6.3 Compounded uncertainties and justified conclusions

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS percentage uncertainty • compounded uncertainties • percentage difference

Objective

Practise the WPH16 uncertainty arithmetic: single-reading and repeat estimates, the three compounding rules, and interval or percentage-difference conclusions. These are original training tasks grounded in the specification and reviewed assessment demand.

1 Key knowledge and worked examples

The building blocks

A single reading has an uncertainty of half the instrument resolution; repeats have an uncertainty of half the range (the reading furthest from the mean is accepted too). The **percentage uncertainty** is

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{value}} \times 100\%$$

quoted to one or two significant figures. Doubling or halving a measured length leaves its percentage unchanged, which is why percentages are the working currency.

The three compounding rules

1. **A power multiplies**: in $A = \pi r^2$, $\%U(A) = 2 \times \%U(r)$.
2. **Products and quotients add**: in $\rho = m/l^3$, $\%U(\rho) = 3 \times \%U(l) + \%U(m)$.
3. **Sums and differences add absolute uncertainties**: two heights each ± 0.5 mm give a difference ± 1 mm.

Rules 1–2 work on percentages; rule 3 works on absolute values. Combine them as needed: the term $D^2/(2d^2)$ carries $2 \times \%U(D) + 2 \times \%U(d)$.

■ Worked example: density of a cube

A cube has side $l = 24.0\ \text{mm} \pm 0.5\ \text{mm}$ and mass $m = 48.23\ \text{g} \pm 0.05\ \text{g}$.

- Known: l, m and their uncertainties. Why these rules: $\rho = m/l^3$ divides measured quantities and cubes one of them.

$$\rho = \frac{m}{l^3}$$

$$\rho = \frac{48.23 \times 10^{-3}\ \text{kg}}{(24.0 \times 10^{-3}\ \text{m})^3} = 3.49 \times 10^3\ \text{kg m}^{-3}$$

$$\%U(l) = \frac{0.5}{24.0} \times 100\% = 2.1\% \qquad \%U(m) = \frac{0.05}{48.23} \times 100\% = 0.1\%$$

$$\%U(\rho) = 3 \times 2.1\% + 0.1\% = 6.4\%$$

So $\rho = 3.49 \times 10^3\ \text{kg m}^{-3} \pm 6\%$. The side dominates: improving the mass balance would not help.

■ Using the result in a conclusion

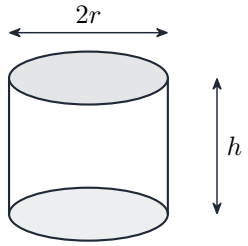
Turn the result into an interval and compare:

$$3.49 \times 10^3 \times (1 \pm 0.064)\ \text{kg m}^{-3} \qquad \Rightarrow \qquad 3.3 \text{ to } 3.7 \times 10^3\ \text{kg m}^{-3}$$

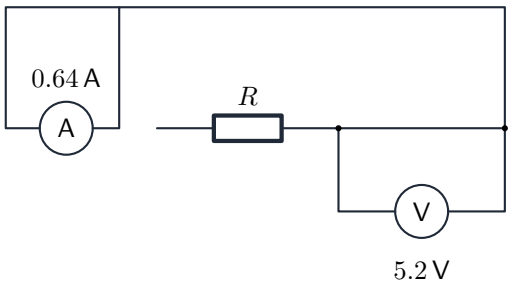
Without an uncertainty, compare percentages instead: a **percentage difference** below about 5% suggests an accurate result. State what the evidence supports and no more: an interval containing a data-book value shows consistency, not proof of identity.

