

6.4 Log graphs and linearisation

Name: _____ Class: _____ Date: _____

Total: 31 marks

KEY WORDS large triangle • best-fit line • intercept

Objective

Practise the WPH16 graph skills: linearise exponentials with natural logs and power laws with base-10 logs, process and plot data, convert gradients and intercepts back into physical quantities, and use a fitted line to predict. These are original training tasks grounded in the specification and reviewed assessment demand.

1 Key knowledge and worked examples

■ Straighten an exponential with $\ln$

For $y = y_0 e^{-kt}$, take natural logs of both sides:

$$\ln y = \ln y_0 - kt$$

Compare with $y = c + mx$: plot $\ln y$ against t and the line is straight with gradient $-k$ and intercept $\ln y_0$. To recover the physical quantities, invert the log: $y_0 = e^{\text{intercept}}$. The minus sign is part of the physics – a decaying quantity slopes down.

■ Straighten a power law with $\lg$

For $y = kx^n$, base-10 logs give

$$\lg y = \lg k + n \lg x$$

so a graph of $\lg y$ against $\lg x$ is straight with gradient n and intercept $\lg k$. Use $\lg$ for power laws, $\ln$ for exponentials, and never mix the two in one analysis.

■ Processing and plotting discipline

- Compute the logged value of **every** reading, to a consistent three decimal places.
- Label the axis with the exact quantity logged: $\ln(A/\text{cm})$ or $\lg(P/W)$ – the unit is divided out **before** the log, never left beside one.
- Choose scales that use more than half of each axis and are easy to subdivide; plot points to within half a small square.

- Take the gradient from a **large triangle** spanning more than half the drawn line, using points **on the line**, not data points off it.
- Read the intercept where the line crosses the vertical axis; if the axis starts away from zero, extend the line or compute the intercept from a point on the line.

■ Worked example: a capacitor discharge

A capacitor discharges through a resistor with $V = V_0 e^{-t/RC}$. A student's line of $\ln V$ against t has intercept 2.71 and gradient -0.032 s^{-1} .

- Known: intercept = $\ln V_0$, gradient = $-1/RC$. Why: the voltages were in volts, so the axis was $\ln(V/V)$.

$$V_0 = e^{2.71} = 15 \text{ V} \quad RC = \frac{1}{0.032 \text{ s}^{-1}} = 31 \text{ s}$$

Both physical answers come from the line, with no single reading trusted on its own.

■ Predict, and round the right way

A fitted line answers questions the table cannot. If $\ln V = \ln A - Bw$ gives $B = 0.0088 \text{ mm}^{-1}$, the thickness needed to dim a display to 60% follows:

$$\ln\left(\frac{V}{A}\right) = -Bw \quad \Rightarrow \quad w = \frac{\ln 0.60}{-0.0088 \text{ mm}^{-1}} = 58 \text{ mm}$$

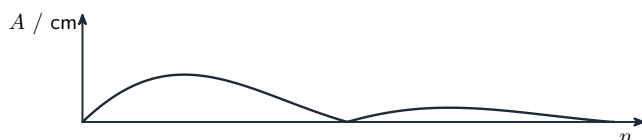
With 2.0 mm filters, $58/2.0 = 29$, so 30 filters are needed: 29 would leave the brightness just above 60%. Round **up** when the requirement is "at most".

■ When is an extrapolation trustworthy?

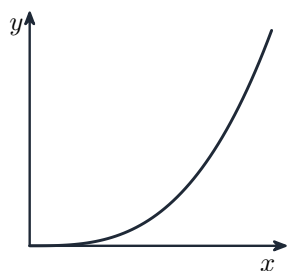
An intercept read where there is no data is only as good as the line: large scatter leaves the fitted line uncertain; a systematic error shifts it; and the model may not hold beyond the measured range. Name the one that applies to the experiment in front of you.

