4.2 Circular motion

Name: _____ Class: _____ Date: _____ **Total: 29 marks**

KEY WORDS radian • Angular velocity • tangential speed • centripetal force

Objective

Develop the methods for circular motion before the curated WPH14 questions. Use $g = 9.81\ \text{m s}^{-2}$ where needed. These are original training questions, not copied past-paper questions.

1 Key knowledge and worked examples

Angle, angular speed and tangential speed

An angle of one **radian** subtends an arc equal to the radius. Hence $\theta = s/r$, with angle in radians. One revolution is 2π radians or 360°.

$$\theta_{\text{rad}} = \theta_{\text{degrees}}\frac{2\pi}{360} \qquad \omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} \qquad v = \omega r$$

Angular velocity describes the rate of angular displacement. In the plane problems here, use its signed rotational rate or magnitude as appropriate. Points on one rigid turntable have the same angular speed. Points farther from its centre have larger **tangential speed** . Their velocity directions also vary around the circle.

Worked example. A wheel rotates at 120 revolutions per minute. Find angular speed and acceleration of a point 0.25 m from its centre.

$$f = \frac{N}{t} = \frac{120}{60\ \text{s}} = 2.0\ \text{Hz}$$

$$\omega = 2\pi f = 2\pi \times 2.0\ \text{s}^{-1} = 12.6\ \text{rad s}^{-1}$$

$$a = r\omega^2 = 0.25\ \text{m} \times (4\pi\ \text{s}^{-1})^2 = 39.5\ \text{m s}^{-2}$$

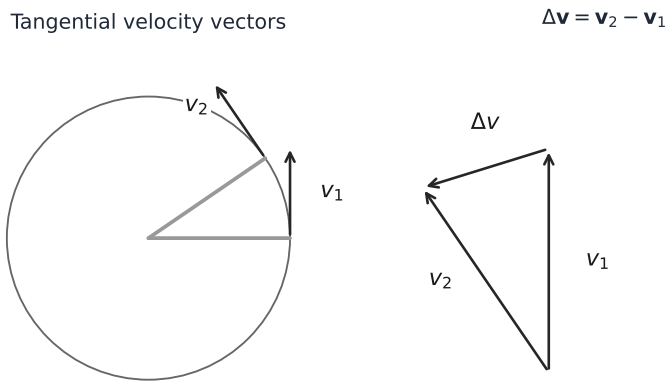
Keep the unrounded angular speed during the calculation. Use radius, not diameter.

Why acceleration points inward

Uniform circular motion has constant speed but changing velocity. Draw the initial and final tangential velocity vectors. The change is $\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$, not their sum. For a small angular change, the velocity triangle and radius triangle are similar:

$$\frac{\Delta v}{v} \approx \frac{\Delta s}{r} \qquad \Delta s \approx v\Delta t$$

$$a = \lim_{\Delta t \to 0}\frac{\Delta v}{\Delta t} = \frac{v^2}{r} = r\omega^2$$



The change in velocity points towards the centre in this limit. A resultant **centripetal force** is therefore needed:

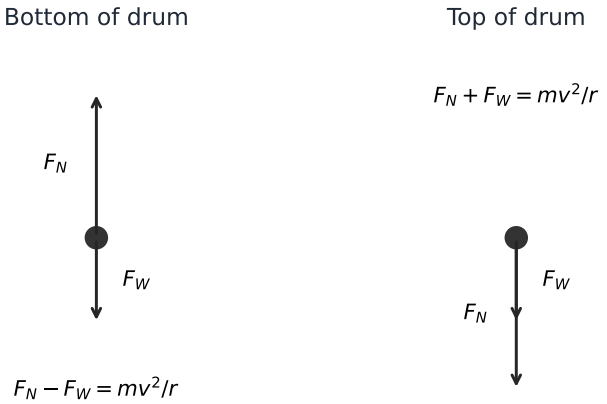
$$\sum F_{\text{inward}} = ma = \frac{mv^2}{r} = mr\omega^2$$

Centripetal force names the resultant of real forces. It is not an extra force to add to a force diagram. Friction can provide it on a turntable. Tension can provide it for a ball on a string.

Write the real-force equation

For clothing against the inside of a vertical drum, weight always points down. The drum's normal reaction points towards the centre. At constant speed:

$$\text{bottom:}\quad F_N - mg = \frac{mv^2}{r} \qquad \text{top:}\quad F_N + mg = \frac{mv^2}{r}$$



The required resultant has the same magnitude, but the reaction is larger at the bottom. If contact is lost, the surface cannot provide a pulling normal reaction. Check this before using a circular-path model.

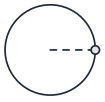
For a horizontal turntable, the largest available friction must be at least $mr\omega^2$. If maximum friction is $mg/25$:

$$mr\omega^2 \le \frac{mg}{25} \quad \Rightarrow \quad r \le \frac{g}{25\omega^2}$$

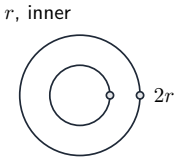
Mass cancels because both required and available forces scale with mass. If the inward force disappears, motion initially follows the tangent. Gravity can then curve the later path; "tangent" describes the release direction.

2 Practice

2.1 A point moves around a circle of radius 0.40 m at 90 revolutions per minute. Calculate angular speed, tangential speed and centripetal acceleration. [5]



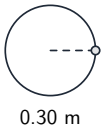
2.2 Two objects rotate on the same rigid disc, at radii r and $2r$. Compare their angular speeds, tangential speeds and accelerations. Explain how an object can accelerate with constant speed. [4]



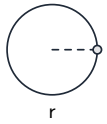
2.3 Use a small-angle velocity-vector triangle to derive $a = v^2/r$. Explain why adding the two velocity vectors does not give the change in velocity. [4]

3 Exam-style questions

3.1 A 0.15 kg piece of clothing follows a vertical drum of radius 0.30 m at constant speed 3.0 m s^{-1}. Calculate the drum's reaction at bottom and top. Explain why these differ even though the required centripetal resultant is equal. State the initial direction of motion if contact and all inward constraint are removed at a point. [6]

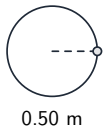


3.2 A small block rests on a horizontal turntable rotating at 2.0 rad s^{-1}. Maximum friction is $mg/20$. Derive and calculate its largest allowed radius. Explain whether doubling the mass changes this radius under the given model. [5]



$\omega = 2.0$ rad/s

3.3 A student calls centripetal force "an extra inward force" and says doubling rotational frequency doubles the needed force. Correct both statements using equations. Then determine the minimum speed for a small object to remain in contact at the top of the inside of a vertical circular track of radius 0.50 m. [5]



vertical track