4.4 Capacitors and RC circuits

Name: _____ Class: _____ Date: _____ **Total: 32 marks**

KEY WORDS capacitor • Capacitance • time constant

Objective

Develop the methods for capacitors and rc circuits before the curated WPH14 questions. Use $g = 9.81$ m s^{-2} where needed. These are original training questions, not copied past-paper questions.

1 Key knowledge and worked examples

Charge storage and energy

A **capacitor** stores separated charge on two conductors. They carry equal and opposite charges in the ideal two-plate model. The quoted charge Q is the magnitude on either plate, not the sum of both magnitudes. **Capacitance** is charge stored per unit p.d.:

$$C = \frac{Q}{V}$$

One farad is one coulomb per volt. For constant capacitance, a V–Q graph is straight through the origin with gradient $1/C$. Its area gives the work needed to transfer charge onto the capacitor:

$$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

The factor one half appears because p.d. rises from zero while charge is transferred. It is not QV at the final voltage throughout charging. For discharge between non-zero voltages, subtract the two stored energies:

$$\Delta W = \frac{1}{2}C(V_{\text{initial}}^2 - V_{\text{final}}^2)$$

This is different from $\frac{1}{2}C(V_{\text{initial}} - V_{\text{final}})^2$.

Charging and discharging

In an ideal series resistor–capacitor charging circuit, initially uncharged, the capacitor p.d. is zero. The resistor initially has the full supply p.d., so current is maximum. As charge builds, capacitor p.d. rises. Resistor p.d. and current fall. At full charge, current is zero and capacitor p.d. equals supply p.d.

For discharge through a fixed resistance, use signed current consistently or work with its magnitude:

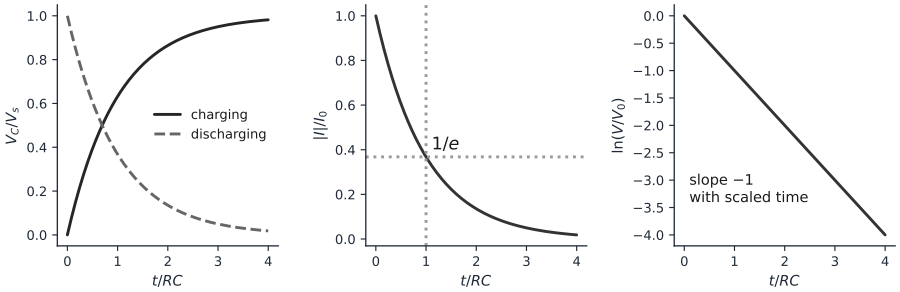
$$Q = Q_0 e^{-t/RC} \qquad V = V_0 e^{-t/RC} \qquad |I| = I_0 e^{-t/RC}$$

The voltage relation follows from $Q = CV$ with constant C. Current magnitude follows from $|I| = V/R$ with constant R. The **time constant** is $\tau = RC$. After one time constant, Q, V and current magnitude are $e^{-1} \approx 0.368$ of their initial values. After three time constants they are about 5%, not zero. Stored energy falls faster because it depends on V^2.

For charging from an ideal constant supply V_s:

$$V_C = V_s(1 - e^{-t/RC}) \qquad V_R = V_s e^{-t/RC} \qquad I = \frac{V_s}{R}e^{-t/RC}$$

A charging capacitor voltage rises towards V_s; charging current falls towards zero. Do not sketch the same curve for both quantities. Equal V_C and V_R means each is $V_s/2$, reached at $t = RC\ln 2$.



Taking natural logs of the discharge equation gives:

$$\ln V = \ln V_0 - \frac{t}{RC}$$

Thus a graph of $\ln V$ against t has gradient $-1/(RC)$ and intercept $\ln V_0$. The same forms hold for Q and current magnitude. Use consistent measurement units inside the logged numerical values.

Worked example. A 100 μF capacitor discharges from 12.0 V through 20.0 kΩ. Find p.d. after 3.0 s and energy lost.

$$\tau = RC = 20.0 \times 10^3\ \Omega \times 100 \times 10^{-6}\ \text{F} = 2.00\ \text{s}$$

