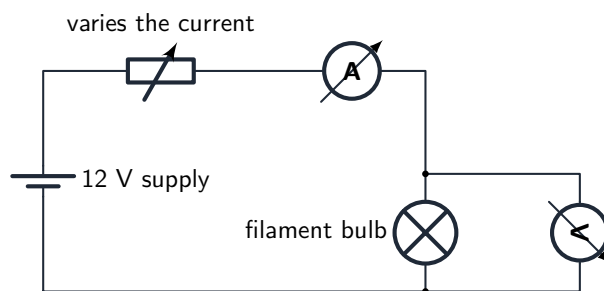
*A power law becomes a straight line on log/log axes; the gradient is the power.*

The specification's own example is a gas: plot an appropriate log/log graph of pressure against volume for a fixed mass at constant temperature. Log graphs are not only for exponential decay.

## ■ Circuits that measure what the investigation needs

An investigation of a component needs both meters in the right places:

- an **ammeter** in series with the component, so the same current flows through it;
- a **voltmeter** in parallel with the component under test **only**, so it reads that component's potential difference;
- a way of changing the current: a variable resistor or potentiometer, in series with a fixed supply.



If the voltmeter straddles two components, the reading no longer belongs to the thing you are investigating. When a two-position switch moves a charged capacitor to a new circuit, draw the voltmeter across the one capacitor the question names, and connect the second capacitor in parallel with it.

Electrolytic capacitors add their own safety rules: connect with the correct polarity, keep the supply below the working voltage, and discharge the capacitor before handling it.

### ■ Data loggers and safety, with reasons

A **data logger** with probes records pairs of readings at the same moment, removes **parallax** from scale reading, and its probes usually have a better **resolution** than a liquid-in-glass thermometer. Say which of these changes improves the measurement; “more accurate” alone names no mechanism.

Safety answers follow the same rule: name the hazard and the action. Hot water and immersion heaters burn – move glass with tongs or heat-proof gloves, and clamp the heater so its leads cannot topple the beaker. Switch off before removing anything electrical.

## Judging the measurements

### ■ Record what you can actually judge

A damped pendulum swings past a rule, and its turning points move quickly. Judging the amplitude to the nearest 1 mm claims a precision the eye cannot deliver, especially viewing the scale at an angle or with the bob far from the rule. Recording to the nearest 5 mm matches the recording to what can honestly be judged.

This is a two-way skill: criticise a recording resolution that is too fine, and justify a coarse one with the reason it is appropriate.

### ■ Timing oscillations

Measure the time for many oscillations, then divide. The reaction-time uncertainty is nearly the same whether you time one swing or ten, so a larger measured time gives a smaller percentage uncertainty in the period.

Two techniques earn marks on their own:

- put a **timing marker** at the centre of the oscillation and time passes through it, because the mass moves fastest there and the moment of passing is clearest;
- start the timer only after several oscillations, when the motion has settled, and use a small initial displacement.

For the period itself, record repeats of the *total* time, take the mean, and only then divide by the number of swings.

## ■ Length instruments at A2

The Unit 3 techniques still earn the marks, now with reasons attached:

- measure a spring or cylinder diameter with calipers at several **orientations** around the coil or cross-section and average, because a real object is not perfectly round (this attacks **random variation**);
- check the zero reading with the jaws closed and subtract a zero error, because it shifts every reading the same way (this attacks a **systematic error**);
- use the micrometer ratchet so the jaws tighten the same amount each time.

Justifying an instrument is quantitative: a micrometer resolves 0.01 mm, so a single 1.21 mm slide has an uncertainty of 0.005 mm. Its **percentage uncertainty** is:

$$\text{percentage uncertainty} = \frac{0.005 \text{ mm}}{1.21 \text{ mm}} \times 100\% = 0.4\%$$

A small percentage uncertainty is the justification; “it is accurate” is not.

## ■ Criticising tables and graphs

For a table of results, check:

- **significant figures** : raw readings use the same decimal places as the instrument’s resolution; processed values for plotting are commonly 3 significant figures. A column mixing 5.1 with 5.143 has a fault to name.
- **units**: every column heading carries its unit, once, in the heading.
- values that stand outside the pattern of the others, judged against the graph as well as the table.

