5.9 Doppler shifts, Hubble law and cosmology

Name: _____ Class: _____ Date: _____ **Total: 28 marks**

KEY WORDS Doppler effect • Redshift • Hubble's law • Hubble constant • Hubble time • Dark matter

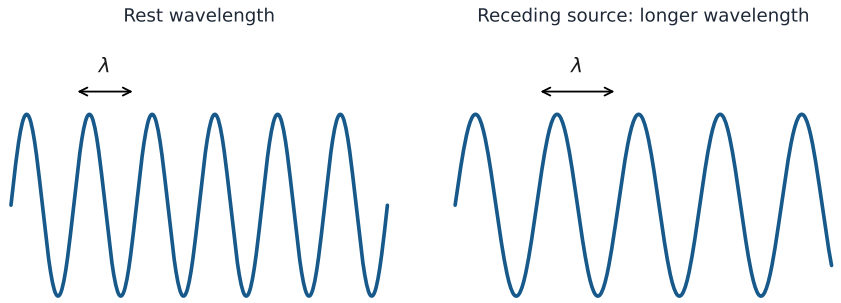
Objective

Develop doppler shifts, hubble law and cosmology from the WPH15 specification and mapped 2025 exam demands. These are original training questions. Use the constants stated in each task.

1 Key knowledge and worked examples

■ Compare the same spectral line

The **Doppler effect** is a change in observed frequency/wavelength caused by relative motion along the line of sight. Successive wavefronts arrive farther apart from a receding source and closer together from an approaching source. Recession gives longer wavelengths and lower frequencies; approach gives shorter wavelengths and higher frequencies. Motion purely perpendicular to the line of sight is not the recession speed in the simple model.



For electromagnetic radiation at low recession speeds:

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} \approx \frac{v}{c_0}$$

Redshift z is positive for recession. For small shifts, its magnitude also approximately equals the fractional decrease of observed frequency relative to emitted frequency. State the sign convention: $(f_{\text{observed}} - f_{\text{rest}})/f_{\text{rest}}$ is negative for recession. Do not equate a positive redshift to a signed frequency increase. The approximation v/c_0 is not a general high-speed relativistic formula.

Worked example. A line emitted at 500 nm is observed at 505 nm.

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = \frac{(505 - 500)\ \text{nm}}{500\ \text{nm}} = 0.010$$

$$v \approx z c_0 = 0.010 \times 3.00 \times 10^8\ \text{m s}^{-1} = 3.00 \times 10^6\ \text{m s}^{-1}$$

To use a star's absorption lines, recall Unit 2: photons are absorbed only when their energies match allowed atomic-level differences. Compare the same identified spectral line, not two different elements.

For opposite limbs of a rotating star, one approaches while the other recedes. If their shifts are equal and opposite, the separation of the two observed wavelengths is twice the shift of either limb. Halve that separation before finding the equatorial speed. Then use $T = 2\pi R/v$. This assumes the observed line-of-sight limb speed represents the equatorial rotation speed under the stated viewing geometry.

■ Hubble law and its limits

On large cosmological scales, recession speed approximately follows **Hubble's law** :

$$v = H_0 d$$

The **Hubble constant** H_0 has units of inverse time. In $\text{km s}^{-1}\,\text{Mpc}^{-1}$, convert kilometres to metres and megaparsecs to metres before taking its reciprocal in seconds. Nearby objects may have local motions that do not follow the large-scale relationship.

Worked example. Use $H_0 = 70\ \text{km s}^{-1}\,\text{Mpc}^{-1}$ and $1\ \text{Mpc} = 3.1 \times 10^{22}$ m to estimate an expansion timescale.

$$H_0 = \frac{70 \times 10^3\ \text{m s}^{-1}}{3.1 \times 10^{22}\ \text{m}} = 2.26 \times 10^{-18}\ \text{s}^{-1}$$

$$t_H = \frac{1}{H_0} = \frac{1}{2.26 \times 10^{-18}\ \text{s}^{-1}} = 4.43 \times 10^{17}\ \text{s}$$

The reciprocal is the **Hubble time** . Interpreting it as an age assumes a model for the past expansion rate; it is not an exact model-independent age. A larger H_0 gives a smaller reciprocal timescale. Evidence for expansion supports an

earlier denser, hotter universe; it does not describe an explosion into an already empty centre-surrounding space.

Dark matter is inferred from gravitational effects that visible matter alone does not explain, such as galaxy rotation and motion. It need not emit detectable light. The amount of gravitating matter affects how expansion changes, so uncertainty in matter content and the expansion model affects predictions of the universe's fate. Do not infer that one measurement of H_0 alone proves perpetual expansion or future collapse. Modern models also consider dark energy; outcome 171 specifically requires understanding the controversy concerning H_0, dark matter, age and fate, rather than memorising one unqualified prediction.