2 Practice

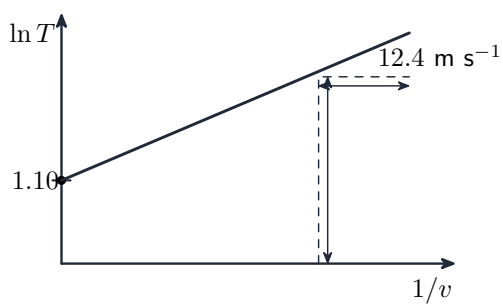
2.1 The amplitudes of a damped oscillator are recorded as 9.5, 7.8, 6.3, 5.1, 4.2, 3.4 cm. Calculate $\ln(A/\text{cm})$ for each value, giving your answers to three decimal places. [2]



2.2 A quantity follows $y = kx^3$. State the graph you would plot to test this, and what its gradient and intercept would represent. [2]



2.3 A line of $\ln T$ against $1/v$ has intercept 1.10 and gradient 12.4 m s^{-1} . The model is $T = Ae^{B/v}$. Determine A and B . [2]

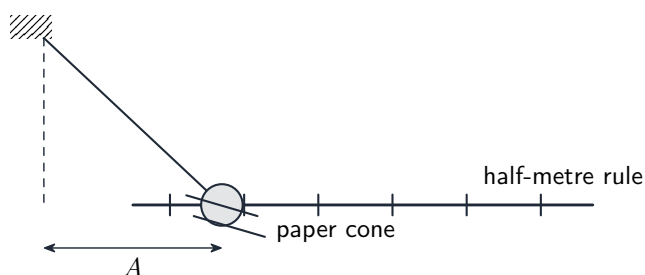
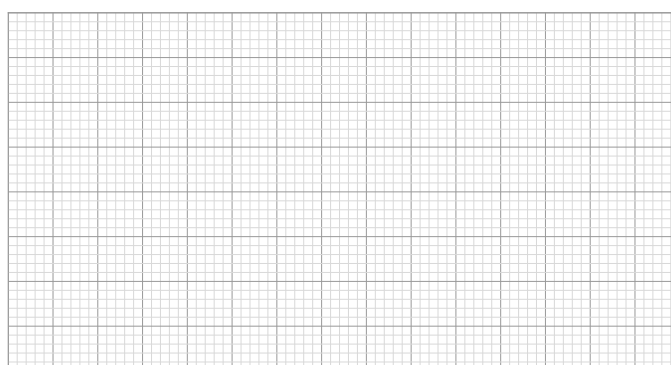


3 Exam-style questions

3.1 A paper cone is attached to a pendulum bob as a damper. The amplitude A follows $A = A_0 e^{-\lambda n}$, where n counts oscillations.

n	A / cm
5	9.5
10	7.8
15	6.3
20	5.1
25	4.2
30	3.4

- Copy and complete the additional column with $\ln(A/\text{cm})$ to three decimal places. [1]
- Plot $\ln(A/\text{cm})$ against n on the grid and draw a best-fit line. [3]
- Use a large triangle to determine λ . [2]
- Determine A_0 from your graph. [2]
- Explain why the value of A_0 from the graph may not be the true initial amplitude. [2]

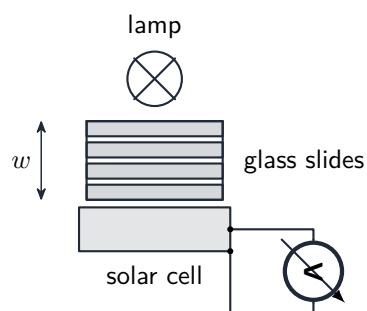


3.2 Light passes through stacks of glass slides of total thickness w . The voltmeter reading across a solar cell follows $V = Ae^{-Bw}$, where A and B are constants.

w / mm	V / mV
2	413
4	406
6	399
8	392
10	385

(a) Explain how a graph of $\ln(V/\text{mV})$ against w determines B . [2]

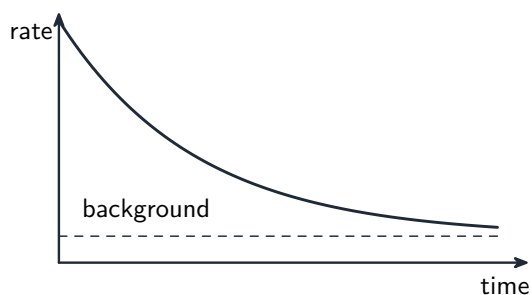
(b) The graph gives a gradient of -0.0088 mm^{-1} . Determine the minimum number of 2.0 mm slides needed to reduce V to at most 60% of its value with no glass. [4]



