5.8 Stellar radiation, distances and evolution

Name: _____ Class: _____ Date: _____ **Total: 30 marks**

KEY WORDS black body • Luminosity • Intensity • Trigonometric parallax • parsec • standard candle • Hertzsprung–Russell diagram • main sequence • Red giants • White dwarfs

Objective

Develop stellar radiation, distances and evolution from the WPH15 specification and mapped 2025 exam demands. These are original training questions. Use the constants stated in each task.

1 Key knowledge and worked examples

■ Temperature, luminosity and received intensity

A **black body** absorbs all incident electromagnetic radiation and is an ideal thermal emitter. Its continuous spectrum has a characteristic shape. Increasing temperature increases total emitted power per unit area and moves the wavelength peak to shorter wavelength. The area under a spectral-intensity curve represents total intensity only with the appropriate spectral-axis definition.

Luminosity $L_\star$ is total power emitted. **Intensity** I received at distance d is power per unit receiving area. For a spherical black-body star of radius R radiating equally in all directions:

$$L_\star = \sigma(4\pi R^2)T^4 \qquad \lambda_{\max}T = 2.898 \times 10^{-3}\ \mathrm{m\,K} \qquad I = \frac{L_\star}{4\pi d^2}$$

The first area is the emitting surface; the second is the expanding receiving sphere. Do not substitute observer distance into the star's surface area. Real emitters may not be perfect black bodies, and intervening material can absorb radiation.

Wien's stated constant describes a spectrum per unit wavelength. A maximum of a spectrum plotted per unit frequency does not transform into that wavelength maximum simply by replacing λ with c_0/f. If an exam gives the frequency corresponding to its specified wavelength peak, convert that given wavelength using $c_0 = f\lambda$. Do not turn that particular task convention into a general rule for every spectral plot.

Worked example. A star's wavelength spectrum peaks at 580 nm and its radius is 7.0×10^8 m. Find temperature, luminosity and intensity at 1.5×10^{11} m.

$$T = \frac{2.898 \times 10^{-3}\ \text{m K}}{\lambda_{\text{max}}} = \frac{2.898 \times 10^{-3}\ \text{m K}}{580 \times 10^{-9}\ \text{m}} = 5.00 \times 10^3\ \text{K}$$

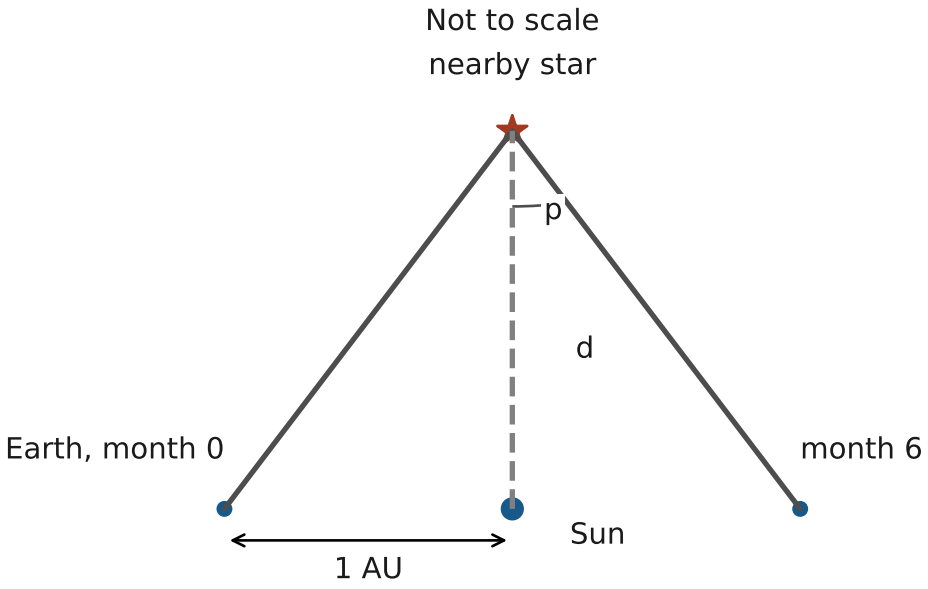
$$L_\star = 4\pi R^2 \sigma T^4 = 4\pi(7.0{\times}10^8\ \text{m})^2(5.67{\times}10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4})(4.997{\times}10^3\ \text{K})^4 = 2.18{\times}10^{26}\ \text{W}$$

$$I = \frac{L_\star}{4\pi d^2} = \frac{2.18 \times 10^{26}\ \text{W}}{4\pi(1.5 \times 10^{11}\ \text{m})^2} = 7.70 \times 10^2\ \text{W m}^{-2}$$

Keep unrounded temperature because it is raised to the fourth power. Compare the received intensity with the stated reference before accepting a planetary-temperature or power claim.