2 Practice

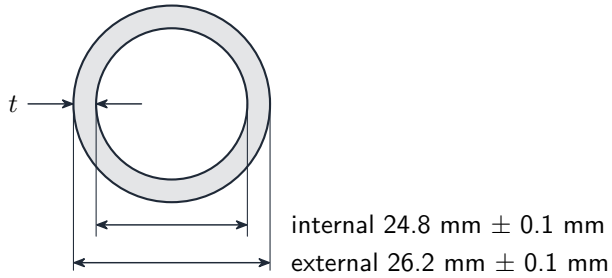
2.1 A cylindrical can has radius $r = 33\ \text{mm} \pm 1\%$ and height $h = 115\ \text{mm} \pm 2\ \text{mm}$. Show that the percentage uncertainty in its volume $V = \pi r^2 h$ is about 4%. [3]



2.2 A resistor's value is found from $R = V/I$, with $V = 5.2\ \text{V} \pm 0.1\ \text{V}$ and $I = 0.64\ \text{A} \pm 0.02\ \text{A}$. Calculate R and its percentage uncertainty. [3]



2.3 The external and internal diameters of a pipe are $26.2\ \text{mm} \pm 0.1\ \text{mm}$ and $24.8\ \text{mm} \pm 0.1\ \text{mm}$. Determine the wall thickness and its absolute uncertainty. [2]



3 Exam-style questions

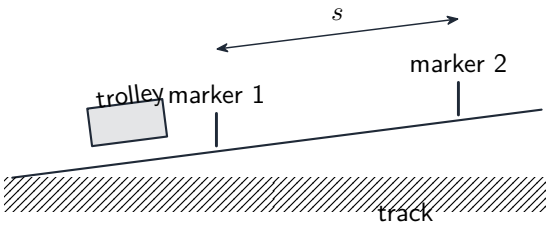
3.1 A trolley accelerates uniformly from rest through a distance s in a time t, with $s = \frac{1}{2}at^2$.

The student records $s = 0.900\ \text{m} \pm 0.002\ \text{m}$ and $t = 1.40\ \text{s} \pm 0.05\ \text{s}$.

(a) Determine the acceleration a. [2]

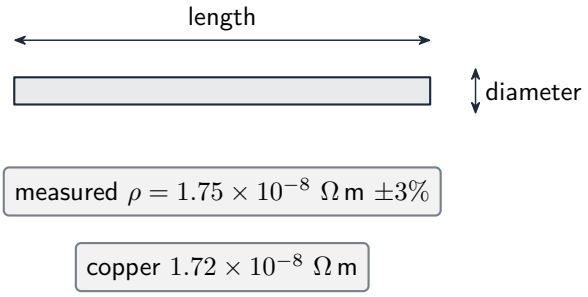
(b) Show that the percentage uncertainty in a is about 8%. [3]

(c) The track is inclined at 5.5° to the horizontal, so the predicted acceleration is $g \sin 5.5° = 0.94\ \text{m s}^{-2}$. Deduce whether the student's value is accurate. [2]



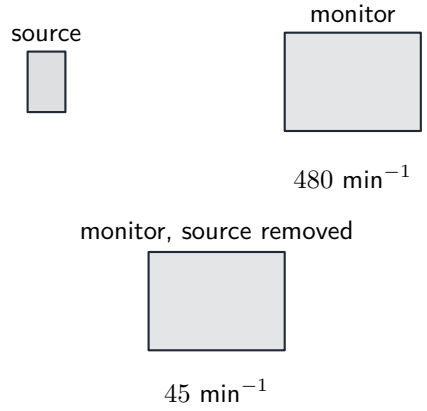
3.2 A student measures the resistivity of a wire as $\rho = 1.75 \times 10^{-8}\ \Omega\ \text{m}$ with a percentage uncertainty of 3%. The data-book value for copper is $1.72 \times 10^{-8}\ \Omega\ \text{m}$.

Determine the interval for the student's value and deduce whether the wire could be copper. [3]



3.3 A radiation monitor reads a gross rate of 480 min^{-1} with the source in place and a background of 45 min^{-1} with the source removed. Each rate is counted for the same time, giving each reading an absolute uncertainty of 10 min^{-1}.

(a) Calculate the corrected count rate and its absolute uncertainty. [3]

(b) Determine the percentage uncertainty in the corrected rate. [2]

(c) The half-life of the source is later determined with a percentage uncertainty of 5%, dominated by the count rates. Explain which measurement most limits this result and one change that would improve it. [3]