For a plotted graph, the examiner’s checklist is short:

- enough data points, spread over more than half of each axis, on scales that are not awkward (multiples of 3 or 7 are hard to read);
- points plotted to within 1 mm (half a small square);
- a thin, continuous best-fit line that balances the scatter;
- whether the points actually support a straight line at all.

Criticising a graph is not listing everything you know. Point at the actual defect: five

points crowded into a quarter of the grid, or a gentle curve drawn as a line.

## Compounding uncertainties

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Unit 6 expects **compounded uncertainties**, which Unit 3 did not. The rules come from the specification's Appendix 10.

### ■ The building blocks

A single reading has an uncertainty of half the instrument's resolution. Repeated readings have an uncertainty of half the range (the reading furthest from the mean is an accepted alternative). A percentage uncertainty is:

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{value}} \times 100\%$$

Quote percentage uncertainties to one or two significant figures. Halving or doubling a measured quantity leaves its percentage uncertainty unchanged, which is why percentages, not absolute values, are the working currency.

### ■ The three compounding rules

1. **A power multiplies the percentage uncertainty.** If  $A = \pi r^2$  and  $r$  has 0.6% uncertainty, then  $A$  has  $2 \times 0.6\% = 1.2\%$ . A diameter and its radius share the same percentage uncertainty.
2. **Products and quotients add percentage uncertainties.** Density  $\rho = m/l^3$  from  $m$  with 0.1% and  $l$  with 2.1% gives  $3 \times 2.1\% + 0.1\% = 6.4\%$ .
3. **Sums and differences add absolute uncertainties.** Subtracting two heights measured to  $\pm 0.5$  mm each gives  $\Delta h = \pm 1$  mm, even though the heights themselves are several hundred millimetres.

The third rule is why differences of similar measurements carry large percentage uncertainties, and it combines with the first two: the percentage uncertainty of  $D^2/(2d^2)$  is  $2 \times \%U(D) + 2 \times \%U(d)$ .

### ■ Worked example: the ramp measurement of $g$

A sphere rolls a distance  $s$  down a ramp of height difference  $\Delta h$  in time  $t$ , where:

$$t^2 = \frac{14s^2}{5g \Delta h} \quad \Rightarrow \quad g = \frac{14s^2}{5t^2 \Delta h}$$

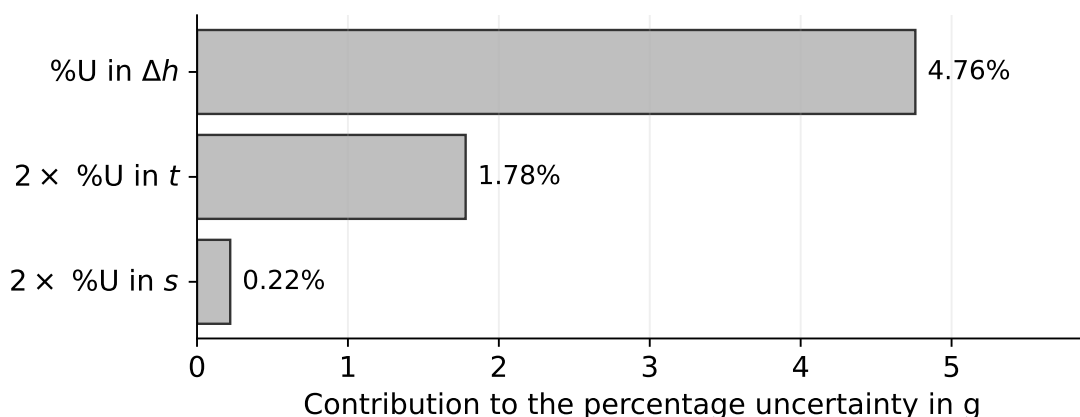
- Known:  $s = 90.0 \text{ cm} \pm 0.1 \text{ cm}$ ,  $\Delta h = 21 \text{ mm} \pm 1 \text{ mm}$ ,  $t = 3.36 \text{ s} \pm 0.03 \text{ s}$ . Why these rules:  $s$  and  $t$  are squared (powers), and the quantities are multiplied and divided.