Two distance methods

Trigonometric parallax measures a nearby star's apparent angular shift against distant background stars as Earth moves around the Sun. Observations six months apart use a baseline of two Earth-orbit radii. The parallax angle p is half the total shift. For small p in radians, distance is $d \approx r_{\text{orbit}}/p$. A **parsec** is the distance giving parallax 1 arcsecond; $d_{\text{pc}} = 1/p_{\text{arcsec}}$. Do not use the diameter of Earth or Sun as the orbital baseline.



At large distance, p becomes too small for the instrument to measure accurately. The limitation is angular resolution/relative uncertainty, not necessarily that the star is invisible.

A **standard candle** is an object of known luminosity. Identify one in a cluster, measure received intensity and use $d = \sqrt{L_\star/(4\pi I)}$. Its apparent brightness is

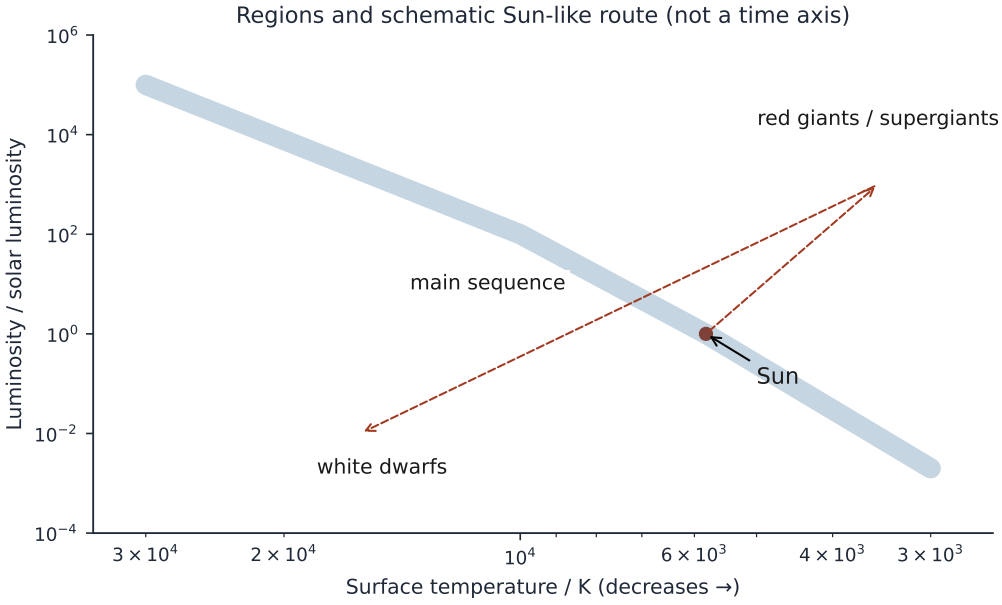
not assumed known beforehand. Cepheid variable stars have a calibrated relationship between period and luminosity. Measure several complete light-curve periods, divide by their number, read the calibration carefully and convert any solar-luminosity units. A straight line on log axes is not automatically a linear relationship between the raw quantities.

Worked example. A standard candle has $L_\star = 4.0 \times 10^{28}$ W and received intensity 2.0×10^{-10} W m^{-2}.

$$d = \sqrt{\frac{L_\star}{4\pi I}} = \sqrt{\frac{4.0 \times 10^{28}\ \text{W}}{4\pi(2.0 \times 10^{-10}\ \text{W m}^{-2})}} = 3.99 \times 10^{18}\ \text{m}$$

Read the Hertzsprung–Russell diagram

A **Hertzsprung–Russell diagram** plots luminosity vertically against surface temperature horizontally, usually with temperature decreasing to the right and logarithmic scales. The **main sequence** runs from hot, bright stars at upper left to cool, faint stars at lower right. **Red giants** are cool but luminous because of their large surface areas. **White dwarfs** are hot but faint because of their small surface areas. Mark the Sun near one solar luminosity and about 5800 K on the main sequence.



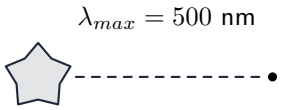
Stars form when gravity contracts a cloud of gas and dust. The core heats until hydrogen fusion can sustain a main-sequence star; outward pressure balances gravity. In a Sun-like star, core hydrogen eventually runs low, fusion there decreases and gravity contracts the core. Core temperature rises; shell hydrogen burning and later helium fusion are associated with expansion to a red giant. Outer layers are lost, leaving a white-dwarf core with no sustained fusion. It cools over a very long time. The Sun does not become a supernova or a neutron star.

A much more massive star can fuse heavier elements through later stages. When

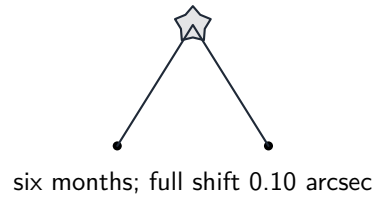
its core can no longer gain energy from fusion, collapse and a supernova can leave a neutron star or black hole, depending on the remnant. A massive main-sequence star has more fuel but consumes it much faster because of its hotter core. Its main-sequence lifetime can therefore be shorter. Explain fusion rate, not just fuel amount. An HR track shows changing temperature/luminosity, not a star travelling across space.