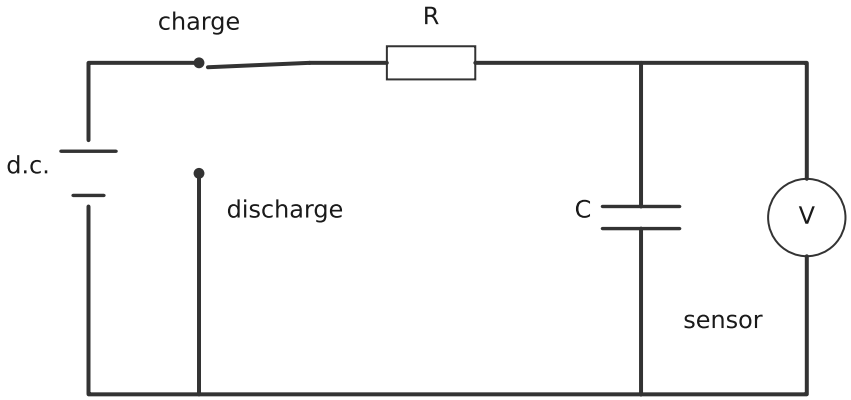
$$V = V_0 e^{-t/RC} = 12.0\ \text{V} \times e^{-3.0/2.00} = 2.68\ \text{V}$$

$$\Delta W = \frac{1}{2}C(V_0^2 - V^2) = \frac{1}{2} \times 100 \times 10^{-6}\ \text{F} \times (12.0^2 - 2.68^2)\ \text{V}^2 = 6.84 \times 10^{-3}\ \text{J}$$

The lost stored energy becomes mainly heat in the discharge resistance. Use unrounded voltage in the final calculation.

■ Core practical 11 and useful extensions

Use a low-voltage d.c. supply, resistor, capacitor and switch. Charge through the resistor, then disconnect the supply and discharge through the known resistance. Connect a voltage sensor, data logger or oscilloscope across the capacitor. Record time and p.d.; choose a sampling interval short compared with RC. Use an instrument input resistance large compared with the discharge resistance, otherwise it changes the discharge path.



Keep R and C fixed, repeat after restoring the same initial charge, and compare measured curves with exponential predictions. Determine RC from the $1/e$ level or a log-fit gradient. Avoid taking logs of readings near zero where relative uncertainty is large. For a resistance-tolerance test, calculate $R = -\Delta t/[C \ln(V_2/V_1)]$ during one uninterrupted discharge and compare with both limits of the permitted interval.

Use correct polarity for an electrolytic capacitor and remain below its voltage rating. Discharge it safely through a resistor before altering connections; do not short a charged capacitor. Leakage, sensor loading and resistor heating can change the measured curve.

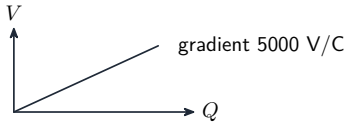
A smoothing capacitor charges near supply peaks and discharges into the load between them. A smaller load resistance gives a smaller time constant, faster discharge and less smoothing. For two identical series capacitors, the same magnitude of charge appears on each. Their equal p.d.s share the supply, so each stores half the charge of one identical capacitor connected alone. This sharing is a useful bridge to the selected paper; it does not replace the main capacitance model.

2 Practice

2.1 A 220 μF capacitor is charged to 9.0 V. Calculate charge on either plate and stored energy. Explain why summing the magnitudes on both plates does not give the quoted capacitor charge. [5]

$C = 220\ \mu\text{F};\ V = 9.0\ \text{V}$

2.2 A capacitor's V–Q graph is a straight line through the origin with gradient 5000 V C^{-1}. Find capacitance. Use the graph to derive the stored-energy formula. [4]



2.3 A 100 μF capacitor discharges from 10.0 V through 30.0 kΩ. Calculate its time constant, p.d. and current magnitude after 6.0 s. [5]

$C = 100\ \mu$F; $R = 30.0$ kΩ

3 Exam-style questions

3.1 An initially uncharged capacitor charges through resistance R from supply V_s. Explain why current is initially maximum and decreases. Derive the time when resistor and capacitor p.d.s are equal. Explain why replacing the resistor with a smaller resistance worsens smoothing when the charged capacitor discharges into that load between supply peaks. [5]

C, R, V_s

3.2 A 470 μF capacitor's p.d. falls from 8.0 V to 3.0 V in 4.0 s. Assume the stated capacitance is exact for this test. Determine the resistance and whether it lies within 10.0 k$\Omega \pm 10\%$. Calculate energy delivered to the resistance during this interval. [6]

470 μF; 8.0 V to 3.0 V in 4.0 s

3.3 Plan a data-logger investigation of capacitor discharge that obtains RC from a log graph. Include the circuit, graph axes and gradient, repeats, sampling, input resistance and safety. Then state how instrument loading can affect the inferred time constant. [7]

data logger | capacitor discharge