$$g = \frac{14 \times (0.900 \text{ m})^2}{5 \times (3.36 \text{ s})^2 \times 21 \times 10^{-3} \text{ m}} = 9.6 \text{ m s}^{-2}$$

Each percentage contribution:

$$\%U(s) = \frac{0.1}{90.0} \times 100\% = 0.11\% \quad \%U(t) = \frac{0.03}{3.36} \times 100\% = 0.89\% \quad \%U(\Delta h) = \frac{1}{21} \times 100\% = 4.8\%$$

$$\%U(g) = 2(0.11\%) + 2(0.89\%) + 4.8\% = 6.8\%$$



*The height difference dominates. Improving the timing would barely change the result; improving  $\Delta h$  would.*

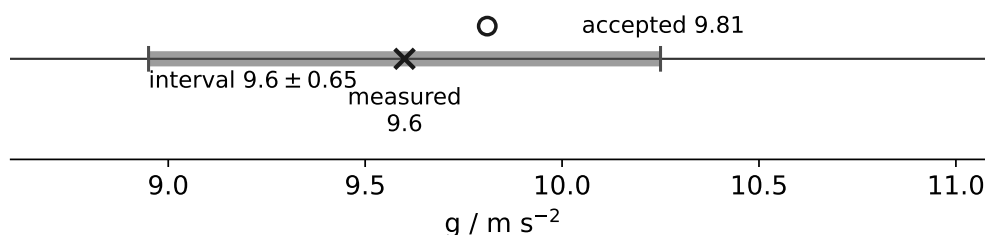
Compounding is also an improvement finder: the largest term names the measurement to fix first.

### ■ Using the uncertainty in a conclusion

With a percentage uncertainty, turn the result into an interval and compare:

$$g = 9.6 \text{ m s}^{-2} \times (1 \pm 0.068) \quad \Rightarrow \quad 8.9 \text{ to } 10.3 \text{ m s}^{-2}$$

The accepted  $9.81 \text{ m s}^{-2}$  lies inside the interval, so the measurement is accurate to the precision claimed.



Without an uncertainty for the measured value, compare percentages instead: a percentage difference below 5% suggests an accurate result:

$$\% \text{ difference} = \frac{9.81 - 9.6}{9.81} \times 100\% = 2.1\% < 6.8\%$$

Both routes end in a sentence that states what the evidence supports. A value of  $\nu = 0.276 \pm 6\%$  spans 0.259 to 0.293; a data-book 0.265 for steel falls inside, so the spring **could** be steel. That is the honest strength of the claim – the interval does not prove it is steel, only that steel is not excluded.

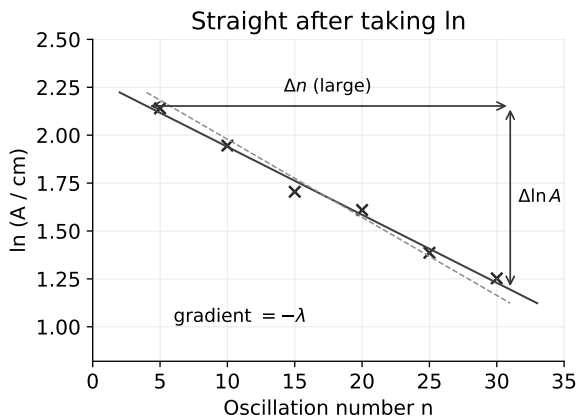
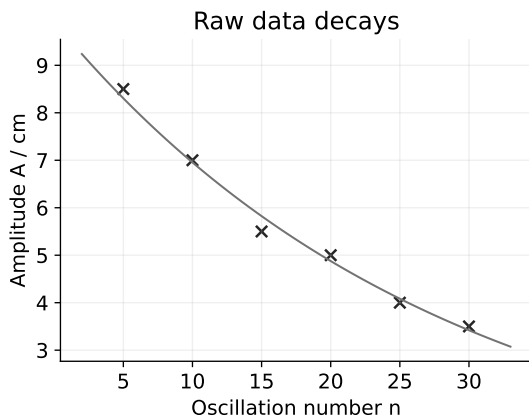
## Log graphs and linearisation

