

6.3 Compounded uncertainties and justified conclusions

Name: _____ Class: _____ Date: _____

Total: 26 marks**KEY WORDS** percentage uncertainty
• percentage difference

• compounded uncertainties

Objective

Practise the WPH16 uncertainty arithmetic: single-reading and repeat estimates, the three compounding rules, and interval or percentage-difference conclusions. These are original training tasks grounded in the specification and reviewed assessment demand.

1 Key knowledge and worked examples

■ The building blocks

A single reading has an uncertainty of half the instrument resolution; repeats have an uncertainty of half the range (the reading furthest from the mean is accepted too). The **percentage uncertainty** is

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{value}} \times 100\%$$

quoted to one or two significant figures. Doubling or halving a measured length leaves its percentage unchanged, which is why percentages are the working currency.

■ The three compounding rules

1. **A power multiplies:** in $A = \pi r^2$, $\%U(A) = 2 \times \%U(r)$.
2. **Products and quotients add:** in $\rho = m/l^3$, $\%U(\rho) = 3 \times \%U(l) + \%U(m)$.
3. **Sums and differences add absolute uncertainties:** two heights each ± 0.5 mm give a difference ± 1 mm.

Rules 1–2 work on percentages; rule 3 works on absolute values. Combine them as needed: the term $D^2/(2d^2)$ carries $2 \times \%U(D) + 2 \times \%U(d)$.

■ Worked example: density of a cube

A cube has side $l = 24.0 \text{ mm} \pm 0.5 \text{ mm}$ and mass $m = 48.23 \text{ g} \pm 0.05 \text{ g}$.

- Known: l , m and their uncertainties. Why these rules: $\rho = m/l^3$ divides measured quantities and cubes one of them.

$$\rho = \frac{m}{l^3}$$

$$\rho = \frac{48.23 \times 10^{-3} \text{ kg}}{(24.0 \times 10^{-3} \text{ m})^3} = 3.49 \times 10^3 \text{ kg m}^{-3}$$

$$\%U(l) = \frac{0.5}{24.0} \times 100\% = 2.1\% \quad \%U(m) = \frac{0.05}{48.23} \times 100\% = 0.1\%$$

$$\%U(\rho) = 3 \times 2.1\% + 0.1\% = 6.4\%$$

So $\rho = 3.49 \times 10^3 \text{ kg m}^{-3} \pm 6\%$. The side dominates: improving the mass balance would not help.

■ Using the result in a conclusion

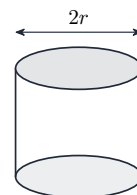
Turn the result into an interval and compare:

$$3.49 \times 10^3 \times (1 \pm 0.064) \text{ kg m}^{-3} \Rightarrow 3.3 \text{ to } 3.7 \times 10^3 \text{ kg m}^{-3}$$

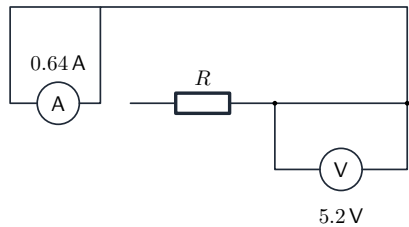
Without an uncertainty, compare percentages instead: a **percentage difference** below about 5% suggests an accurate result. State what the evidence supports and no more: an interval containing a data-book value shows consistency, not proof of identity.

2 Practice

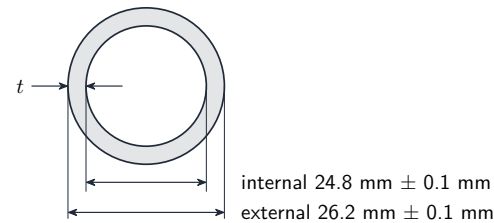
2.1 A cylindrical can has radius $r = 33 \text{ mm} \pm 1\%$ and height $h = 115 \text{ mm} \pm 2 \text{ mm}$. Show that the percentage uncertainty in its volume $V = \pi r^2 h$ is about 4%. [3]



2.2 A resistor's value is found from $R = V/I$, with $V = 5.2 \text{ V} \pm 0.1 \text{ V}$ and $I = 0.64 \text{ A} \pm 0.02 \text{ A}$. Calculate R and its percentage uncertainty. [3]



2.3 The external and internal diameters of a pipe are $26.2 \text{ mm} \pm 0.1 \text{ mm}$ and $24.8 \text{ mm} \pm 0.1 \text{ mm}$. Determine the wall thickness and its absolute uncertainty. [2]