2 Practice

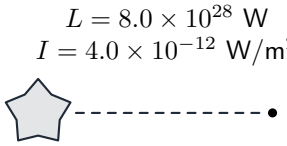
2.1 A star's wavelength spectrum peaks at 500 nm. Find surface temperature using $\lambda_{max}T = 2.898 \times 10^{-3}$ m K. Explain the effect of higher temperature on peak wavelength and emitted power per unit area. [4]



2.2 A nearby star shows total angular shift 0.10 arcsecond between observations six months apart. Calculate distance in parsecs and explain why half the total shift is used. [3]



2.3 A standard candle has luminosity 8.0×10^{28} W and received intensity 4.0×10^{-12} W m^{-2}. Find distance and state one limitation of the isotropic, transparent-space model. [4]



3 Exam-style questions

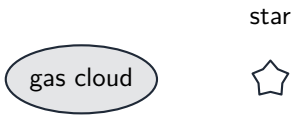
3.1 A spherical black-body star has radius 6.0×10^8 m, surface temperature 6000 K and an orbiting planet at 2.0×10^{11} m. Calculate stellar luminosity and intensity at the planet using $\sigma = 5.67 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$. Explain why the two areas differ. [6]

$R = 6.0 \times 10^8$ m; $T = 6000$ K
$d = 2.0 \times 10^{11}$ m

3.2 On an HR diagram, temperature decreases to the right. Describe where to place the Sun, a red giant and a white dwarf. Explain how a red giant can be cooler but brighter than the Sun. Distinguish an evolutionary track from spatial motion. [5]

Sun | red giant | white dwarf

3.3 Explain a Sun-like star's development from a gas cloud through main sequence and red giant to its remnant. Contrast a very massive star's late outcome and explain why more initial fuel need not mean a longer main-sequence lifetime. [8]



Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 Use Wien's wavelength law, converting nm to m.

$$T = \frac{2.898 \times 10^{-3}\ \text{m K}}{\lambda_{max}} = \frac{2.898 \times 10^{-3}\ \text{m K}}{500 \times 10^{-9}\ \text{m}} = 5796\ \text{K}$$

Temperature. Higher T moves the wavelength peak shorter and increases total emitted power per area as T^4 for a black body.

2.2 Parallax is half the full six-month shift,0.050arcsecond.

$$d_{pc} = \frac{1}{p_{arcsec}} = \frac{1}{0.050} = 20\ \text{pc}$$

Distance. The full baseline is two Earth-orbit radii, while parallax uses one radius.

2.3 Rearrange the inverse-square intensity relation.

$$d = \sqrt{\frac{L}{4\pi I}} = \sqrt{\frac{8.0 \times 10^{28}\ \text{W}}{4\pi(4.0 \times 10^{-12}\ \text{W m}^{-2})}} = 3.99 \times 10^{19}\ \text{m}$$

Substitution and value. Absorption by dust or anisotropic emission can invalidate the assumed relation.

3.1 Use the emitting surface for luminosity.

$$L = 4\pi R^2 \sigma T^4 = 4\pi(6.0 \times 10^8\ \text{m})^2(5.67 \times 10^{-8}\ \text{W m}^{-2}\,\text{K}^{-4})(6000\ \text{K})^4 = 3.32 \times 10^{26}\ \text{W}$$

Luminosity and unit. Spread that power over a sphere at observer distance.

$$I = \frac{L}{4\pi d^2} = \frac{3.324 \times 10^{26}\ \text{W}}{4\pi(2.0 \times 10^{11}\ \text{m})^2} = 661\ \text{W m}^{-2}$$

Intensity. R is stellar radius; d is star-to-planet distance, describing different physical spheres.

3.2 Sun near5800K, one solar luminosity, on main sequence. Red giant upper right, cool but luminous; white dwarf lower left, hot but faint. Large giant radius/surface area can more than offset lower T^4 in $L = 4\pi R^2 \sigma T^4$. A track records changing temperature and luminosity, not physical travel across space.

3.3 Gravity contracts a gas/dust cloud and heats its core. Hydrogen fusion sustains main sequence, with pressure balancing gravity. Depletion of core hydrogen causes core contraction and heating. Shell hydrogen burning and later helium fusion accompany red-giant expansion. Outer layers are lost, leaving a white dwarf without sustained fusion, which cools. A very massive star can fuse heavier nuclei before core collapse/supernova, leaving a neutron star or black hole rather than the Sun's white-dwarf outcome. Its hotter core consumes fuel much faster, so greater fuel supply need not outweigh its much greater fusion rate.