### ■ Straighten an exponential with $\ln$

A damped pendulum's amplitude follows  $A = A_0 e^{-\lambda n}$ , where  $n$  counts oscillations. Take the natural logarithm of both sides:

$$\ln A = \ln A_0 - \lambda n$$

Compare this with  $y = c + mx$ : a graph of  $\ln A$  against  $n$  is straight, with gradient  $-\lambda$  and intercept  $\ln A_0$ . The minus sign is part of the physics – amplitude decays, so the line slopes down.



**Worked example.** The line's intercept is 2.295 and its gradient is  $-0.0355$ .

- Known: intercept =  $\ln A_0$ , gradient =  $-\lambda$ . Why:  $A$  was measured in centimetres, so  $\ln(A/\text{cm})$  was plotted.

$$\lambda = 0.0355 \quad A_0 = e^{2.295} = 9.9 \text{ cm}$$

$\lambda$  has no unit because  $n$  is a count. Had  $n$  been time in seconds,  $\lambda$  would carry  $\text{s}^{-1}$ .

Processing discipline: compute  $\ln$  of every value to a consistent 3 decimal places, label the axis with the exact quantity logged –  $\ln(A/\text{cm})$ , not “ $\ln A$ ” with a unit bolted on – and take the gradient from a **large triangle** covering more than half the drawn line, using points on the line rather than data points off it.

### ■ When the intercept is an extrapolation

$A_0$  sits at  $n = 0$ , where there is no data if the first reading was taken at  $n = 5$ . The intercept is then only as good as the fitted line:

- large scatter leaves the best-fit line uncertain, moving the intercept;
- a systematic error in amplitude shifts the whole line;
- the pendulum’s first swings may not follow the model (a large starting angle stretches the period and distorts early amplitudes).

“Extrapolating beyond the data assumes the model still holds” is the sentence that earns the mark. Say which of these applies to the experiment in front of you.

### ■ Use the fitted line to predict

A fitted relation answers questions the raw table cannot. If  $\ln V = \ln A - Bw$  for light passing through glass, and the line gives  $B = 0.0104 \text{ mm}^{-1}$ , the thickness  $w$  needed to dim a reading to 75% follows without touching the apparatus again:

$$\ln\left(\frac{V}{A}\right) = -Bw \quad \Rightarrow \quad \ln 0.75 = -(0.0104 \text{ mm}^{-1})w \quad \Rightarrow \quad w = 27.7 \text{ mm}$$

With slides of 1.22 mm each, that is  $27.7/1.22 = 22.7$ , so 23 slides – round **up**, because 22 slides would not reach the required dimming. Prediction questions test the direction of the final rounding as much as the algebra.

### ■ Straighten a power law with $\lg$

For  $y = kx^n$ , taking base-10 logarithms gives:

$$\lg y = \lg k + n \lg x$$

A graph of  $\lg y$  against  $\lg x$  is straight with gradient  $n$  and intercept  $\lg k$ . Use it when no power of  $x$  makes a convenient linear axis, and keep the base consistent:  $\lg$  for the power-law plot,  $\ln$  for the exponential one. Both operate on the number after the unit is divided out, so the axis label is  $\lg(P/W)$ , never  $\lg P$  with watts attached.

## Check yourself

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Before attempting a Unit 6 question, check that you can:

- choose the processing graph first and write a method that feeds it;
- plan an extrapolation with its control variables, safety and reading plan;
- place both meters correctly and vary the current in a component investigation;
- justify an instrument or recording resolution with a percentage uncertainty;
- time oscillations and criticise tables and graphs against the examiner's checklist;
- compound uncertainties for powers, products and differences, and name the dominant term;
- turn a result into an interval and state honestly what it supports;
- linearise an exponential with  $\ln$  and a power law with  $\lg$ , and convert gradient and intercept back into physical quantities;
- round a prediction the right way and say when an extrapolation is trustworthy.