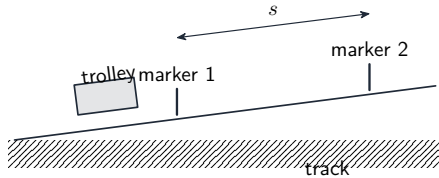


3 Exam-style questions

3.1 A trolley accelerates uniformly from rest through a distance s in a time t , with $s = \frac{1}{2}at^2$.

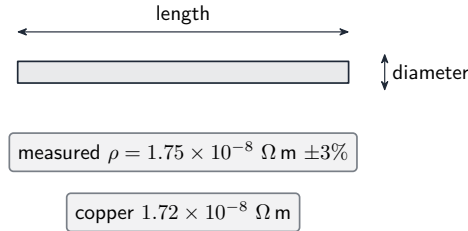
The student records $s = 0.900 \text{ m} \pm 0.002 \text{ m}$ and $t = 1.40 \text{ s} \pm 0.05 \text{ s}$.

- (a) Determine the acceleration a . [2]
- (b) Show that the percentage uncertainty in a is about 8%. [3]
- (c) The track is inclined at 5.5° to the horizontal, so the predicted acceleration is $g \sin 5.5^\circ = 0.94 \text{ m s}^{-2}$. Deduce whether the student's value is accurate. [2]



3.2 A student measures the resistivity of a wire as $\rho = 1.75 \times 10^{-8} \Omega \text{ m}$ with a percentage uncertainty of 3%. The data-book value for copper is $1.72 \times 10^{-8} \Omega \text{ m}$.

Determine the interval for the student's value and deduce whether the wire could be copper. [3]



3.3 A radiation monitor reads a gross rate of 480 min^{-1} with the source in place and a background of 45 min^{-1} with the source removed. Each rate is counted for the same time, giving each reading an absolute uncertainty of 10 min^{-1} .

- (a) Calculate the corrected count rate and its absolute uncertainty. [3]
- (b) Determine the percentage uncertainty in the corrected rate. [2]
- (c) The half-life of the source is later determined with a percentage uncertainty of 5%, dominated by the count rates. Explain which measurement most limits this result and one change that would improve it. [3]



Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 Height percentage: $\%U(h) = \frac{2}{115} \times 100\% = 1.7\%$. Volume rule: $\%U(V) = 2 \times \%U(r) + \%U(h)$. $\%U(V) = 2 \times 1\% + 1.7\% = 3.7\% \approx 4\%$.

2.2 $R = V/I = 5.2/0.64 = 8.125 \approx 8.1 \Omega$. $\%U(R) = \frac{0.1}{5.2} \times 100\% + \frac{0.02}{0.64} \times 100\% = 1.9\% + 3.1\% = 5.0\%$.

2.3 Thickness $= (26.2 - 24.8)/2 = 0.7$ mm. Diameters are subtracted then halved: absolute uncertainties add, $\pm(0.1 + 0.1) = \pm 0.2$ mm on the diameter difference, so the thickness is $0.7 \text{ mm} \pm 0.1 \text{ mm}$. (Halving halves the absolute uncertainty; the percentage is unchanged.)

3.1 (a) Rearranged: $a = 2s/t^2$. $a = 2 \times 0.900/1.40^2 = 1.80/1.96 = 0.918 \text{ m s}^{-2}$. (b) $\%U(s) = \frac{0.002}{0.900} \times 100\% = 0.22\%$ and $\%U(t) = \frac{0.05}{1.40} \times 100\% = 3.6\%$. a divides by t^2 , so $\%U(a) = 0.22\% + 2 \times 3.6\% = 7.4\%$, about 8%. (c) Interval: $0.918 \times (1 \pm 0.074) = 0.85$ to 0.99 m s^{-2} . The predicted 0.94 m s^{-2} lies inside the interval, so the measurement is accurate to the precision claimed.

3.2 Absolute uncertainty: $0.03 \times 1.75 \times 10^{-8} = 0.0525 \times 10^{-8} \Omega \text{ m}$. Interval: 1.70 to $1.80 \times 10^{-8} \Omega \text{ m}$. The copper value $1.72 \times 10^{-8} \Omega \text{ m}$ lies inside, so the wire could be copper; the data does not distinguish copper from any other material whose value also falls inside the interval.

3.3 (a) Corrected rate $R = 480 - 45 = 435 \text{ min}^{-1}$. Subtracted readings add absolute uncertainties: $\Delta R = 10 + 10 = 20 \text{ min}^{-1}$, so $R = 435 \pm 20 \text{ min}^{-1}$. (b) $\%U(R) = \frac{20}{435} \times 100\% = 4.6\%$. (c) The corrected rate's 4.6% is the dominant contribution to the stated 5%. Count for a longer time at each distance: more counts means a smaller statistical uncertainty in each rate, which shrinks the absolute uncertainty of both the gross and background readings.