3.3 A source decays with activity $R = R_0 e^{-\lambda t}$. The background rate is a constant 20 min^{-1} . The gross monitor rate is recorded every ten minutes:

t / min	Gross rate / min^{-1}
0	470
10	367
20	288
30	227
40	179
50	143

- (a) Explain why the graph must use $\ln(R_{\text{corr}}/\text{min}^{-1})$ rather than the gross rate. [2]
- (b) Process the data and describe how the plotted graph determines λ . [3]
- (c) The line gives $\lambda = 0.026 \text{ min}^{-1}$. Determine the half-life. [2]
- (d) The source's data sheet states a half-life of 25 minutes. The percentage uncertainty in the half-life is estimated at 5%. Deduce whether the measurement agrees with the data sheet. [2]



Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 Each value passed through the natural log: 2.251, 2.054, 1.841, 1.629, 1.435, 1.224, all to three decimal places and consistent.

2.2 Plot $\lg y$ against $\lg x$; gradient = 3 and intercept = $\lg k$. (Plotting y against x^3 with a line through the origin is an acceptable alternative.)

2.3 $A = e^{1.10} = 3.0$; matching $\ln T = \ln A + B/v$ to $y = c + mx$ gives $B = +12.4 \text{ m s}^{-1}$ – the sign is positive because the exponent is $+B/v$ here.

3.1 (a) $\ln(A/\text{cm})$: 2.251, 2.054, 1.841, 1.629, 1.435, 1.224. (b) Axes labelled $\ln(A/\text{cm})$ and n , sensible scales, points plotted to half a small square, thin best-fit line. (c) Large triangle on the line, e.g. from (5, 2.25) to (30, 1.22): $\lambda = -\text{gradient} = (2.25 - 1.22)/(30 - 5) = 0.041$, accept 0.040–0.043, no unit. (d) Intercepts: extend the line to $n = 0$ and read $\ln A_0 \approx 2.46$, then $A_0 = e^{2.46} = 11.7 \text{ cm}$ (accept 11–12 cm, unit matching the logged quantity). (e) Any paired reasoning: there is no data below $n = 5$, so the line's position at $n = 0$ is an extrapolation that scatter makes uncertain; or a systematic error in judging amplitude shifts the whole line; or the earliest swings may depart from the model if the starting angle was large.

3.2 (a) Taking logs: $\ln V = \ln A - Bw$, so the plot is straight with gradient $-B$. (b) With no glass $V = A$, so the target is $V/A = 0.60$:

$$\ln 0.60 = -Bw \quad \Rightarrow \quad w = \frac{\ln 0.60}{-0.0088 \text{ mm}^{-1}} = 58 \text{ mm}$$

Dividing by the 2.0 mm thickness gives $58/2.0 = 29$, so at least 30 slides: 29 leave the reading marginally above 60%.

3.3 (a) The gross rate is the source's activity plus a constant background; $\ln$ of a sum is not linear in t . Subtracting the background first leaves $R_{\text{corr}} = R_0 e^{-\lambda t}$, whose log is linear. (b) Corrected rates: 450, 347, 268, 207, 159, 123 min^{-1} ; logs to three decimal places: 6.109, 5.849, 5.591, 5.333, 5.069, 4.812; plot $\ln(R_{\text{corr}}/\text{min}^{-1})$ against t , best-fit line, gradient = $-\lambda$. (c) The half-life follows from the decay constant:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{0.026 \text{ min}^{-1}} = 26.7 \text{ min}$$

so the half-life is 26.7 minutes. (d) Interval: $26.7 \times (1 \pm 0.05) = 25.4$ to 28.0 min . The data-sheet 25 minutes lies just below the lower limit, so strictly the measurement does not agree at that uncertainty; alternatively a percentage difference of $\frac{26.7-25}{25} \times 100\% = 6.8\%$ exceeds 5%, again not in agreement. A conclusion that follows the chosen comparison earns the mark.