2 Practice

2.1 A spectral line has rest wavelength 600 nm and observed wavelength 606 nm. Calculate redshift and recession speed using $c_0 = 3.00 \times 10^8\ \mathrm{m\,s^{-1}}$. State the sign of the observed frequency change. [4]

rest 600 nm | observed 606 nm

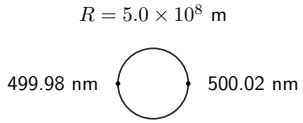
2.2 Convert $H_0 = 65\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ to $\mathrm{s^{-1}}$, using 1 Mpc $= 3.1 \times 10^{22}$ m. Find the reciprocal timescale in seconds and years, using 1 year $= 3.15 \times 10^7$ s. [4]

$H_0 = 65$ km/s/Mpc | 1 Mpc $= 3.1 \times 10^{22}$ m

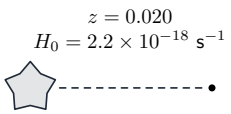
2.3 Explain how relative motion produces a Doppler shift. Distinguish a measured wavelength increase from identification of a different atomic line. [3]

3 Exam-style questions

3.1 Opposite equatorial limbs of a rotating star show wavelengths 499.98 and 500.02 nm for the same line. Assume equal opposite line-of-sight velocities and the equator viewed edge-on. Star radius is 5.0×10^8 m. Find limb speed and rotation period using c from 2.1. Explain why using the full line separation overestimates speed. [6]



3.2 A galaxy has redshift 0.020. Estimate its distance using c from 2.1 and $H_0 = 2.2 \times 10^{-18}$ s^{-1}. Explain two limitations of treating Hubble's law and $1/H_0$ as exact distance and age formulas in every situation. [5]



3.3 Explain how observed stellar/galactic motions can provide evidence for matter not seen in luminous objects. Explain why uncertainty about cosmic mass/energy content prevents a simple certain prediction of the universe's fate from present expansion alone. Distinguish dark matter from dark energy. [6]

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 Compare the same line with its rest wavelength.

$$z = \frac{\lambda_o - \lambda_r}{\lambda_r} = \frac{(606 - 600)\ \text{nm}}{600\ \text{nm}} = 0.010$$

Redshift.

$$v \approx zc_0 = 0.010(3.00 \times 10^8\ \text{m s}^{-1}) = 3.00 \times 10^6\ \text{m s}^{-1}$$

Speed. Observed frequency decreases.

2.2 Convert km to m and Mpc to m.

$$H_0 = \frac{65 \times 10^3\ \text{m s}^{-1}}{3.1 \times 10^{22}\ \text{m}} = 2.10 \times 10^{-18}\ \text{s}^{-1}$$

Rate.

$$t_H = \frac{1}{H_0} = \frac{1}{2.0968 \times 10^{-18}\ \text{s}^{-1}} = 4.77 \times 10^{17}\ \text{s}$$

Seconds.

$$t_{H,years} = \frac{t_H}{3.15 \times 10^7\ \text{s/year}} = \frac{4.7692 \times 10^{17}\ \text{s}}{3.15 \times 10^7\ \text{s/year}} = 1.51 \times 10^{10}\ \text{years}$$

Years.

2.3 Motion changes spacing/arrival interval of successive wavefronts along the line of sight. Recession stretches wavelength and reduces frequency; approach does the reverse. Identify the same atomic transition before comparing observed and rest wavelengths; different lines already have different rest wavelengths.

3.1 Rest wavelength is the midpoint 500.00nm under the given symmetry. Each limb shift is half the full separation.

$$\Delta\lambda = \frac{\lambda_+ - \lambda_-}{2} = \frac{(500.02 - 499.98)\ \text{nm}}{2} = 0.020\ \text{nm}$$

$$v \approx \frac{\Delta\lambda}{\lambda_0}c_0 = \frac{0.020\ \text{nm}}{500.00\ \text{nm}}(3.00 \times 10^8\ \text{m s}^{-1}) = 1.20 \times 10^4\ \text{m s}^{-1}$$

Speed. Use circumference per period.

$$T = \frac{2\pi R}{v} = \frac{2\pi(5.0 \times 10^8\ \text{m})}{1.20 \times 10^4\ \text{m s}^{-1}} = 2.62 \times 10^5\ \text{s}$$

Period. Full separation includes one red and one blue shift, so it doubles either limb's speed.

3.2 Use the small-redshift approximation.

$$v \approx zc_0 = 0.020(3.00 \times 10^8\ \text{m s}^{-1}) = 6.00 \times 10^6\ \text{m s}^{-1}$$

$$d = \frac{v}{H_0} = \frac{6.00 \times 10^6\ \text{m s}^{-1}}{2.2 \times 10^{-18}\ \text{s}^{-1}} = 2.73 \times 10^{24}\ \text{m}$$

Distance method/value. Nearby peculiar motion can dominate recession, and high redshift needs a fuller cosmological relation. Reciprocal present expansion rate is an age scale; actual age depends on expansion history rather than assuming constant-rate extrapolation.

3.3 Infer gravitational mass needed for observed orbital/rotation speeds. Motions can require more mass than luminous material accounts for, supporting a non-luminous gravitating component called dark matter. Dark energy describes the component/effect associated with accelerated cosmic expansion, not unseen gravitating mass alone. Future expansion depends on cosmic matter/energy content and how it evolves; current expansion plus uncertain content does not by itself establish certain recollapse or one exact future history.